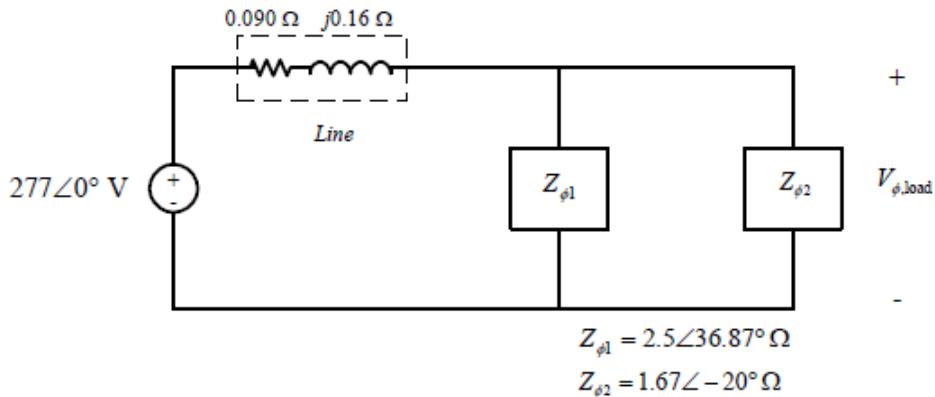


Q1)

SOLUTION To solve this problem, first convert the delta-connected load 2 to an equivalent wye (by dividing the impedance by 3), and get the per-phase equivalent circuit.



- (a) The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{\mathbf{V}_{\phi,\text{load}} - 277 \angle 0^\circ \text{ V}}{0.09 + j0.16 \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{2.5 \angle 36.87^\circ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{1.67 \angle -20^\circ \Omega} = 0$$

$$(5.443 \angle -60.6^\circ) (\mathbf{V}_{\phi,\text{load}} - 277 \angle 0^\circ \text{ V}) + (0.4 \angle -36.87^\circ) \mathbf{V}_{\phi,\text{load}} + (0.6 \angle 20^\circ) \mathbf{V}_{\phi,\text{load}} = 0$$

$$(5.955 \angle -53.34^\circ) \mathbf{V}_{\phi,\text{load}} = 1508 \angle -60.6^\circ$$

$$\mathbf{V}_{\phi,\text{load}} = 253.2 \angle -7.3^\circ \text{ V}$$

Therefore, the line voltage at the loads is $V_L\sqrt{3}$ $V_\phi = 439$ V.

(b) The voltage drop in the transmission lines is

$$\Delta \mathbf{V}_{\text{line}} = \mathbf{V}_{\phi, \text{gen}} - \mathbf{V}_{\phi, \text{load}} = 277 \angle 0^\circ \text{ V} - 253.2 \angle -7.3^\circ = 41.3 \angle 52^\circ \text{ V}$$

(c) The real and reactive power of each load is

$$P_1 = 3 \frac{V_\phi^2}{Z} \cos \theta = 3 \frac{(253.2 \text{ V})^2}{2.5 \Omega} \cos 36.87^\circ = 61.6 \text{ kW}$$

$$Q_1 = 3 \frac{V_\phi^2}{Z} \sin \theta = 3 \frac{(253.2 \text{ V})^2}{2.5 \Omega} \sin 36.87^\circ = 46.2 \text{ kvar}$$

$$P_2 = 3 \frac{V_\phi^2}{Z} \cos \theta = 3 \frac{(253.2 \text{ V})^2}{1.67 \Omega} \cos (-20^\circ) = 108.4 \text{ kW}$$

$$Q_2 = 3 \frac{V_\phi^2}{Z} \sin \theta = 3 \frac{(253.2 \text{ V})^2}{1.67 \Omega} \sin (-20^\circ) = -39.5 \text{ kvar}$$

(d) The line current is

$$\mathbf{I}_{\text{line}} = \frac{\Delta \mathbf{V}_{\text{line}}}{Z_{\text{line}}} = \frac{41.3 \angle 52^\circ \text{ V}}{0.09 + j0.16 \Omega} = 225 \angle -8.6^\circ \text{ A}$$

Therefore, the losses in the transmission line are

$$P_{\text{line}} = 3 I_{\text{line}}^2 R_{\text{line}} = 3 (225 \text{ A})^2 (0.09 \Omega) = 13.7 \text{ kW}$$

$$Q_{\text{line}} = 3 I_{\text{line}}^2 X_{\text{line}} = 3 (225 \text{ A})^2 (0.16 \Omega) = 24.3 \text{ kvar}$$

(e) The real and reactive power supplied by the generator is

$$P_{\text{gen}} = P_{\text{line}} + P_1 + P_2 = 13.7 \text{ kW} + 61.6 \text{ kW} + 108.4 \text{ kW} = 183.7 \text{ kW}$$

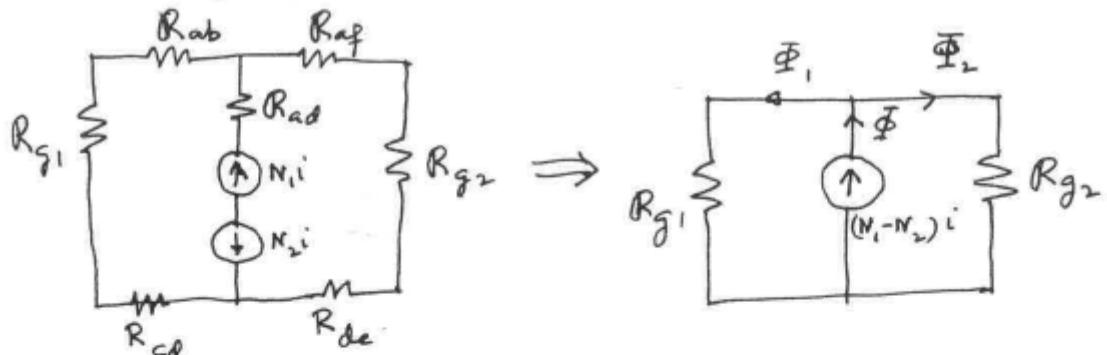
$$Q_{\text{gen}} = Q_{\text{line}} + Q_1 + Q_2 = 24.3 \text{ kvar} + 46.2 \text{ kvar} - 39.5 \text{ kvar} = 31 \text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos \left[\tan^{-1} \frac{Q_{\text{gen}}}{P_{\text{gen}}} \right] = \cos \left[\tan^{-1} \frac{31 \text{ kvar}}{183.7 \text{ kW}} \right] = 0.986 \text{ lagging}$$

Q2)

1.5 MMFs of the two coils oppose each other.



$$A_{g1} = A_{g2} = 2.5 \times 2.5 \times 10^{-4} = 6.25 \times 10^{-4} \text{ m}^2$$

$$R_{g1} = \frac{0.05 \times 10^{-2}}{4\pi 10^{-7} \times 6.25 \times 10^{-4}} = 0.637 \times 10^6 \text{ At/Wb.}$$

$$R_{g2} = \frac{0.1 \times 10^{-2}}{4\pi 10^{-7} \times 6.25 \times 10^{-4}} = 1.274 \times 10^6 \text{ At/Wb}$$

$$\Phi_1 = \frac{(700 - 200) 0.5}{0.637 \times 10^6} = 0.392 \times 10^{-3} \text{ Wb.}$$

$$\Phi_2 = \frac{500 \times 0.5}{1.274 \times 10^6} = 0.196 \times 10^{-3} \text{ Wb}$$

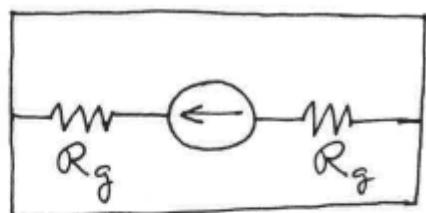
$$\Phi = \Phi_1 + \Phi_2 = 0.588 \times 10^{-3} \text{ Wb}$$

$$B_{g1} = \frac{0.392 \times 10^{-3}}{6.25 \times 10^{-4}} = 0.627 \text{ Wb/m}^2$$

$$B_{g2} = \frac{0.196 \times 10^{-3}}{6.25 \times 10^{-4}} = 0.3135 \text{ Wb/m}^2$$

Q3)

1.7 (a)



$$(b) R_g = \frac{Ig}{\mu_0 A_g} = \frac{0.25 \times 10^{-3}}{4\pi \times 10^{-7} \times 500 \times 10^3} = 39.7886 \times 10^3 \text{ At/Wb.}$$

$$\Phi = \frac{N_i}{2 R_g} = \frac{500 \times 5}{2 \times 39.7886 \times 10^3} = 0.0314 \text{ Wb.}$$

$$B_g = \frac{0.0314}{500 \times 10^{-4}} = 0.628 \text{ T.}$$

Q4)

1.8 $B = 1.4 \text{ T}$ throughout. $H_c = 0.$

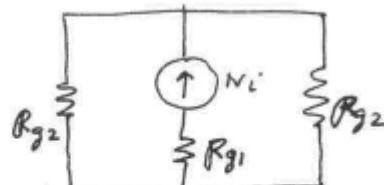
$$N_i = H_{g1} g_1 + H_{g2} g_2$$

$$H_{g1} = H_{g2} = \frac{B}{\mu_0}$$

$$g_1 = g_2 = g$$

$$N_i = 2 \frac{B}{\mu_0} g$$

$$g = \frac{\mu_0 N_i}{2 B} = \frac{4\pi \times 10^{-7} \times 500 \times 20}{2 \times 1.4} \rightarrow 4.5 \text{ mm}$$



Q5)

For Constant B

$$P_h \propto f \quad \text{i.e.} \quad P_h = Af \quad \text{where } A \text{ is const.}$$
$$P_e \propto f^2 \quad \text{i.e.} \quad P_e = Bf^2 \quad \sim B \sim$$

at 60 Hz

$$1800 = A * 60 + B * (60)^2 \rightarrow ①$$

at 90 Hz

$$3000 = A * 90 + B * (90)^2 \rightarrow ②$$

① by 3 and ② by ② and Subtract

$$3 * 1800 - 2 * 3000 = 3B * (60)^2 - 2B * (90)^2$$
$$-600 = -5400B$$

$$\text{So, } B = 0.111 \quad \text{sub. in } ①$$

$$A = 23.333$$

Hence; at 60 Hz $P_h = 23.333 * 60 = 1400 \text{ W}$

$$P_e = 0.111 * (60)^2 = 400 \text{ W}$$

At 90 Hz $P_h = 23.333 * 90 = 2100 \text{ W}$

$$P_e = 0.111 * (90)^2 = 900 \text{ W}$$

Q6)

An air-gap flux density of 0.5 T requires a total flux of

$$\phi = BA_{\text{eff}} = (0.5 \text{ T})(0.05 \text{ m})(0.05 \text{ m})(1.05) = 0.00131 \text{ Wb}$$

This flux requires a flux density in the right-hand leg of

$$B_{\text{right}} = \frac{\phi}{A} = \frac{0.00131 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 0.524 \text{ T}$$

The flux density in the other three legs of the core is

$$B_{\text{top}} = B_{\text{left}} = B_{\text{bottom}} = \frac{\phi}{A} = \frac{0.00131 \text{ Wb}}{(0.10 \text{ m})(0.05 \text{ m})} = 0.262 \text{ T}$$

The magnetizing intensity required to produce a flux density of 0.5 T in the air gap can be found from the equation $B_{\text{ag}} = \mu_0 H_{\text{ag}}$:

$$H_{\text{ag}} = \frac{B_{\text{ag}}}{\mu_0} = \frac{0.5 \text{ T}}{4\pi \times 10^{-7} \text{ H/m}} = 398 \text{ kA} \cdot \text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.524 T in the right-hand leg of the core can be found from Figure P1-9 to be

$$H_{\text{right}} = 410 \text{ A} \cdot \text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.262 T in the top, left, and bottom legs of the core can be found from Figure P1-9 to be

$$H_{\text{top}} = H_{\text{left}} = H_{\text{bottom}} = 240 \text{ A} \cdot \text{t/m}$$

The total MMF required to produce the flux is

$$\mathcal{F}_{\text{TOT}} = H_{\text{ag}} l_{\text{ag}} + H_{\text{right}} l_{\text{right}} + H_{\text{top}} l_{\text{top}} + H_{\text{left}} l_{\text{left}} + H_{\text{bottom}} l_{\text{bottom}}$$

$$\mathcal{F}_{\text{TOT}} = (398 \text{ kA} \cdot \text{t/m})(0.0005 \text{ m}) + (410 \text{ A} \cdot \text{t/m})(0.40 \text{ m}) + 3(240 \text{ A} \cdot \text{t/m})(0.40 \text{ m})$$

$$\mathcal{F}_{\text{TOT}} = 278.6 + 164 + 288 = 651 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}_{\text{TOT}}}{N} = \frac{651 \text{ A} \cdot \text{t}}{1000 \text{ t}} = 0.651 \text{ A}$$

The flux densities in the four sides of the core and the total flux present in the air gap were calculated above.