EE-465 (Term 162)

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Solution of HW2

5.1 (a) $\bar{A} = \bar{D} = 1.0\angle 0°\,\text{pu};\ \bar{C} = 0.\,\text{S}$

$\bar{B} = \bar{Z} = (0.19 + j0.34)25 = 9.737\angle 60.8°\ \Omega$

(b) $\bar{V}_R = \dfrac{33}{\sqrt{3}}\angle 0° = 19.05\angle 0°\,\text{kV}_{\text{LN}}$

$\bar{I}_R = \dfrac{S_R\ \angle -\cos^{-1}(pf)}{\sqrt{3}\,V_{R\,L\text{-}L}} = \dfrac{10}{\sqrt{3}(33)}\angle -\cos^{-1}(0.9)$

$= 0.175\angle -25.84°\,\text{kA}$

$\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = (1.0)(19.05) + (9.737\angle 60.8°)(0.175\angle -25.84°)$

$= 20.45 + j0.976 = 20.47\angle 2.732°\,\text{kV}_{\text{L-N}}$

$V_{S_{L\text{-}L}} = 20.47\sqrt{3} = 35.45\,\text{kV}$

(c) $\bar{I}_R = 0.175\angle 25.84°$

$\bar{V}_S = \bar{A}\bar{V}_R + \bar{B}\bar{I}_R = (1.0)(19.05) + (9.737\angle 60.8°)(0.175\angle 25.84°)$

$= 19.15 + j1.701 = 19.23\angle 5.076°\,\text{kV}_{\text{L-N}}$

$V_{S_{L\text{-}L}} = 19.23\sqrt{3} = 33.3\,\text{kV}$

5.2 (a) $\overline{A}=\overline{D}=1+\frac{\overline{Y}\overline{Z}}{2}=1+\frac{1}{2}(3.33\times10^{-6}\times200\angle90^\circ)(0.08+j0.48)(200)$

$=1+(0.0324\angle170.5^\circ)=0.968+j0.00533=0.968\angle0.315^\circ\text{pu}$

$\overline{C}=\overline{Y}\left(1+\frac{\overline{Y}\overline{Z}}{4}\right)=(6.66\times10^{-4}\angle90^\circ)(1+.0.0162\angle170.5^\circ)$

$=6.553\times10^{-4}\angle90.155^\circ\text{ S}$

$\overline{B}=\overline{Z}=97.32\angle80.54^\circ\ \Omega$

(b) $\overline{V}_R=\frac{220}{\sqrt{3}}\angle0^\circ=127\angle0^\circ\text{kV}_{\text{L-N}}$

$\overline{I}_R=\frac{P_R\angle-\cos^{-1}(pf)}{\sqrt{3}V_{R_{L-L}}(pf)}=\frac{250\angle-\cos^{-1}0.99}{\sqrt{3}(220)(0.99)}=0.6627\angle-8.11^\circ\text{kA}$

$\overline{V}_S=\overline{A}\overline{V}_R+\overline{B}\overline{I}_R=(0.968\angle0.315^\circ)(127\angle0^\circ)+(97.32\angle80.54^\circ)(0.6627\angle-8.11^\circ)$

$=142.4+j62.16=155.4\angle23.58^\circ\text{ kV}_{\text{L-N}}$

$\overline{V}_{S_{L-L}}=155.4\sqrt{3}=269.2\text{ kV}$

$\overline{I}_S=\overline{C}\overline{V}_R+\overline{D}\overline{I}_R=(6.553\times10^{-4}\angle90.155^\circ)(127)+(0.968\angle-0.315^\circ)(0.6627\angle-8.11^\circ)$

$=0.6353-j3.786\times10^{-3}=0.6353\angle-0.34^\circ\text{kA}$

(c) $V_{RNL}=V_S/A=269.2/0.968=278.1\text{kV}_{\text{LL}}$

$\%VR=\frac{V_{RNL}-V_{RFL}}{V_{RFL}}\times100=\left(\frac{278.1-220}{220}\right)100=26.4\%$

5.7 $V_S = \frac{3300}{\sqrt{3}} = 1905.3\text{ V}$ (Line-to-neutral)

$0.5\angle 53.13° = 0.5(0.6 + j0.8) = 0.3 + j0.4$

$$I = \frac{(900/3)10^3}{0.8 \times V_R} = \frac{375 \times 10^3}{V_R}\text{ A}$$

From the phasor diagram drawn below with $\overline{I}$ as reference,

$$V_S^2 = (V_R \cos\phi_R + IR)^2 + (V_R \sin\phi_R + IX)^2 \quad (1)$$

$$(1905.3)^2 = \left(0.8V_R + \frac{375 \times 10^3 \times 0.3}{V_R}\right)^2 + \left(0.6V_R + \frac{375 \times 10^3 \times 0.4}{V_R}\right)^2$$

From which one gets $V_R = 1805\text{ V}$

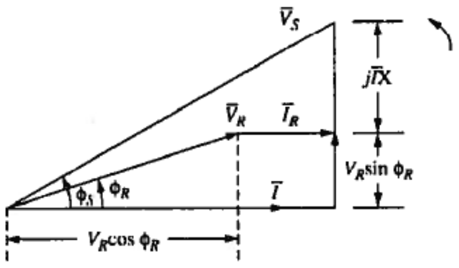
(a) Line-to-line voltage at receiving end $= 1805\sqrt{3}$

$= 3126\text{ V}$ ←

$= 3.126\text{ kV}$

(b) Line current is given by

$I = \underline{375 \times 10^3} = 207.76\text{ A}$ ←



Phasor diagram

5.14 (a) $\bar{Z}_C = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{0.03 + j0.35}{4.4\times10^{-6}\angle 90^\circ}} = 282.6\angle -2.45^\circ\ \Omega$

(b) $\bar{\gamma}l = \sqrt{\bar{z}\,\bar{y}}\,(l) = \sqrt{(0.35128\angle 85.101^\circ)(4.4\times10^{-6}\angle 90^\circ)}\,(400)$

$= 0.4973\angle 87.55^\circ = 0.02126 + j0.4968$ pu

(c) $\bar{A} = \bar{D} = \cosh\bar{\gamma}l = \cosh(0.02126 + j0.4968)$

$= (\cosh 0.02126)(\cos 0.4968 \text{ radians}) + j(\sinh 0.02126)(\sin 0.4968 \text{ radians})$

$= (1.00023)(0.87911) + j(0.02126)(0.47661)$

$= 0.87931 + j0.01013 = 0.8794\angle 0.66^\circ$ pu

$\sinh\bar{\gamma}l = \sinh(0.02126 + j0.4968)$

$= \sinh(0.02126)\cos(0.4968 \text{ radians}) + j(\cosh 0.02126)(\sin 0.4968 \text{ radians})$

$= (0.02126)(0.87911) + j(1.00023)(0.47661)$

$= 0.01869 + j0.4767 = 0.4771\angle 87.75^\circ$

$\bar{B} = \bar{Z}_C \sinh(\bar{\gamma}l) = (282.6\angle -2.45^\circ)(0.4771\angle 87.75^\circ)$

$= 134.8\angle 85.3^\circ\ \Omega$

$\bar{C} = \frac{1}{\bar{Z}_C}\sinh(\bar{\gamma}l) = \frac{0.4771\angle 87.75^\circ}{282.6\angle -2.45^\circ} = 1.688\times10^{-3}\angle 90.2^\circ \text{S}$

5.26 (a) $\bar{Z}_C = \sqrt{\frac{\bar{z}}{\bar{y}}} = \sqrt{\frac{j0.34}{j4.5\times10^{-6}}} = 274.9\angle 0^\circ = 274.9\ \Omega$

(b) $\bar{\gamma}l = \sqrt{\bar{z}\,\bar{y}}\,(l) = \sqrt{(j0.34)(j4.5\times10^{-6})}\,(300) = j0.3711$pu

(c) $\bar{\gamma}l = j(\beta l);\quad \beta l = 0.3711$pu

$A = D = \cos\ (\beta l) = \cos\ (0.3711 \text{ radians}) = 0.9319\angle 0^\circ$pu

$\bar{B} = j\,Z_C \sin\ (\beta l) = j(274.9)\sin\ (0.3711 \text{ radians})$

$= j99.68\ \Omega$

$\bar{C} = j\left(\frac{1}{\bar{Z}_C}\right)\sin\ (\beta l) = j\left(\frac{1}{274.9}\right)\sin\ (0.3711 \text{ radians})$

$= j1.319\times10^{-3}$ S

(d) $\beta = 0.3711/300 = 1.237\times10^{-3}$ radians/ km

$\lambda = 2\pi/\beta = 5079$ km

(e) $\text{SIL} = \frac{V^2_{rated\ L\text{-}L}}{\bar{Z}_C} = \frac{(500)^2}{274.9} = 909.4\ \text{MW}(3\phi)$

5.28 (a) $V_R = V_S / A = 500/0.9319 = 536.5\,\text{kV}$

(b) $V_R = V_S = 500\,\text{kV}$

(c) $$\bar{V}_S = \cos(\beta l).\bar{V}_R + (j\mathcal{Z}_C \sin\beta l)\left(\frac{\bar{V}_R}{\frac{1}{1}\mathcal{Z}_C}\right)$$

$$\bar{V}_S = \left|\cos\beta l + jZ\sin\beta l\right| V_R$$

$$V_S = 500/\left|\cos 0.3711 \text{ radians} + j2\sin 0.3711 \text{ radians}\right|$$

$$= 500/1.18 = 423.4\ \text{kV}$$

(d) $$P_{\max 3\phi} = \frac{V_S V_R}{X'} = \frac{(500)(500)}{99.68} = 2508\,\text{MW}$$

5.38 From Problem 5.14

$\bar{A} = 0.8794\angle 0.66°$ pu; $A = 0.8794$ and $\theta_A = 0.66°$

$\bar{B} = \bar{\mathcal{Z}}' = 134.8\angle 85.3°\ \Omega$; $\mathcal{Z}' = 134.8$ and $\theta_{\mathcal{Z}} = 85.3°$

Using Eq. (5.5.6)

$$P_{R\max} = \frac{500 \times 500}{134.8} - \frac{(0.8794)(500)^2}{134.8}\cos(85.3° - 0.66°)$$

$$= 1854.6 - 152.4 = 1702\ \text{MW}\,(3\phi)$$

For this loading at unity power factor,

$$I_R = \frac{P_{R\max}}{\sqrt{3}\,V_{RLL}(PF)} = \frac{1702}{\sqrt{3}(500)(1.0)} = 1.966\,\text{kA/Phase}$$

From Table A.4, the thermal limit for 3 ACSR 1113 kcmil conductors is $3 \times 1.11 = 3.33$ kA/phase. The current 1.966 kA corresponding to the theoretical steady-state stability limit is well below the thermal limit of 3.33 kA.

5.48 (a) $SIL = (345)^2/300 = 396.8\,\text{MW}$

Neglecting losses and using Eq. (5.4.29)

$$P = \frac{1\times 0.95(SIL)\sin 35^\circ}{\sin\left(\frac{2\pi(300)}{5000}\text{radians}\right)} = 1.48(SIL) = 1.48(396.8) = 587.3\ \text{MW/line}$$

$$\text{Number of 345-kV lines} = \frac{2000}{587.3} + 1 = 3.4 + 1 \simeq 5\,\text{Lines}$$

(b) For 500-kV lines, $SIL = \frac{(500)^2}{275} = 909.1\,\text{MW}$

$$P = 1.48\,(SIL) = 1.48\times 909.1 = 1345.6\,\text{MW/ Line}$$

$$\text{Number of 500-kV Lines} = \frac{2000}{1345.6} + 1 = 1.49 + 1 \simeq 3\,\text{Lines}$$

(c) For 765-kV lines, $SIL = \frac{(765)^2}{260} = 2250.9\,\text{MW}$

$$P = 1.48(SIL) = 1.48\times 2250.9 = 3331.3\,\text{MW/ Line}$$

$$\text{Number of 765-kV lines} = \frac{2200}{3331.3} + 1 = 0.6 + 1 = 2\,\text{Lines}$$

5.57 From Problem 5.23

$Z' = R' + jX' = (11.0 + j134.3)\,\Omega$

Based on 40% series compensation, half at each end of the line, the impedance of each series capacitor is

$$Z_{CAP} = -jX_{CAP} = -j\frac{1}{2}(0.4)(134.3) = -j26.86\ \Omega\text{ / phase (at each end)}$$

Using the $\overline{ABCD}$ parameters from Problem 5.14, the equivalent $\overline{ABCD}$ parameters of the compensated line are

$$\underbrace{\begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix}}_{\text{compensated line}} = \underbrace{\begin{bmatrix} 1 & -j26.86 \\ 0 & 1 \end{bmatrix}}_{\text{Sending-end series capacitors}} \underbrace{\begin{bmatrix} 0.8794\angle 0.66^\circ & 134.8\angle 85.3^\circ \\ 1.688\times 10^{-3}\angle 90.2^\circ & 0.8794\angle 0.66^\circ \end{bmatrix}}_{\text{uncompensated line}}$$

$$\underbrace{\begin{bmatrix} 1 & -j26.86 \\ 0 & 1 \end{bmatrix}}_{\text{receiving-end series capacitors}}$$

$$\begin{bmatrix} A_{eq} & B_{eq} \\ C_{eq} & D_{eq} \end{bmatrix} = \begin{bmatrix} 0.9248\angle 0.64^\circ & 86.66\angle 82.31^\circ \\ 1.688\angle 90.2^\circ & 0.9248\angle 0.64^\circ \end{bmatrix}$$