HW#5: Symmetrical Three-Phase Fault

8.1. A sinusoidal voltage given by $v(t) = 390\sin(315t + \alpha)$ is suddenly applied to a series *RL* circuit. $R = 32\ \Omega$ and $L = 0.4$ H.

(a) The switch is closed at such a time as to permit no transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.

(b) The switch is closed at such a time as to permit maximum transient current. What value of α corresponds to this instant of closing the switch? Obtain the instantaneous expression for $i(t)$. Use *MATLAB* to plot $i(t)$ up to 80 ms in steps of 0.01 ms.

(c) What is the maximum value of current in part (b) and at what time does this occur after the switch is closed?

$$v(t) = 390\sin(315t + \alpha)$$
$$i(t) = I_m \sin(315t + \alpha - \gamma) - I_m e^{-t/\tau}\sin(\alpha - \gamma)$$

(a) For no transient $\alpha = \gamma$

$$\gamma = \tan^{-1}\frac{(315)(0.4)}{32} = 75.75^\circ \quad \Rightarrow \quad \alpha = 75.75^\circ$$
$$Z = 32 + j(315)(0.4) = 32 + j126 = 130\angle 75.75^\circ \ \ \Omega$$
$$I = \frac{390}{130} = 3 \ \ \text{A} \quad \Rightarrow \quad i(t) = 3\sin 315t$$

(b) For maximum transient current $\alpha - \gamma = -90^\circ$. Therefore, $\alpha = 75.75 - 90 = -14.25^\circ$, and $\tau = \frac{L}{R} = 0.0125$ sec, and the current is

$$i(t) = 3\sin(315t - \frac{\pi}{2}) + 3e^{-80t}$$

(c)

$$\frac{di(t)}{dt} = (3)(315)\cos(315t - \frac{\pi}{2}) - 240e^{-80t} = 0$$

Use the command `[Imax, k] = max(i), tmax = t(k)` to find the maximum value of current, and the corresponding time.

$$i_{max} = 4.371 \ \ \text{A}$$
$$t_{max} = 0.0096 \ \ \text{sec}$$

9.2. The system shown in Figure 67 shows an existing plant consisting of a generator of 100 MVA, 30 kV, with 20 percent subtransient reactance and a generator of 50 MVA, 30 kV with 15 percent subtransient reactance, connected in parallel to a 30-kV bus bar. The 30-kV bus bar feeds a transmission line via the circuit breaker

C which is rated at 1250 MVA. A grid supply is connected to the station bus bar through a 500-MVA, 400/30-kV transformer with 20 percent reactance. Determine the reactance of a current limiting reactor in ohm to be connected between the grid system and the existing bus bar such that the short-circuit MVA of the breaker C does not exceed.

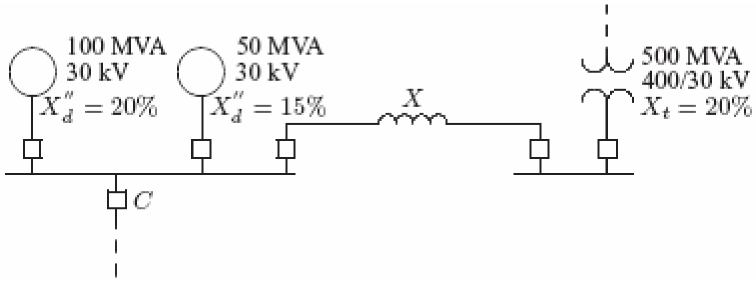


FIGURE 67
One-line diagram for Problem 9.2.

The base impedance for line is

$$Z_B = \frac{(30)^2}{100} = 9 \ \ \Omega$$

The reactances on a common 100 MVA base are

$$X''_{dg1} = \frac{100}{100}(0.2) = 0.2 \ \text{pu}$$
$$X''_{dg2} = \frac{100}{50}(0.15) = 0.3 \ \text{pu}$$
$$X_t = \frac{100}{500}(0.2) = 0.04 \ \text{pu}$$

The impedance diagram is as shown in Figure 68.
Reactance to the point of fault is

$$X_f = \frac{S_B}{\text{SCMVA}} = \frac{100}{1250} = 0.08 \ \text{pu}$$

Parallel reactance of the generators is

$$X_{||} = \frac{(0.2)(0.3)}{0.2 + 0.3} = 0.12 \ \text{pu}$$

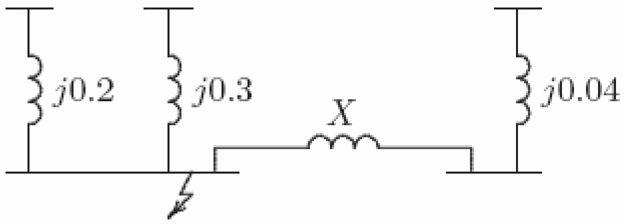


FIGURE 68
The impedance diagram for Problem 9.2.

From Figure 68, reactance to the point of fault is

$$\frac{(0.12)(X+0.04)}{0.12+(X+0.04)} = 0.08$$

Solving for X, we get $X = 0.2$ pu., or

$$X_\Omega = (X)(Z_B) = (0.2)(9) = 1.8\ \ \Omega$$

9.3. The one-line diagram of a simple power system is shown in Figure 69. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common MVA base. All resistances and shunt capacitances are neglected. The generators are operating on no load at their rated voltage with their emfs in phase. A three-phase fault occurs at bus 1 through a fault impedance of $Z_f = j0.08$ per unit.
(a) Using Thévenin's theorem obtain the impedance to the point of fault and the fault current in per unit.
(b) Determine the bus voltages and line currents during fault.

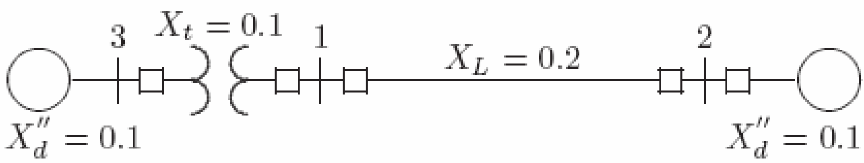


FIGURE 69
One-line diagram for Problem 9.3.

The impedance diagram is as shown in Figure 70.
(a) Impedance to the point of fault is

$$X = j\frac{(0.2)(0.3)}{0.2+0.3} = j0.12\ \ \text{pu}$$

The fault current is

$$I_f = \frac{1}{j0.12 + j0.08} = 5\angle{-90^\circ}\ \ \text{pu}$$

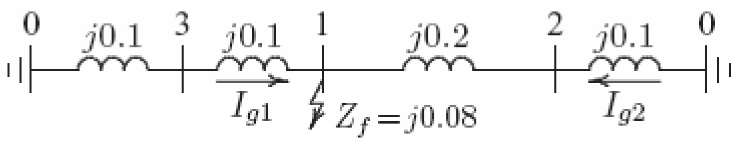


FIGURE 70
The impedance diagram for Problem 9.3.

(b)

$$
\begin{aligned}
V_1 &= (j0.08)(-j5) = 0.4 \text{ pu}\\
I_{g1} &= \frac{j0.3}{j0.5}(5)\angle{-90^\circ} = 3\angle{-90^\circ} \text{ pu}\\
I_{g2} &= \frac{j0.2}{j0.5}(5)\angle{-90^\circ} = 2\angle{-90^\circ} \text{ pu}\\
V_2 &= 0.4 + (j0.2)(-j2) = 0.8 \text{ pu}\\
V_3 &= 0.4 + (j0.1)(-j3) = 0.7 \text{ pu}
\end{aligned}
$$

9.8. Obtain the bus impedance matrix for the network of Problem 9.4.

Add branch 1, $z_{10} = j0.05$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.05$$

Next, add branch 2, $z_{20} = j0.075$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 \\ 0 & j0.075 \end{bmatrix}$$

Add branch 3, $z_{13} = j0.3$ between the new node $q = 3$ and the existing node $p = 1$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{11} \\ Z_{21} & Z_{22} & Z_{21} \\ Z_{11} & Z_{12} & Z_{11} + z_{13} \end{bmatrix} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix}$$

Add link 4, $z_{12} = j0.75$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$
\begin{aligned}
\mathbf{Z}_{bus}^{(4)} &= \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}\\
&= \begin{bmatrix} j0.05 & 0 & j0.05 & -j0.05 \\ 0 & j0.075 & 0 & j0.075 \\ j0.05 & 0 & j0.35 & -j0.05 \\ -j0.05 & j0.075 & -j0.05 & Z_{44} \end{bmatrix}
\end{aligned}
$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = j0.75 + j0.05 + j0.075 - 2(j0) = j0.875$$

and

$$\frac{\mathbf{\Delta Z\, \Delta Z}^T}{Z_{44}} = \frac{1}{j0.875}\begin{bmatrix} -j0.05 \\ j0.075 \\ -j0.05 \end{bmatrix}\begin{bmatrix} -j0.05 & j0.075 & -j0.05 \end{bmatrix}$$
$$= \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.05 & 0 & j0.05 \\ 0 & j0.075 & 0 \\ j0.05 & 0 & j0.35 \end{bmatrix} - \begin{bmatrix} j0.002857 & -j0.004286 & j0.002857 \\ -j0.004286 & j0.006428 & -j0.004286 \\ j0.002857 & -j0.004286 & j0.002857 \end{bmatrix}$$
$$= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix}$$

Add link 5, $z_{23} = j0.45$ between node $q = 3$ and node $p = 2$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(5)} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{13} - Z_{12} \\ Z_{21} & Z_{22} & Z_{23} & Z_{23} - Z_{22} \\ Z_{31} & Z_{32} & Z_{33} & Z_{33} - Z_{32} \\ Z_{31} - Z_{21} & Z_{32} - Z_{22} & Z_{33} - Z_{23} & Z_{44} \end{bmatrix}$$
$$= \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 & j0.042857 \\ j0.004286 & j0.068571 & j0.004286 & -j0.064286 \\ j0.047143 & j0.004286 & j0.347142 & j0.342857 \\ j0.042857 & -j0.064286 & j0.342857 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{23} + Z_{22} + Z_{33} - 2Z_{23} = j0.45 + j0.068571 + j0.347142 - 2(j0.004286) = j0.85714$$

and

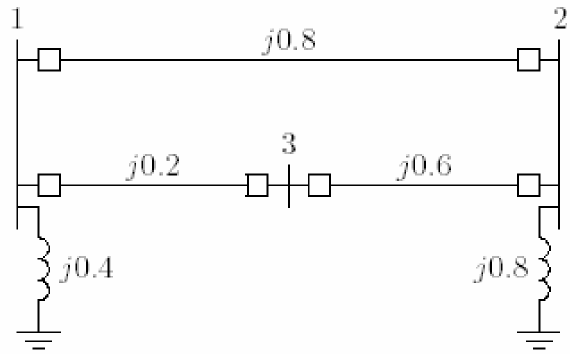
$$\frac{\mathbf{\Delta Z\, \Delta Z}^T}{Z_{44}} = \frac{1}{j0.85714}\begin{bmatrix} j0.042857 \\ -j0.064286 \\ j0.342857 \end{bmatrix}\begin{bmatrix} j0.042857 & -j0.064286 & j0.342857 \end{bmatrix}$$
$$= \begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137143 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.047143 & j0.004286 & j0.047143 \\ j0.004286 & j0.068571 & j0.004286 \\ j0.047143 & j0.004286 & j0.347142 \end{bmatrix} -$$
$$\begin{bmatrix} j0.002143 & -j0.003214 & j0.017143 \\ -j0.003214 & j0.004821 & -j0.025714 \\ j0.017143 & -j0.025714 & j0.137142 \end{bmatrix} = \begin{bmatrix} j0.0450 & j0.00750 & j0.030 \\ j0.0075 & j0.06375 & j0.030 \\ j0.0300 & j0.03000 & j0.210 \end{bmatrix}$$

9.9. The bus impedance matrix for the network shown in Figure 77 is given by

$$Z_{bus} = j\begin{bmatrix} 0.300 & 0.200 & 0.275 \\ 0.200 & 0.400 & 0.250 \\ 0.275 & 0.250 & 0.41875 \end{bmatrix}$$



There is a line outage and the line from bus 1 to 2 is removed. Using the method of building algorithm determine the new bus impedance matrix.

The line between buses 1 and 2 with impedance $Z_{12} = j0.8$ is removed. The removal of this line is equivalent to connecting a link having an impedance equal to the negated value of the original impedance. Therefore, we add link $z_{12} = -j0.8$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{12} - Z_{11} \\ Z_{21} & Z_{22} & Z_{23} & Z_{22} - Z_{21} \\ Z_{31} & Z_{32} & Z_{33} & Z_{32} - Z_{31} \\ Z_{21} - Z_{11} & Z_{22} - Z_{12} & Z_{23} - Z_{13} & Z_{44} \end{bmatrix}$$

Thus, we get

$$\mathbf{Z}_{bus}^{(1)} = \begin{bmatrix} j0.300 & j0.200 & j0.27500 & -j0.100 \\ j0.200 & j0.400 & j0.25000 & j0.200 \\ j0.275 & j0.250 & j0.41875 & -j0.025 \\ -j0.100 & j0.200 & -j0.02500 & Z_{44} \end{bmatrix}$$

From (9.58)

$$Z_{44} = z_{12} + Z_{11} + Z_{22} - 2Z_{12} = -j0.8 + j0.3 + j0.4 - 2(j0.2) = -j0.5$$

and

$$\frac{\mathbf{\Delta Z \, \Delta Z}^T}{Z_{44}} = \frac{1}{-j0.5}\begin{bmatrix} -j0.100 \\ j0.200 \\ -j0.025 \end{bmatrix}\begin{bmatrix} -j0.10 & j0.20 & -j0.025 \end{bmatrix}$$

$$= \begin{bmatrix} -j0.020 & j0.040 & -j0.0050 \\ j0.040 & -j0.080 & j0.0100 \\ -j0.005 & j0.010 & -j0.0013 \end{bmatrix}$$

From (9.59), the new bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.300 & j0.200 & j0.27500 \\ j0.200 & j0.400 & j0.25000 \\ j0.275 & j0.250 & j0.41875 \end{bmatrix} - \begin{bmatrix} -j0.020 & j0.040 & -j0.00500 \\ j0.040 & -j0.080 & j0.01000 \\ -j0.005 & j0.010 & -j0.00125 \end{bmatrix}$$

$$= \begin{bmatrix} j0.320 & j0.160 & j0.280 \\ j0.160 & j0.480 & j0.240 \\ j0.280 & j0.240 & j0.420 \end{bmatrix}$$

9.11. The per unit bus impedance matrix for the power system of Problem 9.5 is given by

$$Z_{bus} = j\begin{bmatrix} 0.240 & 0.140 & 0.200 & 0.200 \\ 0.140 & 0.2275 & 0.175 & 0.175 \\ 0.200 & 0.175 & 0.310 & 0.310 \\ 0.200 & 0.1750 & 0.310 & 0.500 \end{bmatrix}$$

(a) A bolted three-phase fault occurs at bus 4. Using the bus impedance matrix calculate the fault current, bus voltages, and line currents during fault.
(b) Repeat (a) for a three-phase fault at bus 2 with a fault impedance of $Z_f = j0.0225$.

From (9.22), for a solid fault at bus 4 the fault current is

$$I_4(F) = \frac{V_4(0)}{Z_{44}} = \frac{1.0}{j0.5} = -j2 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{14}I_4(F) = 1.0 - (j0.200)(-j2) = 0.60 \ \text{pu} \\ V_2(F) &= V_2(0) - Z_{24}I_4(F) = 1.0 - (j0.175)(-j2) = 0.65 \ \text{pu} \\ V_3(F) &= V_3(0) - Z_{34}I_4(F) = 1.0 - (j0.310)(-j2) = 0.38 \ \text{pu} \\ V_4(F) &= V_4(0) - Z_{44}I_4(F) = 1.0 - (j0.500)(-j2) = 0 \ \text{pu} \end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{21}(F) &= \frac{V_2(F) - V_1(F)}{z_{12}} = \frac{0.65 - 0.60}{j0.5} = -j0.1 \ \text{pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.60 - 0.38}{j0.2} = -j1.1 \ \text{pu} \\ I_{23}(F) &= \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.65 - 0.38}{j0.3} = -j0.9 \ \text{pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.38 - 0}{j0.19} = -j2 \ \text{pu} \end{aligned}$$

(c) From (9.22), for a fault at bus 2 with fault impedance $Z_f = j0.0225$ per unit, the fault current is

$$I_2(F) = \frac{V_2(0)}{Z_{22} + Z_f} = \frac{1.0}{j0.2275 + j0.0225} = -j4 \quad \text{pu}$$

From (9.23), bus voltages during the fault are

$$\begin{aligned} V_1(F) &= V_1(0) - Z_{12}I_2(F) = 1.0 - (j0.140)(-j4) = 0.44 \ \text{pu} \\ V_2(F) &= V_2(0) - Z_{22}I_2(F) = 1.0 - (j0.2275)(-j4) = 0.09 \ \text{pu} \\ V_3(F) &= V_3(0) - Z_{32}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \ \text{pu} \\ V_4(F) &= V_4(0) - Z_{42}I_2(F) = 1.0 - (j0.175)(-j4) = 0.30 \ \text{pu} \end{aligned}$$

From (9.25), the short circuit currents in the lines are

$$\begin{aligned} I_{12}(F) &= \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.44 - 0.09}{j0.5} = -j0.7 \ \text{pu} \\ I_{13}(F) &= \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.44 - 0.30}{j0.2} = -j0.7 \ \text{pu} \\ I_{32}(F) &= \frac{V_3(F) - V_2(F)}{z_{23}} = \frac{0.30 - 0.09}{j0.3} = -j0.7 \ \text{pu} \\ I_{34}(F) &= \frac{V_3(F) - V_4(F)}{z_{34}} = \frac{0.30 - 0.30}{j0.19} = 0 \ \text{pu} \end{aligned}$$