Solution of Home Work #1

2.14. Three parallel three-phase loads are supplied from a 207.85-V rms, 60-Hz three-phase supply. The loads are as follows:

Load 1: A 15 HP motor operating at full-load, 93.25 percent efficiency, and 0.6 lagging power factor.
Load 2: A balanced resistive load that draws a total of 6 kW.
Load 3: A Y-connected capacitor bank with a total rating of 16 kvar.

(a) What is the total system kW, kvar, power factor, and the supply current per phase?
(b) What is the system power factor and the supply current per phase when the resistive load and induction motor are operating but the capacitor bank is switched off?

The real power input to the motor is

$$P_1 = \frac{(15)(746)}{0.9325} = 12 \text{ kW}$$

$$S_1 = \frac{12}{0.6} \angle 53.13° \text{ kVA} = 12 \text{ kW} + j16 \text{ kvar}$$

$$S_2 = 6 \text{ kW} + j0 \text{ kvar}$$

$$S_3 = 0 \text{ kW} - j16 \text{ kvar}$$

(a) The total complex power is

$$S = 18\angle 0° \text{ kVA} = 18 \text{ kW} + j0 \text{ kvar}$$

The supply current is

$$I = \frac{18000}{(3)(120)} = 50\angle 0° \text{ A}, \qquad \text{at unity power factor}$$

(b) With the capacitor switched off, the total power is

$$S = 18 + j16 = 24.08\angle 41.63° \text{ kVA}$$

$$I = \frac{24083\angle -41.63}{(3)(120\angle 0°)} = 66.89\angle -41.63° \text{ A}$$

The power factor is $\cos 41.63° = 0.747$ lagging.

3.12. A 40-MVA, 20-kV/400-kV, single-phase transformer has the following series impedances:

$$Z_1 = 0.9 + j1.8 \ \Omega \quad \text{and} \quad Z_2 = 128 + j288 \ \Omega$$

Using the transformer rating as base, determine the per unit impedance of the transformer from the ohmic value referred to the low-voltage side. Compute the per unit impedance using the ohmic value referred to the high-voltage side.

The transformer equivalent impedance referred to the low-voltage side is

$$Z_{e1} = 0.9 + j1.8 + \left(\frac{20}{400}\right)^2 (128 + j288) = 1.22 + j2.52 \ \Omega$$

The low-voltage base impedance is

$$Z_{B1} = \frac{(20)^2}{40} = 10 \ \Omega$$

$$Z_{pu_1} = \frac{1.22 + j2.52}{10} = 0.122 + j0.252 \ \text{pu}$$

The transformer equivalent impedance referred to the high-voltage side is

$$Z_{e2} = \left(\frac{400}{20}\right)^2 (0.9 + j1.8) + (128 + j288) = 488 + j1008 \ \Omega$$

The high-voltage base impedance is

$$Z_{B2} = \frac{(400)^2}{40} = 4000 \ \Omega$$

$$Z_{pu_2} = \frac{488 + j1008}{4000} = 0.122 + j0.252 \ \text{pu}$$

We note that the transformer per unit impedance has the same value regardless of whether it is referred to the primary or the secondary side.

3.13. Draw an impedance diagram for the electric power system shown in Figure 26 showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

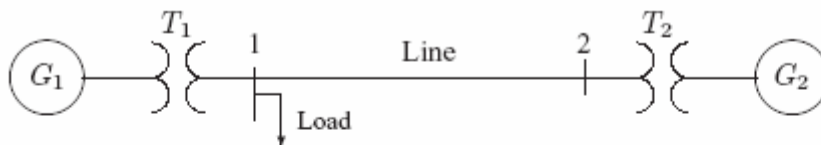|			
G_1 :	90 MVA	20 kV	$X = 9\%$
T_1 :	80 MVA	20/200 kV	$X = 16\%$
T_2 :	80 MVA	200/20 kV	$X = 20\%$
G_2 :	90 MVA	18 kV	$X = 9\%$

FIGURE 26
One-line diagram for Problem 3.13

The base voltage V_{BG1} on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20 \left(\frac{200}{20}\right) = 200 \ \text{kV}$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$V_{BG2} = 200 \left(\frac{20}{200}\right) = 20 \ \ \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$G: \quad X = 0.09 \left(\frac{100}{90}\right) = 0.10 \ \ \text{pu}$$

$$T_1: \quad X = 0.16 \left(\frac{100}{80}\right) = 0.20 \ \ \text{pu}$$

$$T_2: \quad X = 0.20 \left(\frac{100}{80}\right) = 0.25 \ \ \text{pu}$$

$$G_2: \quad X = 0.09 \left(\frac{100}{90}\right) \left(\frac{18}{20}\right)^2 = 0.081 \ \ \text{pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \ \Omega$$

The per unit line reactance is

$$Line: \quad X = \left(\frac{120}{400}\right) = 0.30 \ \ \text{pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(200)^2}{48 - j64} = 300 + j400 \ \ \Omega$$

The load impedance in per unit is

$$Z_{L\,(pu)} = \frac{300 + j400}{400} = 0.75 + j1.0 \ \ \text{pu}$$
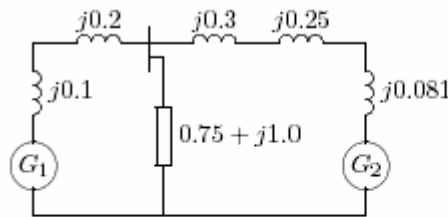
The per unit equivalent circuit is shown in Figure 27.

FIGURE 27
Per unit impedance diagram for Problem 3.11.

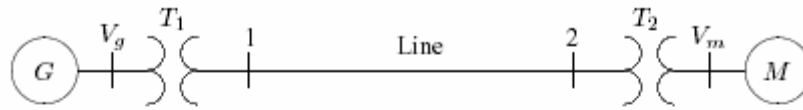**3.15.** The three-phase power and line-line ratings of the electric power system shown in Figure 30 are given below.

FIGURE 30
One-line diagram for Problem 3.15

$$
\begin{array}{llll}
G_1 : & 60\text{ MVA} & 20\text{ kV} & X = 9\% \\
T_1 : & 50\text{ MVA} & 20/200\text{ kV} & X = 10\% \\
T_2 : & 50\text{ MVA} & 200/20\text{ kV} & X = 10\% \\
M : & 43.2\text{ MVA} & 18\text{ kV} & X = 8\% \\
\text{Line:} & & 200\text{ kV} & Z = 120 + j200\ \Omega
\end{array}
$$

(a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

(b) The motor is drawing 45 MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

The base voltage V_{BG1} on the LV side of T_1 is 20 kV. Hence the base on its HV side is

$$
V_{B1} = 20\left(\frac{200}{20}\right) = 200\ \text{kV}
$$

This fixes the base on the HV side of T_2 at $V_{B2} = 200$ kV, and on its LV side at

$$
V_{Bm} = 200\left(\frac{20}{200}\right) = 20\ \text{kV}
$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69)and (3.70) are

$$
G: \quad X = 0.09\left(\frac{100}{60}\right) = 0.15\ \text{pu}
$$

$$
T_1: \quad X = 0.10\left(\frac{100}{50}\right) = 0.20\ \text{pu}
$$

$$
T_2: \quad X = 0.10\left(\frac{100}{50}\right) = 0.20\ \text{pu}
$$

$$
M: \quad X = 0.08\left(\frac{100}{43.2}\right)\left(\frac{18}{20}\right)^2 = 0.15\ \text{pu}
$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line impedance is

$$\text{Line:} \quad Z_{line} = (\frac{120 + j200}{400}) = 0.30 + j0.5 \ \text{pu}$$
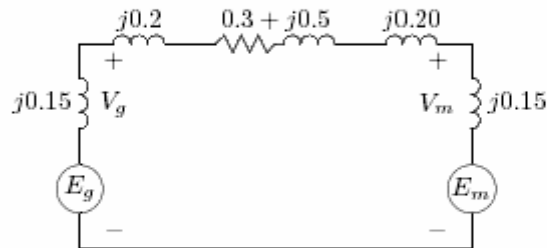
The per unit equivalent circuit is shown in Figure 31.

FIGURE 31
Per unit impedance diagram for Problem 3.15.

(b) The motor complex power in per unit is

$$S_m = \frac{45\angle 36.87°}{100} = 0.45\angle 36.87° \ \text{pu}$$

and the motor terminal voltage is

$$V_m = \frac{18\angle 0°}{20} = 0.90\angle 0° \ \text{pu}$$

$$I = \frac{0.45\angle -36.87°}{0.90\angle 0°} = 0.5\angle -36.87° \ \text{pu}$$

$$V_g = 0.90\angle 0° + (0.3 + j0.9)(0.5\angle -36.87° = 1.31795\angle 11.82° \ \text{pu}$$

Thus, the generator line-to-line terminal voltage is

$$V_g = (1.31795)(20) = 26.359 \ \text{kV}$$

$$E_g = 0.90\angle 0° + (0.3 + j1.05)(0.5\angle -36.87° = 1.375\angle 13.88° \ \text{pu}$$

Thus, the generator line-to-line internal emf is

$$E_g = (1.375)(20) = 27.5 \ \text{kV}$$

3.16. The one-line diagram of a three-phase power system is as shown in Figure 32. Impedances are marked in per unit on a 100-MVA, 400-kV base. The load at bus 2 is $S_2 = 15.93$ MW $-j33.4$ Mvar, and at bus 3 is $S_3 = 77$ MW $+j14$ Mvar. It is required to hold the voltage at bus 3 at $400\angle 0°$ kV. Working in per unit, determine the voltage at buses 2 and 1.
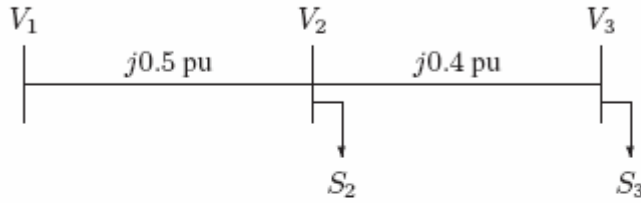

FIGURE 32
One-line diagram for Problem 3.16

$$S_2 = 15.93 \text{ MW} - j33.4 \text{ Mvar} = 0.1593 - j0.334 \text{ pu}$$
$$S_3 = 77.00 \text{ MW} + j14.0 \text{ Mvar} = 0.7700 + j0.140 \text{ pu}$$
$$V_3 = \frac{400\angle 0°}{400} = 1.0\angle 0° \text{ pu}$$
$$I_3 = \frac{S_3^*}{V_3^*} = \frac{0.77 - j0.14}{1.0\angle 0°} = 0.77 - j0.14 \text{ pu}$$
$$V_2 = 1.0\angle 0° + (j0.4)(0.77 - j0.14) = 1.1\angle 16.26° \text{ pu}$$

Therefore, the line-to-line voltage at bus 2 is

$$V_2 = (400)(1.1) = 440 \text{ kV}$$
$$I_2 = \frac{S_2^*}{V_2^*} = \frac{0.1593 + j0.334}{1.1\angle -16.26°} = 0.054 + j0.332 \text{ pu}$$
$$I_{12} = (0.77 - j0.14) + (0.054 + j0.332) = 0.824 + j0.192 \text{ pu}$$
$$V_1 = 1.1\angle 16.26° + (j0.5)(0.824 + j0.192) = 1.2\angle 36.87° \text{ pu}$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (400)(1.2) = 480 \text{ kV}$$