Solution of Home Work #7: Unsymmetrical Faults

10.11. Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown in Figure 83. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 Ω. The generator data and the reactance of the reactors are tabulated below. A line-to-ground fault occurs on phase a of the common bus bar. Neglect prefault currents and assume generators are operating at their rated voltage. Determine the fault current in phase a.

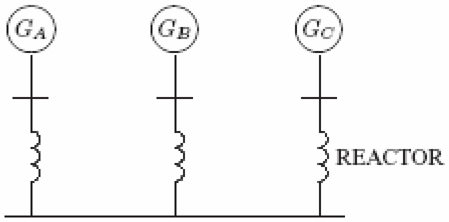


FIGURE 83
Circuit for Problem 10.11.

Item	X^1	X^2	X^0
G_A	0.25 pu	0.155 pu	0.056 pu
G_B	0.20 pu	0.155 pu	0.056 pu
G_C	0.20 pu	0.155 pu	0.060 pu
Reactor	6.0 Ω	6.0 Ω	6.0 Ω

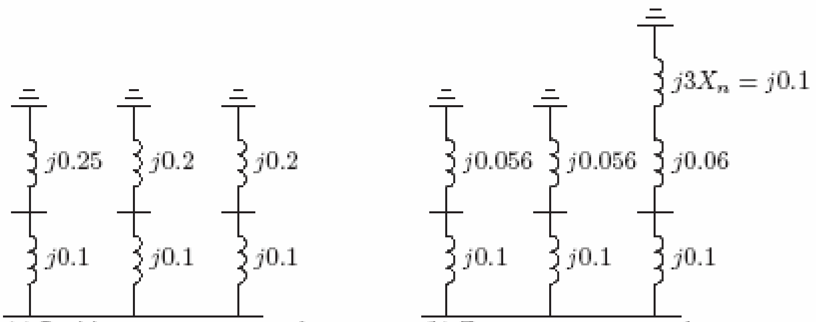
The generator base impedance is

$$Z_B = \frac{(30)^2}{15} = 60 \ \ \Omega$$

The reactor per-unit reactance, and the per-unit generator C neutral reactor are

$$X_R = \frac{6}{60} = 0.1 \ \ \text{pu}$$
$$X_n = \frac{2}{60} = 0.3333 \ \ \text{pu}$$

The positive-sequence impedance network is shown in Figure 84(a), and the zero-sequence impedance network is shown in Figure 84(b).



(a) Positive sequence network (b) Zero sequence network

FIGURE 84
Positive- and zero-sequence impedance networks for Problem 10.11.

The positive-sequence impedance is

$$\frac{1}{X^1} = \frac{1}{0.35} + \frac{1}{0.3} + \frac{1}{0.3} \quad \text{or} \quad X^1 = 0.105$$

The negative-sequence impedance network is the same as the positive-sequence impedance network, except for the value of the generator negative-sequence reactance. Therefore, the negative-sequence impedance is

$$\frac{1}{X^2} = \frac{1}{0.255} + \frac{1}{0.255} + \frac{1}{0.255} \quad \text{or} \quad X^2 = 0.085$$

The zero-sequence impedance is

$$\frac{1}{X^0} = \frac{1}{0.156} + \frac{1}{0.156} + \frac{1}{0.26} \quad \text{or} \quad X^0 = 0.06$$

The line-to-ground fault current in phase a is

$$I_a = 3I_a^0 = \frac{3(1)}{j(0.105 + 0.085 + 0.06)} = 12\angle{-90^\circ} \text{ pu}$$

10.14. The zero-, positive-, and negative-sequence bus impedance matrices for a three-bus power system are

$$\mathbf{Z}_{bus}^0 = j\begin{bmatrix} 0.20 & 0.05 & 0.12 \\ 0.05 & 0.10 & 0.08 \\ 0.12 & 0.08 & 0.30 \end{bmatrix} \quad \text{pu}$$

$$\mathbf{Z}_{bus}^1 = \mathbf{Z}_{bus}^2 = j\begin{bmatrix} 0.16 & 0.10 & 0.15 \\ 0.10 & 0.20 & 0.12 \\ 0.15 & 0.12 & 0.25 \end{bmatrix} \quad \text{pu}$$

Determine the per unit fault current and the bus voltages during fault for
(a) A bolted three-phase fault at bus 2.
(b) A bolted single line-to-ground fault at bus 2.
(c) A bolted line-to-line fault at bus 2.
(d) A bolted double line-to-ground fault at bus 2.

(a) The symmetrical components of fault current for a bolted balanced three-phase fault at bus 2 is given by

$$I_2^{012}(F) = \begin{bmatrix} 0 \\ \frac{1}{Z_{22}^1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{j0.20} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5\angle -90^\circ \\ 5\angle 150^\circ \\ 5\angle 30^\circ \end{bmatrix}$$

For balanced fault we only have the positive-sequence component of voltage. Thus, from (10.98), bus voltages during fault for phase a are

$$\begin{aligned} V_1(F) &= 1 - Z_{12}^1 I_2(F) = 1 - j0.10(-j5) = 0.5 \\ V_2(F) &= 1 - Z_{22}^1 I_2(F) = 1 - j0.20(-j5) = 0.0 \\ V_3(F) &= 1 - Z_{32}^1 I_2(F) = 1 - j0.12(-j5) = 0.4 \end{aligned}$$

(b) From (10.90), the symmetrical components of fault current for a single line-to-ground fault at bus 2 is given by

$$I_2^0(F) = I_2^1(F) = I_2^2(F) = \frac{1.0}{Z_{22}^1 + Z_{22}^2 + Z_{22}^0}$$

$$= \frac{1.0}{j0.20 + j0.20 + j0.10} = -j2$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j2 \\ -j2 \\ -j2 \end{bmatrix} = \begin{bmatrix} 6\angle -90^\circ \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(-j2) \\ 1 - j0.10(-j2) \\ 0 - j0.10(-j2) \end{bmatrix} = \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(-j2) \\ 1 - j0.20(-j2) \\ 0 - j0.20(-j2) \end{bmatrix} = \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(-j2) \\ 1 - j0.12(-j2) \\ 0 - j0.12(-j2) \end{bmatrix} = \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix} = \begin{bmatrix} 0.50\angle 0^\circ \\ 0.9539\angle -114.79^\circ \\ 0.9539\angle +114.79^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix} = \begin{bmatrix} 0.0\angle 0^\circ \\ 0.9165\angle -109.11^\circ \\ 0.9165\angle +109.11^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix} = \begin{bmatrix} 0.36\angle 0^\circ \\ 0.9625\angle -115.87^\circ \\ 0.9625\angle +115.87^\circ \end{bmatrix}$$

(c) From (10.92) and (10.93), the symmetrical components of fault current for line-to-line fault at bus 2 are

$$I_2^0 = 0$$
$$I_2^1 = -I_2^2 = \frac{V_2(0)}{Z_{22}^1 + Z_{22}^2} = \frac{1}{j0.20 + j0.20} = -j2.5$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.5 \\ j2.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.33 \\ 4.33 \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.10(-j2.5) \\ 0 - j0.10(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.20(-j2.50) \\ 0 - j0.20(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.12(-j2.5) \\ 0 - j0.12(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6614\angle -139.11^\circ \\ 0.614\angle +130.11^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.50\angle 180^\circ \\ 0.50\angle +180^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6083\angle -145.285^\circ \\ 0.6083\angle +145.285^\circ \end{bmatrix}$$

(d) From (10.94)–(10.96), the symmetrical components of fault current for a double line-to-ground fault at bus 2 is given by

$$I_2^1 = \frac{V_2(0)}{Z_{22}^1 + \frac{Z_{22}^2(Z_{22}^0)}{Z_{22}^2 + Z_{22}^0}} = -\frac{1}{j0.20 + \frac{j0.20(j0.10)}{j0.20 + j0.10}} = -j3.75$$

$$I_2^2 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^2} - \frac{1 - j0.20(-j3.75)}{j0.20} = j1.25$$

$$I_2^0 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^0} = -\frac{1 - j0.20(-j3.75)}{j0.20} = j2.5$$

The phase currents at the faulted bus are

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j2.5 \\ -j3.75 \\ j1.25 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.7282\angle 139.11^\circ \\ 5.7282\angle 40.89^\circ \end{bmatrix}$$

and the total fault current is

$$I_2^b + I_2^c = 5.7282\angle 139.11^\circ + 5.7282\angle 40.89^\circ = 7.5\angle 90^\circ$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(j2.5) \\ 1 - j0.10(-j3.75) \\ 0 - j0.10(j1.25) \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(j2.5) \\ 1 - j0.20(-j3.75) \\ 0 - j0.20(j1.25) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(j2.5) \\ 1 - j0.12(-j3.75) \\ 0 - j0.12(j1.25) \end{bmatrix} = \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 0.875\angle 0^\circ \\ 0.50\angle -120^\circ \\ 0.50\angle +120^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.75\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.90\angle 0^\circ \\ 0.3775\angle -113.413^\circ \\ 0.3775\angle +113.413^\circ \end{bmatrix}$$

10.15. The reactance data for the power system shown in Figure 85 in per unit on a common base is as follows:

FIGURE 85
The impedance diagram for Problem 10.15.

Obtain the Thévenin sequence impedances for the fault at bus 1 and compute the fault current in per unit for the following faults:

(a) A bolted three-phase fault at bus 1.
(b) A bolted single line-to-ground fault at bus 1.
(c) A bolted line-to-line fault at bus 1.
(d) A bolted double line-to-ground fault at bus 1.

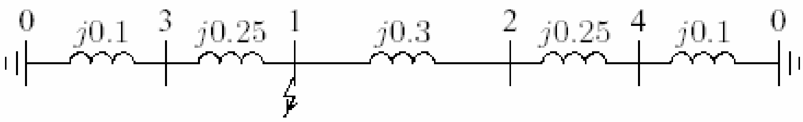


FIGURE 86
Positive-sequence impedance network for Problem 10.15.

The positive-sequence impedance network is shown in Figure 86, and impedance to the point of fault is

$$Z^1 = j\frac{(0.35)(0.65)}{0.35 + 0.65} = j0.2275 \text{ pu}$$

Since negative-sequence reactances are the same as positive-sequence reactances, $X^2 = X^1 = 0.2275$. The zero-sequence impedance network is shown in Figure 87, and impedance to the point of fault is

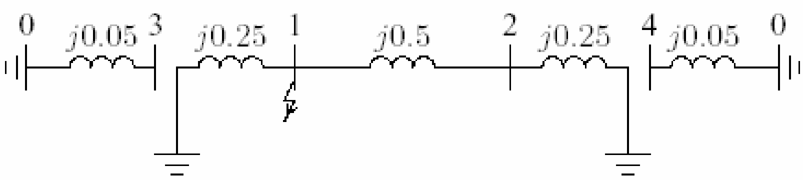


FIGURE 87

zero-sequence impedance network for Problem 10.15.

$$Z^0 = j\frac{(0.25)(0.75)}{0.25 + 0.75} = j0.1875 \text{ pu}$$

(a) For a bolted three-phase fault at bus 1, the fault current is

$$I_f = \frac{1}{j0.2275} = 4.3956\angle{-90^\circ} \text{ pu}$$

(b) For a bolted single-line to ground fault at bus 1, the fault current is

$$I_f = 3I_a^0 = \frac{3}{j(0.2275 + 0.2275 + 0.1875)} = 4.669\angle{-90^\circ} \text{ pu}$$

(c) For a bolted line-to-line fault at bus 1, the fault current in phase b is

$$I_a^1 = \frac{1}{j(0.2275 + 0.2275)} = -j2.1978 \text{ pu}$$
$$I_b(F) = -j\sqrt{3}I_a^1 = -3.8067 \text{ pu}$$

(d) For a bolted double line-to-line fault at bus 1, we have

$$I_a^1 = \frac{1}{j0.2275 + +j\frac{(0.2275)(0.1875)}{0.2275+0.1875}} = -j3.02767 \text{ pu}$$
$$I_a^0 = \frac{1 - (0.2275)(-j3.02767)}{j0.1875} = j1.65975 \text{ pu}$$
$$I(F) = 3I_a^0 = 4.979\angle 90^\circ \text{ pu}$$