EE-465 (Term 162)

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Solution of HW3

13.1 From the results of Example 13.2:

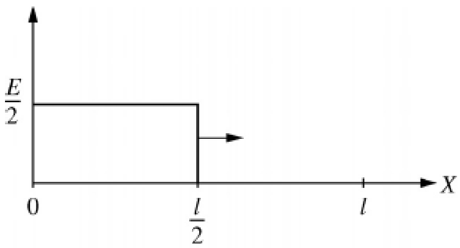
$$V(x,t)=\frac{E}{2}U_{-1}\left(t-\frac{x}{v}\right)+\frac{E}{2}U_{-1}\left(t+\frac{x}{v}-2\tau\right)$$

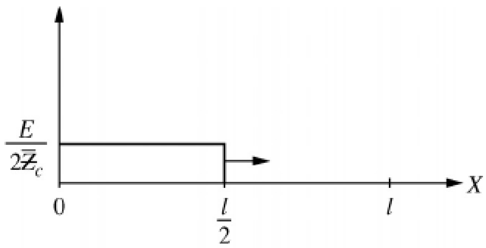
$$i(x,t)=\frac{E}{2Z_c}U_{-1}\left(t-\frac{x}{v}\right)-\frac{E}{2Z_c}U_{-1}\left(t+\frac{x}{v}-2\tau\right)$$

For $t=\tau/2=\frac{l}{2v}$:

$$V\left(x,\frac{\tau}{2}\right)=\frac{E}{2}U_{-1}\left(\frac{\frac{l}{2}-x}{v}\right)+\frac{E}{2}U_{-1}\left(\frac{x-\frac{3}{2}l}{v}\right)$$

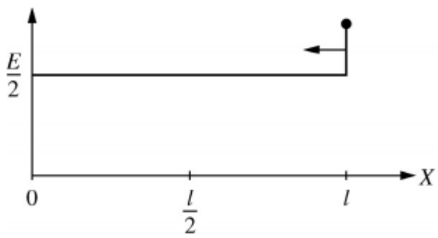
$$i\left(x,\frac{\tau}{2}\right)=\frac{E}{2Z_c}U_{-1}\left(\frac{\frac{l}{2}-x}{v}\right)-\frac{E}{2Z_c}U_{-1}\left(\frac{x-\frac{3}{2}l}{v}\right)$$

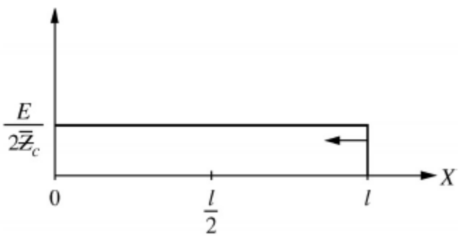




For $t=\tau=\frac{l}{v}$:

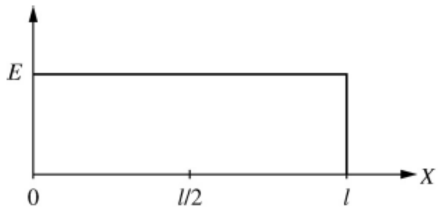
$$V(x,\tau)=\frac{E}{2}U_{-1}\left(\frac{l-x}{v}\right)+\frac{E}{2}U_{-1}\left(\frac{x-l}{v}\right) \quad i(x,\tau)=\frac{E}{2Z_c}U_{-1}\left(\frac{l-x}{v}\right)-\frac{E}{2Z_c}U_{-1}\left(\frac{x-l}{v}\right)$$

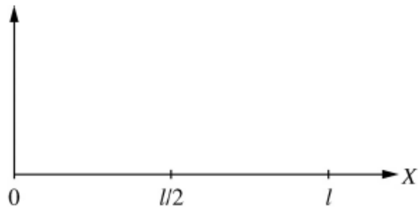




For $t = 2\tau = \dfrac{2l}{w}$:

$$V(x,2\tau) = \frac{E}{2}U_{-1}\left(\frac{2l-x}{v}\right) + \frac{E}{2}U_{-1}\left(\frac{x}{v}\right) \quad i(x,2\tau) = \frac{E}{2Z_c}U_{-1}\left(\frac{2l-x}{v}\right) - \frac{E}{2Z_c}U_{-1}\left(\frac{x}{v}\right)$$





13.5 $\Gamma_R = \dfrac{4-1}{4+1} = 0.6 \qquad \Gamma_S = \dfrac{\frac{1}{3}-1}{\frac{1}{3}+1} = -0.5$

$$E_G(S) = \frac{E}{S}$$

$$V(x,S) = \frac{E}{S}\left[\frac{1}{\frac{1}{3}+1}\right]\frac{\left[e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)}\right]}{1-(0.6)(-0.5)e^{-2S\tau}}$$

$$V(x,S) = \frac{3E}{4S}\left[\frac{e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)}}{1+0.3e^{-2S\tau}}\right]$$

$$V(x,S) = \frac{3E}{4S}\left[e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)}\right]\left[1-0.3e^{-2S\tau} + (0.3)^2 e^{-4S\tau}\ldots\right]$$

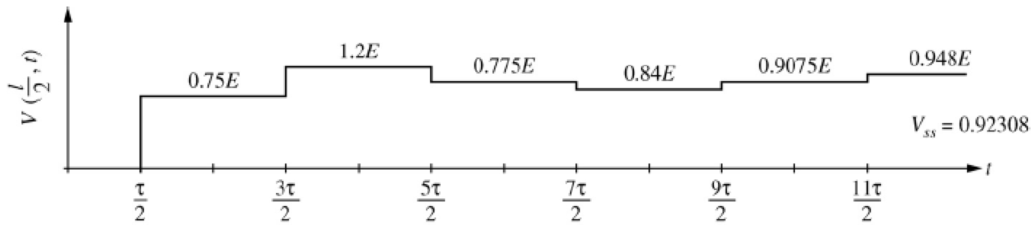
$$V(x,S) = \frac{3E}{4S}\left[e^{-\frac{Sx}{v}} + 0.6e^{S\left(\frac{x}{v}-2\tau\right)} - 0.3e^{-S\left(\frac{x}{v}+2\tau\right)} - 0.18e^{S\left(\frac{x}{v}-4\tau\right)}\right.$$

$$\left. + 0.09e^{-S\left(\frac{x}{v}+4\tau\right)} + 0.054e^{S\left(\frac{x}{v}-6\tau\right)}\ldots\right]$$

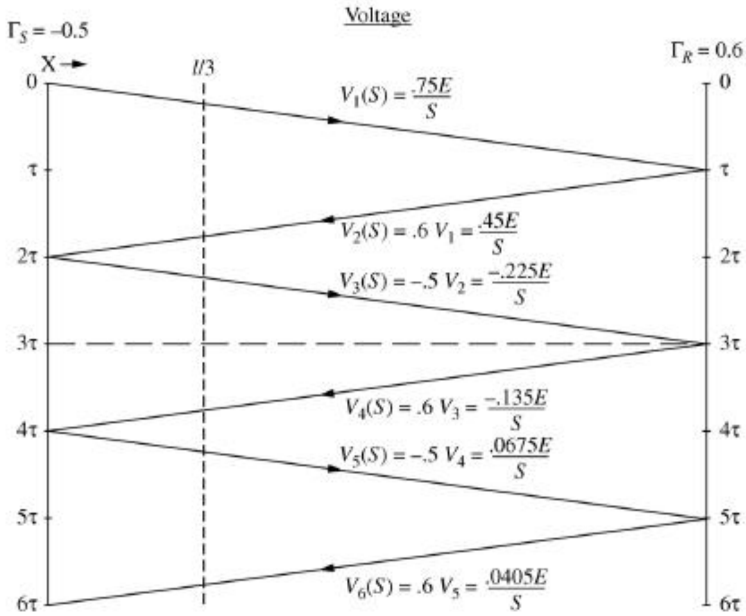
$$V(x,t)=\frac{3E}{4}\left[U_{-1}\left(t-\frac{x}{v}\right)+0.6U_{-1}\left(t+\frac{x}{v}-2\tau\right)\right.$$
$$-0.3U_{-1}\left(t-\frac{x}{v}-2\tau\right)-0.18U_{-1}\left(t+\frac{x}{v}-4\tau\right)$$
$$\left.+0.09U_{-1}\left(t-\frac{x}{v}-4\tau\right)+0.054U_{-1}\left(t+\frac{x}{v}-6\tau\right)\cdots\right]$$

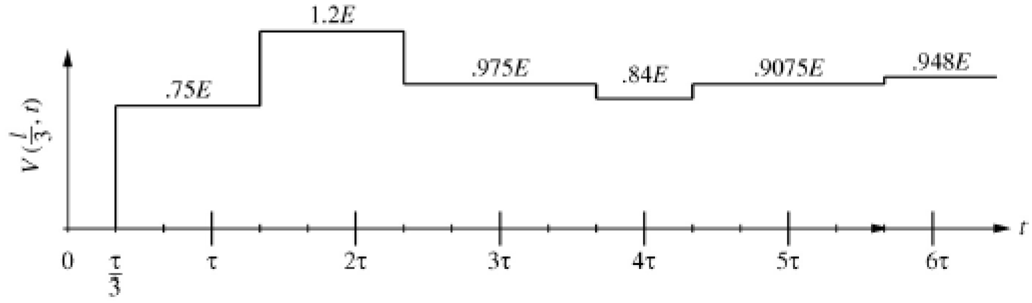
At the center of the line, where $x=\frac{l}{2}$:

$$V\left(\frac{l}{2},t\right)=\frac{3E}{4}\left[U_{-1}\left(t-\frac{\tau}{2}\right)+0.6U_{-1}\left(t-\frac{3\tau}{2}\right)-0.3U_{-1}\left(t-\frac{5}{2}\tau\right)\right.$$
$$\left.-0.18U_{-1}\left(t-\frac{7\tau}{2}\right)+0.09U_{-1}\left(t-\frac{9\tau}{2}\right)+0.054U_{-1}\left(t-\frac{11\tau}{2}\right)\cdots\right]$$

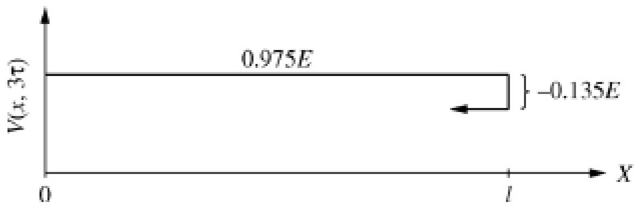


13.9



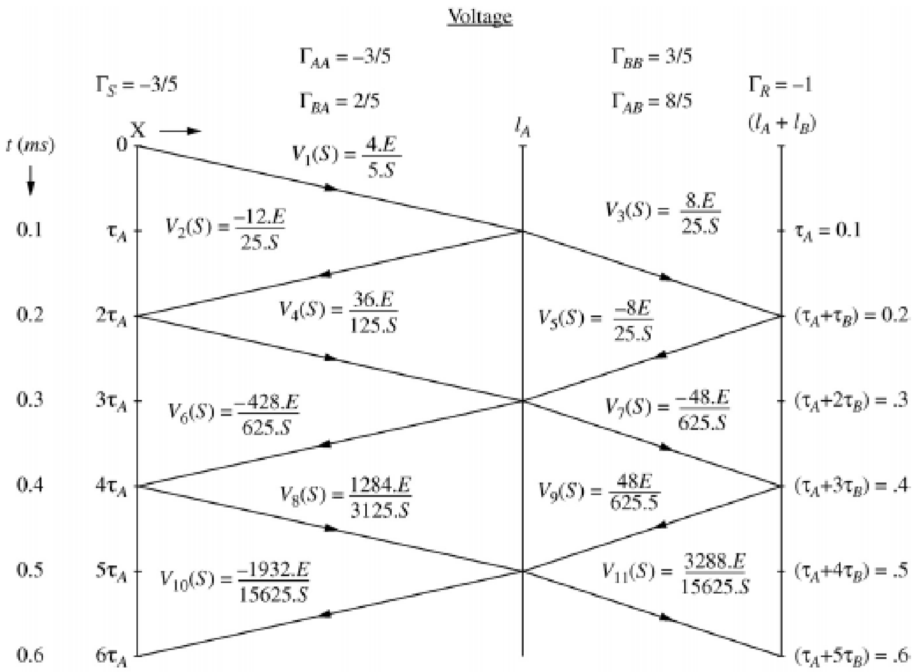


1.2E
.75E
.975E
.84E
.9075E
.948E
V(l/3, t)
0
τ/3
τ
2τ
3τ
4τ
5τ
6τ
t

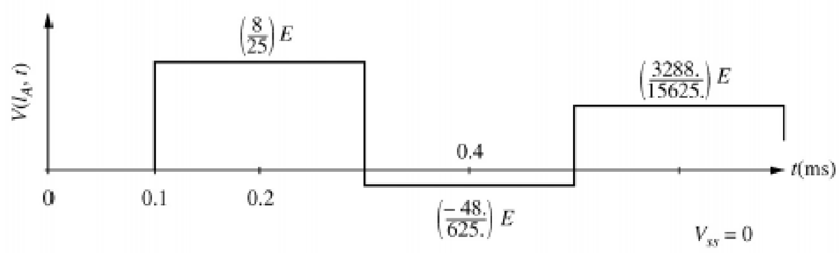


0.975E
−0.135E
V(x, 3τ)
0
l
X

13.11

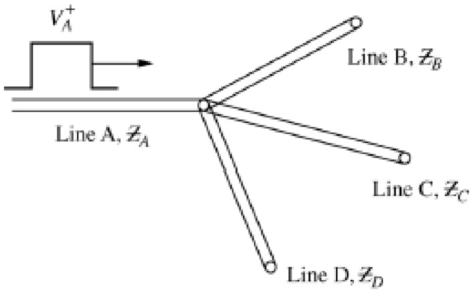


Voltage
Γ_AA = −3/5
Γ_BB = 3/5
Γ_S = −3/5
Γ_BA = 2/5
Γ_AB = 8/5
Γ_R = −1
(l_A + l_B)
l_A
t (ms)
X
V_1(S) = 4.E/5.S
V_2(S) = −12.E/25.S
V_3(S) = 8.E/25.S
V_4(S) = 36.E/125.S
V_5(S) = −8E/25.S
V_6(S) = −428.E/625.S
V_7(S) = −48.E/625.S
V_8(S) = 1284.E/3125.S
V_9(S) = 48E/625.5
V_10(S) = −1932.E/15625.S
V_11(S) = 3288.E/15625.S
0.1
0.2
0.3
0.4
0.5
0.6
τ_A
2τ_A
3τ_A
4τ_A
5τ_A
6τ_A
τ_A = 0.1
(τ_A+τ_B) = 0.2
(τ_A+2τ_B) = .3
(τ_A+3τ_B) = .4
(τ_A+4τ_B) = .5
(τ_A+5τ_B) = .6



(8/25) E
(3288./15625.) E
0.4
V(l_A, t)
0
0.1
0.2
t(ms)
(−48./625.) E
V_ss = 0

13.14 For a voltage wave V_A^+ arriving at the junction from line A,



$$KVL \quad V_A^+ + V_A^- = V_B^+ \qquad (1)$$

$$V_B^+ = V_C^+ \qquad (2)$$

$$V_B^+ = V_D^+ \qquad (3)$$

$$KCL \quad I_A^+ + I_A^- = I_B^+ + I_C^+ + I_D^+$$

$$\frac{V_A^+}{Z_A} - \frac{V_A^-}{Z_A} = \frac{V_B^+}{Z_B} + \frac{V_C^+}{Z_C} + \frac{V_D^+}{Z_D} \qquad (4)$$

Using Eqs (2) and (3) in Eq (4):

$$\frac{V_A^+}{Z_A} - \frac{V_A^-}{Z_A} = V_B^+\left(\frac{1}{Z_B} + \frac{1}{Z_C} + \frac{1}{Z_D}\right) = \frac{V_B^+}{Z_{eq}} \qquad (5)$$

Where $Z_{eq} = Z_B // Z_C // Z_D = \dfrac{1}{\dfrac{1}{Z_B} + \dfrac{1}{Z_C} + \dfrac{1}{Z_D}}$

Solving Eqs (1) and (5):

$$V_A^- = \left[\frac{\dfrac{Z_{eq}}{Z_A} - 1}{(Z_{eq}/Z_A) + 1}\right] V_A^+ = \Gamma_{AA} V_A^+ \qquad V_B^+ = \left[\frac{2(Z_{eq}/Z_A)}{(Z_{eq}/Z_A) + 1}\right] V_A^+ = \Gamma_{BA} V_A^+$$

Also $V_C^+ = \Gamma_{CA} V_A^+ \qquad V_D^+ = \Gamma_{DA} V_A^+ \qquad \Gamma_{CA} = \Gamma_{DA} = \Gamma_{BA}$