KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
ELECTRICAL ENGINEERING DEPARTMENT

EE 306 – Term 192

HW # 5: Synchronous Machines

Key Solutions

Problem # 1:

A 3-phase, 120 MVA, 13.8 kV, 0.8 PF lagging, 60 Hz and Y-connected synchronous generator has synchronous reactance of 1.2 Ω per phase, and its armature resistance is 0.1 Ω per phase.

(a) Determine the voltage regulation at rated conditions,

(b) Determine the voltage and apparant power rating if this generator is operated at 50 Hz with the same armature and field losses at it had at 60 Hz,

Solution:

(a) $$I_A = I_L = \frac{S}{\sqrt{3}\,V_T} = \frac{120\text{ MVA}}{\sqrt{3}(13.8\text{ kV})} = 5020\text{ A}$$

The power factor is 0.8 lagging, so $\mathbf{I}_A = 5020\angle -36.87°\text{ A}$. The phase voltage is 13.8 kV / $\sqrt{3}$ = 7967 V. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 7967\angle 0° + (0.1\,\Omega)(5020\angle -36.87°\text{ A}) + j(1.2\,\Omega)(5020\angle -36.87°\text{ A})$$

$$\mathbf{E}_A = 12{,}800\angle 20.7°\text{ V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{12{,}800 - 7967}{7967} \times 100\% = 60.7\%$$

(b) If the generator is to be operated at 50 Hz with the same armature and field losses as at 60 Hz (so that the windings do not overheat), then its armature and field currents must not change. Since the voltage of the generator is directly proportional to the speed of the generator, the voltage rating (and hence the apparent power rating) of the generator will be reduced by a factor of 5/6.

$$V_{T,\text{rated}} = \frac{5}{6}(13.8 \text{ kV}) = 11.5 \text{ kV}$$

$$S_{\text{rated}} = \frac{5}{6}(120 \text{ MVA}) = 100 \text{ MVA}$$

Also, the synchronous reactance will be reduced by a factor of 5/6.

$$X_S = \frac{5}{6}(1.2\ \Omega) = 1.00\ \Omega$$

Problem # 2:

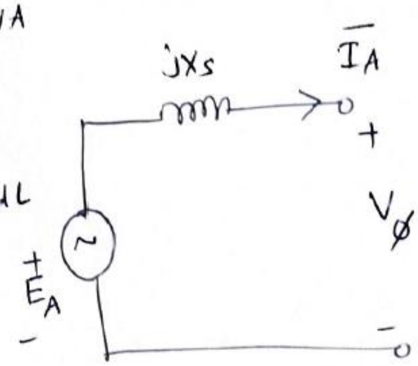
A 3-phase, 40 MVA, 13.8 kV, 60 Hz, Δ-connected alternator requires field current of 250 A to produce an open circuit rated voltage while a field current of 150 A is required to produce short circuit rated armature current. Neglect armature resistance.

Determine the Synchronous Reactance (X_s).

Solution:

$$V_{rated\,L} = V_{rated\,ph} = 13.8\,kV$$

$$S_{rated} = 40\,MVA$$

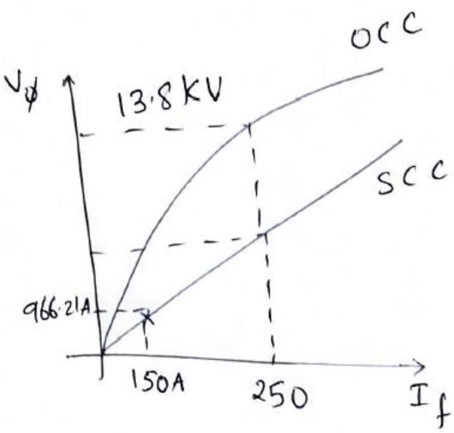


From,

$$S_{rated} = \sqrt{3}\, V_{rated\,L}\, I_{A\,rated\,L}$$

$$\Rightarrow I_{A\,rated} = 1673.53\,A$$

$$I_{A\,rated\,ph} = 966.21\,A$$



$$I_A\Big|_{250A} = 966.21 \times \frac{250}{150}$$

$$= 1610.35\,A.$$

Now,

$$X_S = \frac{V_\phi}{I_{Aph}}\Bigg|_{same\ I_f}$$

$$= \frac{13.8 \times 10^3}{1610.35}$$

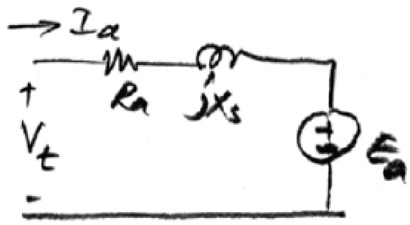
[* one observation: $X_{S\Delta} = 3 X_{SY}$]

$$X_S = 8.569\ \Omega/phase$$

Problem # 3:

A 380 V, 40 hp, 4 pole, 60 Hz, Y-connected synchronous motor has a synchronous reactance of 2.5 Ω per phase and negligible armature resistance. The total rotational losses of the machine are 1.5 kW. If the shaft load is 10-hp, find the **Armature Current, Excitation Voltage and Torque Angle**. The power factor of the motor at this load is 0.8 leading.

Solution:



$$V_t = \frac{380}{\sqrt{3}}\angle 0^\circ = 219.393\angle 0^\circ$$

$$P_{in} = 10 \times 746 + 1500 = 8960\ W$$

$$I_a = \frac{8960}{\sqrt{3}\times 380 \times 0.8}\angle \cos^{-1} 0.8 = 17.0166\angle 36.87^\circ\ A$$

$$E_a = \frac{380}{\sqrt{3}}\angle 0^\circ - j\,2.5\times 17.0166\angle 36.87^\circ = 247.2713\angle -7.9^\circ\ V$$

Torque angle 7.9°

Problem # 4:

A 230 V, 50 Hz, two-pole, synchronous motor draws 40 A from the line at unity power factor and full load. Determine the following assuming that the motor is lossless:

(a) Output torque of the motor,

(b) What should be done to change the power factor to 0.85 leading? explain using phaser diagram.

(c) Magnitude of the line current if the power factor is adjusted to 0.85 leading.

Solution:

(a) If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{IN} = \sqrt{3} V_T I_L \cos\theta = \sqrt{3}(230 \text{ V})(40 \text{ A})(1.0) = 15.93 \text{ kW}$$

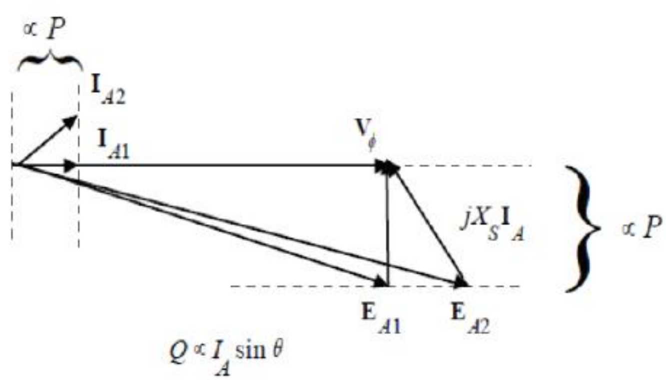
The rotational speed of the motor is

$$n_m = \frac{120 f_{se}}{P} = \frac{120(50 \text{ Hz})}{4} = 1500 \text{ r/min}$$

The output torque would be

$$\tau_{LOAD} = \frac{P_{OUT}}{\omega_m} = \frac{15.93 \text{ kW}}{(1500 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 101.4 \text{ N}\cdot\text{m}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in field current increases $|\mathbf{E}_A|$ while keeping the distance $E_A \sin\delta$ constant. This increase in E_A changes the angle of the current $\mathbf{I}_A$, eventually causing it to reach a power factor of 0.8 leading.



(c) The magnitude of the line current will be

$$I_L = \frac{P}{\sqrt{3}\, V_T \text{ PF}} = \frac{15.93 \text{ kW}}{\sqrt{3}(230 \text{ V})(0.8)} = 50.0 \text{ A}$$