10.10. What reactance must be placed in the neutral of the generator of Problem 9 to limit the magnitude of the fault current for a bolted double line-to-ground fault to that for a bolted three-phase fault?

The generator base impedance is

$$Z_B = \frac{(30)^2}{50} = 18 \ \ \Omega$$

From (10.86) the positive sequence component of fault current is

$$I_a^1 = \frac{1}{0.25 + \frac{(0.15)(0.05+3X_n)}{0.2+3X_n}} = \frac{0.2 + 3X_n}{0.0575 + 1.2X_n}$$

and from (10.86) the zero sequence component of fault current is

$$I_a^0 = -\frac{1 - \frac{(0.25)(0.2+3X_n)}{0.0575+1.2X_n}}{0.05 + 3X_n} = -\frac{0.0075 + 0.45X}{3.6X^2 + 0.2325X + 0.002875}$$

The double line-to-ground fault current is

$$I_{f\,DLG} = 3I_a^0$$

Since the magnitude of the fault current is to be equal to 4, we get

$$|I_a^0| = \frac{4}{3}$$

Substituting for I_a^0, we have

$$\frac{4}{3} = \frac{0.0075 + 0.45X}{3.6X^2 + 0.2325X + 0.002875}$$

which results in the following second-order polynomial equation

$$14.4x^2 - 0.42X - 0.011 = 0$$

The positive root of the above polynomial is $X = 0.0458333$ or

$$X_n = (0.0458333)(18) = 0.825 \ \ \Omega$$

10.14. The zero-, positive-, and negative-sequence bus impedance matrices for a three-bus power system are

$$\mathbf{Z}_{bus}^{0} = j\begin{bmatrix} 0.20 & 0.05 & 0.12 \\ 0.05 & 0.10 & 0.08 \\ 0.12 & 0.08 & 0.30 \end{bmatrix} \quad \text{pu}$$

$$\mathbf{Z}_{bus}^{1} = \mathbf{Z}_{bus}^{2} = j\begin{bmatrix} 0.16 & 0.10 & 0.15 \\ 0.10 & 0.20 & 0.12 \\ 0.15 & 0.12 & 0.25 \end{bmatrix} \quad \text{pu}$$

Determine the per unit fault current and the bus voltages during fault for

(a) A bolted three-phase fault at bus 2.

(b) A bolted single line-to-ground fault at bus 2.

(c) A bolted line-to-line fault at bus 2.

(d) A bolted double line-to-ground fault at bus 2.

(a) The symmetrical components of fault current for a bolted balanced three-phase fault at bus 2 is given by

$$I_2^{012}(F) = \begin{bmatrix} 0 \\ \frac{1}{Z_{22}^1} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{j0.20} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5\angle -90^\circ \\ 5\angle 150^\circ \\ 5\angle 30^\circ \end{bmatrix}$$

For balanced fault we only have the positive-sequence component of voltage. Thus, from (10.98), bus voltages during fault for phase a are

$$\begin{aligned} V_1(F) &= 1 - Z_{12}^1 I_2(F) = 1 - j0.10(-j5) = 0.5 \\ V_2(F) &= 1 - Z_{22}^1 I_2(F) = 1 - j0.20(-j5) = 0.0 \\ V_3(F) &= 1 - Z_{32}^1 I_2(F) = 1 - j0.12(-j5) = 0.4 \end{aligned}$$

(b) From (10.90), the symmetrical components of fault current for a single line-to-ground fault at bus 2 is given by

$$\begin{aligned} I_2^0(F) = I_2^1(F) = I_2^2(F) &= \frac{1.0}{Z_{22}^1 + Z_{22}^2 + Z_{22}^0} \\ &= \frac{1.0}{j0.20 + j0.20 + j0.10} = -j2 \end{aligned}$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j2 \\ -j2 \\ -j2 \end{bmatrix} = \begin{bmatrix} 6\angle -90^\circ \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(-j2) \\ 1 - j0.10(-j2) \\ 0 - j0.10(-j2) \end{bmatrix} = \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(-j2) \\ 1 - j0.20(-j2) \\ 0 - j0.20(-j2) \end{bmatrix} = \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(-j2) \\ 1 - j0.12(-j2) \\ 0 - j0.12(-j2) \end{bmatrix} = \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.10 \\ 0.80 \\ -0.20 \end{bmatrix} = \begin{bmatrix} 0.50\angle 0^\circ \\ 0.9539\angle -114.79^\circ \\ 0.9539\angle +114.79^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.20 \\ 0.60 \\ -0.40 \end{bmatrix} = \begin{bmatrix} 0.0\angle 0^\circ \\ 0.9165\angle -109.11^\circ \\ 0.9165\angle +109.11^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.16 \\ 0.76 \\ -0.24 \end{bmatrix} = \begin{bmatrix} 0.36\angle 0^\circ \\ 0.9625\angle -115.87^\circ \\ 0.9625\angle +115.87^\circ \end{bmatrix}$$

(c) From (10.92) and (10.93), the symmetrical components of fault current for line-to-line fault at bus 2 are

$$I_2^0 = 0$$
$$I_2^1 = -I_2^2 = \frac{V_2(0)}{Z_{22}^1 + Z_{22}^2} = \frac{1}{j0.20 + j0.20} = -j2.5$$

The fault current is

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.5 \\ j2.5 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.33 \\ 4.33 \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.10(-j2.5) \\ 0 - j0.10(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.20(-j2.50) \\ 0 - j0.20(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - j0.12(-j2.5) \\ 0 - j0.12(j2.5) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.75 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6614\angle -139.11^\circ \\ 0.614\angle +130.11^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.50 \\ 0.50 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.50\angle 180^\circ \\ 0.50\angle +180^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.70 \\ 0.30 \end{bmatrix} = \begin{bmatrix} 1\angle 0^\circ \\ 0.6083\angle -145.285^\circ \\ 0.6083\angle +145.285^\circ \end{bmatrix}$$

(d) From (10.94)–(10.96), the symmetrical components of fault current for a double line-to-ground fault at bus 2 is given by

$$I_2^1 = \frac{V_2(0)}{Z_{22}^1 + \frac{Z_{22}^2(Z_{22}^0)}{Z_{22}^2 + Z_{22}^0}} = -\frac{1}{j0.20 + \frac{j0.20(j0.10)}{j0.20 + j0.10}} = -j3.75$$

$$I_2^2 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^2} - \frac{1 - j0.20(-j3.75)}{j0.20} = j1.25$$

$$I_2^0 = -\frac{V_2(0) - Z_{22}^1 I_2^1}{Z_{22}^0} = -\frac{1 - j0.20(-j3.75)}{j0.20} = j2.5$$

The phase currents at the faulted bus are

$$I_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j2.5 \\ -j3.75 \\ j1.25 \end{bmatrix} = \begin{bmatrix} 0 \\ 5.7282\angle 139.11^\circ \\ 5.7282\angle 40.89^\circ \end{bmatrix}$$

and the total fault current is

$$I_2^b + I_2^c = 5.7282\angle 139.11^\circ + 5.7282\angle 40.89^\circ = 7.5\angle 90^\circ$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{12}^0 I_2^0 \\ V_1^1(0) - Z_{12}^1 I_2^1 \\ 0 - Z_{12}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.05(j2.5) \\ 1 - j0.10(-j3.75) \\ 0 - j0.10(j1.25) \end{bmatrix} = \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{22}^0 I_2^0 \\ V_2^1(0) - Z_{22}^1 I_2^1 \\ 0 - Z_{22}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.10(j2.5) \\ 1 - j0.20(-j3.75) \\ 0 - j0.20(j1.25) \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{32}^0 I_2^0 \\ V_3^1(0) - Z_{32}^1 I_2^1 \\ 0 - Z_{32}^2 I_2^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.08(j2.5) \\ 1 - j0.12(-j3.75) \\ 0 - j0.12(j1.25) \end{bmatrix} = \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.125 \\ 0.625 \\ 0.125 \end{bmatrix} = \begin{bmatrix} 0.875\angle 0^\circ \\ 0.50\angle -120^\circ \\ 0.50\angle +120^\circ \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.75\angle 0^\circ \\ 0 \\ 0 \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.20 \\ 0.55 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.90\angle 0^\circ \\ 0.3775\angle -113.413^\circ \\ 0.3775\angle +113.413^\circ \end{bmatrix}$$

(d) For a bolted double line-to-line fault at bus 1, we have

$$I_a^1 = \frac{1}{j0.2275 + +j\frac{(0.2275)(0.1875)}{0.2275+0.1875}} = -j3.02767 \text{ pu}$$

$$I_a^0 = \frac{1-(0.2275)(-j3.02767)}{j0.1875} = j1.65975 \text{ pu}$$

$$I(F) = 3I_a^0 = 4.979\angle 90^\circ \text{ pu}$$

10.16. For Problem 10.15, obtain the bus impedance matrices for the sequence networks. A bolted single line-to-ground fault occurs at bus 1. Find the fault current, the three-phase bus voltages during fault, and the line currents in each phase. Check your results using the **zbuild** and **lgfault** programs.

First, we obtain the positive-sequence bus impedance matrix. Add branch 1, $z_{30} = j0.1$ between node $q = 3$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{33} = z_{30} = j0.1$$

Next, add branch 2, $z_{40} = j0.1$ between node $q = 4$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{33} & 0 \\ 0 & Z_{44} \end{bmatrix} = \begin{bmatrix} j0.1 & 0 \\ 0 & j0.1 \end{bmatrix}$$

Add branch 3, $z_{24} = j0.25$ between the new node $q = 2$ and the existing node $p = 4$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(3)} = \begin{bmatrix} j0.35 & 0 & j0.1 \\ 0 & j0.1 & 0 \\ j0.1 & 0 & j0.1 \end{bmatrix}$$

Add branch 4, $z_{13} = j0.25$ between the new node $q = 1$ and the existing node $p = 3$. According to rule 2, we get

$$\mathbf{Z}_{bus}^{(4)} = \begin{bmatrix} j0.35 & 0 & j0.1 & 0 \\ 0 & j0.35 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & 0 \\ 0 & j0.1 & 0 & j0.1 \end{bmatrix}$$

Add link 5, $z_{12} = j0.3$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(5)} = \left[\begin{array}{cccc|c} j0.35 & 0 & j0.1 & 0 & -j0.35 \\ 0 & j0.35 & 0 & j0.1 & j0.35 \\ j0.1 & 0 & j0.1 & 0 & -j0.1 \\ 0 & j0.1 & 0 & j0.1 & j0.1 \\ \hline -j0.35 & j0.35 & -j0.1 & j0.1 & j1 \end{array}\right]$$

From (9.58)

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j1}\begin{bmatrix} -j0.35 \\ j0.35 \\ -j0.1 \\ j0.1 \end{bmatrix}\left[\, -j0.35 \;\; j0.35 \;\; -j0.1 \;\; j0.1 \,\right]$$

$$= \begin{bmatrix} j0.1225 & -j0.1225 & j0.0350 & -j0.0350 \\ -j0.1225 & j0.1225 & -j0.0350 & j0.0350 \\ j0.0350 & -j0.0350 & j0.0100 & -j0.0100 \\ -j0.0350 & j0.0350 & -j0.0100 & j0.0100 \end{bmatrix}$$

From (9.59), the new positive-sequence bus impedance matrix is

$$\mathbf{Z}_{bus}^1 = \begin{bmatrix} j0.35 & 0 & j0.1 & 0 \\ 0 & j0.35 & 0 & j0.1 \\ j0.1 & 0 & j0.1 & 0 \\ 0 & j0.1 & 0 & j0.1 \end{bmatrix} - \begin{bmatrix} j0.1225 & -j0.1225 & j0.0350 & -j0.0350 \\ -j0.1225 & j0.1225 & -j0.0350 & j0.0350 \\ j0.0350 & -j0.0350 & j0.0100 & -j0.0100 \\ -j0.0350 & j0.0350 & -j0.0100 & j0.0100 \end{bmatrix}$$

$$= \begin{bmatrix} j0.2275 & j0.1225 & j0.0650 & j0.0350 \\ j0.1225 & j0.2275 & j0.0350 & j0.0650 \\ j0.0650 & j0.0350 & j0.0900 & j0.0100 \\ j0.0350 & j0.0650 & j0.0100 & j0.0900 \end{bmatrix}$$

Next, we obtain the zero-sequence bus impedance matrix. Add branch 1, $z_{10} = j0.25$ between node $q = 1$ and reference node 0. According to rule 1, we have

$$\mathbf{Z}_{bus}^{(1)} = Z_{11} = z_{10} = j0.25$$

Next, add branch 2, $z_{20} = j0.25$ between node $q = 2$ and reference node 0

$$\mathbf{Z}_{bus}^{(2)} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} = \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix}$$

Add link 3, $z_{12} = j0.5$ between node $q = 2$ and node $p = 1$. From (9.57), we have

$$\mathbf{Z}_{bus}^{(3)} = \left[\begin{array}{cc|c} j0.25 & 0 & -j0.25 \\ 0 & j0.25 & j0.25 \\ \hline -j0.25 & j0.25 & j1 \end{array}\right]$$

From (9.58)

$$\frac{\mathbf{\Delta Z}\,\mathbf{\Delta Z}^T}{Z_{44}} = \frac{1}{j1}\begin{bmatrix} -j0.25 \\ j0.25 \end{bmatrix}\left[\, -j0.25 \;\; j0.25 \,\right]$$

$$= \begin{bmatrix} j0.0625 & -j0.0625 \\ -j0.0625 & j0.0625 \end{bmatrix}$$

From (9.59), the new positive-sequence bus impedance matrix is

$$\mathbf{Z}_{bus}^{0} = \begin{bmatrix} j0.25 & 0 \\ 0 & j0.25 \end{bmatrix} - \begin{bmatrix} j0.0625 & -j0.0625 \\ -j0.0625 & j0.0625 \end{bmatrix}$$
$$= \begin{bmatrix} j0.1875 & j0.0625 \\ j0.0625 & j0.1875 \end{bmatrix}$$

For a bolted single line-to-ground fault at bus 1, from (10.90), the symmetrical components of fault current is given by

$$I_1^0(F) = I_1^1(F) = I_1^2(F) = \frac{1.0}{Z_{11}^1 + Z_{11}^2 + Z_{11}^0}$$
$$= \frac{1.0}{j0.2275 + j0.2275 + j0.1875} = -j1.5564$$

The fault current is

$$I_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j1.5564 \\ -j1.5564 \\ -j1.5564 \end{bmatrix} = \begin{bmatrix} 4.6693\angle{-90^\circ} \\ 0\angle 0^\circ \\ 0\angle 0^\circ \end{bmatrix}$$

From (10.98), the symmetrical components of bus voltages during fault are

$$V_1^{012}(F) = \begin{bmatrix} 0 - Z_{11}^0 I_1^0 \\ V_1^1(0) - Z_{11}^1 I_1^1 \\ 0 - Z_{11}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.1875(-j1.5564) \\ 1 - j0.2275(-j1.5564) \\ 0 - j0.2275(-j1.5564) \end{bmatrix} = \begin{bmatrix} -0.2918 \\ 0.6459 \\ -0.3541 \end{bmatrix}$$

$$V_2^{012}(F) = \begin{bmatrix} 0 - Z_{21}^0 I_1^0 \\ V_2^1(0) - Z_{21}^1 I_1^1 \\ 0 - Z_{21}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0.0625(-j1.5564) \\ 1 - j0.1225(-j1.5564) \\ 0 - j0.1225(-j1.5564) \end{bmatrix} = \begin{bmatrix} -0.0973 \\ 0.8093 \\ -0.1907 \end{bmatrix}$$

$$V_3^{012}(F) = \begin{bmatrix} 0 - Z_{31}^0 I_1^0 \\ V_3^1(0) - Z_{31}^1 I_1^1 \\ 0 - Z_{31}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0(-j1.5564) \\ 1 - j0.0650(-j1.5564) \\ 0 - j0.0650(-j1.5564) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8988 \\ -0.1012 \end{bmatrix}$$

$$V_4^{012}(F) = \begin{bmatrix} 0 - Z_{41}^0 I_1^0 \\ V_4^1(0) - Z_{41}^1 I_1^1 \\ 0 - Z_{41}^2 I_1^2 \end{bmatrix} = \begin{bmatrix} 0 - j0(-j1.5564) \\ 1 - j0.0350(-j1.5564) \\ 0 - j0.0350(-j1.5564) \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9455 \\ -0.0545 \end{bmatrix}$$

Bus voltages during fault are

$$V_1^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.2918 \\ 0.6459 \\ -0.3541 \end{bmatrix} = \begin{bmatrix} 0\angle{-180^\circ} \\ 0.9704\angle{-116.815^\circ} \\ 0.9704\angle{+116.815^\circ} \end{bmatrix}$$

$$V_2^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.0973 \\ 0.8093 \\ -0.1907 \end{bmatrix} = \begin{bmatrix} 0.5214\angle 0^\circ \\ 0.9567\angle -115.151^\circ \\ 0.9567\angle +115.151^\circ \end{bmatrix}$$

$$V_3^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8988 \\ -0.1012 \end{bmatrix} = \begin{bmatrix} 0.7977\angle 0^\circ \\ 0.9535\angle -114.727^\circ \\ 0.9535\angle +114.727^\circ \end{bmatrix}$$

$$V_4^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.9455 \\ -0.0545 \end{bmatrix} = \begin{bmatrix} 0.8911\angle 0^\circ \\ 0.9739\angle -117.223^\circ \\ 0.9739\angle +117.7223^\circ \end{bmatrix}$$

The symmetrical components of fault currents in lines for phase a are

$$I_{21}^{012} = \begin{bmatrix} \frac{V_2^0(F)-V_1^0(F)}{z_{12}^0} \\ \frac{V_2^1(F)-V_1^1(F)}{z_{12}^1} \\ \frac{V_2^2(F)-V_1^2(F)}{z_{12}^2} \end{bmatrix} = \begin{bmatrix} \frac{-0.0973-(-0.2918)}{j0.5} \\ \frac{0.8093-0.6459)}{j0.3} \\ \frac{-0.1907-(-0.3541)}{j0.3} \end{bmatrix} = \begin{bmatrix} 0.3891\angle -90^\circ \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix}$$

$$I_{31}^{012} = \begin{bmatrix} \frac{V_3^0(F)-V_1^0(F)}{z_{13}^0} \\ \frac{V_3^1(F)-V_1^1(F)}{z_{13}^1} \\ \frac{V_3^2(F)-V_1^2(F)}{z_{13}^2} \end{bmatrix} = \begin{bmatrix} \frac{0-(-0.2918)}{\infty} \\ \frac{0.8988-0.6459)}{j0.25} \\ \frac{-0.1012-(-0.3541)}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0117\angle -90^\circ \\ 1.0117\angle -90^\circ \end{bmatrix}$$

$$I_{42}^{012} = \begin{bmatrix} \frac{V_4^0(F)-V_2^0(F)}{z_{24}^0} \\ \frac{V_4^1(F)-V_2^1(F)}{z_{24}^1} \\ \frac{V_4^2(F)-V_2^2(F)}{z_{24}^2} \end{bmatrix} = \begin{bmatrix} \frac{0-(-0.0973)}{\infty} \\ \frac{0.9455-0.8093)}{j0.25} \\ \frac{-0.0545-(-0.1907)}{j0.25} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix}$$

The line fault currents are

$$I_{21}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.3891\angle -90^\circ \\ 0.5447\angle -90^\circ \\ 0.5447\angle -90^\circ \end{bmatrix} = \begin{bmatrix} 1.4784\angle -90^\circ \\ 0.1556\angle 90^\circ \\ 0.1556\angle 90^\circ \end{bmatrix}$$

$$I_{31}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.0117\angle 90^\circ \\ 1.0117\angle 90^\circ \end{bmatrix} = \begin{bmatrix} 2.0233\angle -90^\circ \\ 1.0117\angle 90^\circ \\ 1.0117\angle 90^\circ \end{bmatrix}$$

$$I_{42}^{abc}(F) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5447\angle{-90^\circ} \\ 0.5447\angle{-90^\circ} \end{bmatrix} = \begin{bmatrix} 1.0895\angle{-90^\circ} \\ 0.5447\angle 90^\circ \\ 0.5447\angle 90^\circ \end{bmatrix}$$

To check the result we use the following commands

```
zdata1 = [0   3   0    0.10
          0   4   0    0.10
          1   3   0    0.25
          1   2   0    0.3
          2   4   0    0.25];
zdata0 = [0   3   0    0.05
      0   4   0    0.05
      0   1   0    0.25
      0   2   0    0.25
      1   3   9999 9999
      1   2   0    0.50
      2   4   9999 9999];
Zbus1 = zbuild(zdata1)
Zbus0 = zbuild(zdata0)
zdata2 = zdata1;
Zbus2 = Zbus1;
lgfault(zdata0, Zbus0, zdata1, Zbus1, zdata2, Zbus2)
```

which result in

```
Zbus1 =
        0 + 0.2275i   0 + 0.1225i   0 + 0.0650i   0 + 0.0350i
        0 + 0.1225i   0 + 0.2275i   0 + 0.0350i   0 + 0.0650i
        0 + 0.0650i   0 + 0.0350i   0 + 0.0900i   0 + 0.0100i
        0 + 0.0350i   0 + 0.0650i   0 + 0.0100i   0 + 0.0900i
Zbus0 =
        0 + 0.1875i   0 + 0.0625i   0 + 0.0000i   0 + 0.0000i
        0 + 0.0625i   0 + 0.1875i   0 + 0.0000i   0 + 0.0000i
        0 + 0.0000i   0 + 0.0000i   0 + 0.0500i   0 + 0.0000i
        0 + 0.0000i   0 + 0.0000i   0 + 0.0000i   0 + 0.0500i

Line-to-ground fault analysis
Enter Faulted Bus No. -> 1

Enter Fault Impedance Zf = R + j*X in
complex form (for bolted fault enter 0). Zf = 0

Single line to-ground fault at bus No. 1
Total fault current =    4.6693 per unit
```

Bus Voltages during the fault in per unit

Bus	-------Voltage Magnitude-------		
No.	Phase a	Phase b	Phase c
1	0.0000	0.9704	0.9704
2	0.5214	0.9567	0.9567
3	0.7977	0.9535	0.9535
4	0.8911	0.9739	0.9739

Line currents for fault at bus No. 1

From	To	-----Line Current Magnitude----		
Bus	Bus	Phase a	Phase b	Phase c
1	F	4.6693	0.0000	0.0000
2	1	1.4786	0.1556	0.1556
3	1	2.0234	1.0117	1.0117
4	2	1.0895	0.5447	0.5447