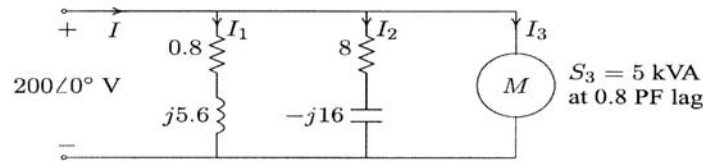


2.7. Two impedances,  $Z_1 = 0.8 + j5.6 \Omega$  and  $Z_2 = 8 - j16 \Omega$ , and a single-phase motor are connected in parallel across a 200-V rms, 60-Hz supply as shown in Figure 8. The motor draws 5 kVA at 0.8 power factor lagging.



**FIGURE 8**  
Circuit for Problem 2.7.

- (a) Find the complex powers  $S_1$ ,  $S_2$  for the two impedances, and  $S_3$  for the motor.  
 (b) Determine the total power taken from the supply, the supply current, and the overall power factor.  
 (c) A capacitor is connected in parallel with the loads. Find the kvar and the capacitance in  $\mu\text{F}$  to improve the overall power factor to unity. What is the new line current?

(a) The load complex power are

$$S_1 = \frac{|V|^2}{Z_1^*} = \frac{(200)^2}{0.8 - j5.6} = 1000 + j7000 \text{ VA}$$

$$S_2 = \frac{|V|^2}{Z_2^*} = \frac{(200)^2}{8 + j16} = 1000 - j2000 \text{ VA}$$

$$S_3 = 5000 \angle 36.87^\circ = 4000 + j3000 \text{ VA}$$

Therefore, the total complex power is

$$S_t = 6 + j8 = 10 \angle 53.13^\circ \text{ kVA}$$

(b) From  $S = VI^*$ , the current is

$$I = \frac{10000 \angle -53.13^\circ}{200 \angle 0^\circ} = 50 \angle -53.13^\circ \text{ A}$$

and the power factor is  $\cos 53.13^\circ = 0.6$  lagging.

(c) For overall unity power factor,  $Q_C = 8000 \text{ Var}$ , and the capacitive impedance is

$$Z_C = \frac{|V|^2}{S_C^*} = \frac{(200)^2}{j8000} = -j5 \Omega$$

and the capacitance is

$$C = \frac{10^6}{(2\pi)(60)(5)} = 530.5 \mu\text{F}$$

The new current is

$$I = \frac{6000 \angle 0^\circ}{200 \angle 0^\circ} = 30 \angle 0^\circ \text{ A}$$

**3.11.** A three-phase, Y-connected, 75-MVA, 27-kV synchronous generator has a synchronous reactance of  $9.0 \Omega$  per phase. Using rated MVA and voltage as base values, determine the per unit reactance. Then refer this per unit value to a 100-MVA, 30-kV base.

The base impedance is

$$Z_B = \frac{(KV_B)^2}{MVA_B} = \frac{(27)^2}{75} = 9.72 \Omega$$

$$X_{pu} = \frac{9}{9.72} = 0.926 \text{ pu}$$

The generator reactance on a 100-MVA, 30-kV base is

$$X_{pu_{new}} = 0.926 \left( \frac{100}{75} \right) \left( \frac{27}{30} \right)^2 = 1.0 \text{ pu}$$

**3.12.** A 40-MVA, 20-kV/400-kV, single-phase transformer has the following series impedances:

$$Z_1 = 0.9 + j1.8 \Omega \quad \text{and} \quad Z_2 = 128 + j288 \Omega$$

Using the transformer rating as base, determine the per unit impedance of the transformer from the ohmic value referred to the low-voltage side. Compute the per unit impedance using the ohmic value referred to the high-voltage side.

The transformer equivalent impedance referred to the low-voltage side is

$$Z_{e1} = 0.9 + j1.8 + \left( \frac{20}{400} \right)^2 (128 + j288) = 1.22 + j2.52 \Omega$$

The low-voltage base impedance is

$$Z_{B1} = \frac{(20)^2}{40} = 10 \Omega$$

$$Z_{pu1} = \frac{1.22 + j2.52}{10} = 0.122 + j0.252 \text{ pu}$$

The transformer equivalent impedance referred to the high-voltage side is

$$Z_{e2} = \left(\frac{400}{20}\right)^2 (0.9 + j1.8) + (128 + j288) = 488 + j1008 \Omega$$

The high-voltage base impedance is

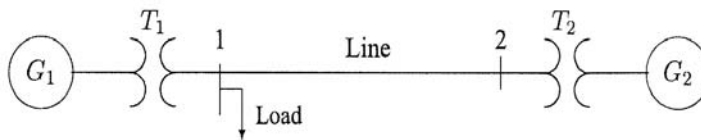
$$Z_{B2} = \frac{(400)^2}{40} = 4000 \Omega$$

$$Z_{pu2} = \frac{488 + j1008}{4000} = 0.122 + j0.252 \text{ pu}$$

We note that the transformer per unit impedance has the same value regardless of whether it is referred to the primary or the secondary side.

**3.13.** Draw an impedance diagram for the electric power system shown in Figure 26 showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

$G_1$ :	90 MVA	20 kV	$X = 9\%$
$T_1$ :	80 MVA	20/200 kV	$X = 16\%$
$T_2$ :	80 MVA	200/20 kV	$X = 20\%$
$G_2$ :	90 MVA	18 kV	$X = 9\%$
Line:		200 kV	$X = 120 \Omega$
Load:		200 kV	$S = 48 \text{ MW} + j64 \text{ Mvar}$



**FIGURE 26**  
One-line diagram for Problem 3.13

The base voltage  $V_{BG1}$  on the LV side of  $T_1$  is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20 \left(\frac{200}{20}\right) = 200 \text{ kV}$$

This fixes the base on the HV side of  $T_2$  at  $V_{B2} = 200$  kV, and on its LV side at

$$V_{BG2} = 200 \left(\frac{20}{200}\right) = 20 \text{ kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69) and (3.70) are

$$G: \quad X = 0.09 \left( \frac{100}{90} \right) = 0.10 \text{ pu}$$

$$T_1: \quad X = 0.16 \left( \frac{100}{80} \right) = 0.20 \text{ pu}$$

$$T_2: \quad X = 0.20 \left( \frac{100}{80} \right) = 0.25 \text{ pu}$$

$$G_2: \quad X = 0.09 \left( \frac{100}{90} \right) \left( \frac{18}{20} \right)^2 = 0.081 \text{ pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line reactance is

$$\text{Line:} \quad X = \left( \frac{120}{400} \right) = 0.30 \text{ pu}$$

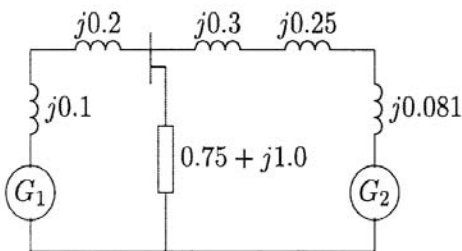
The load impedance in ohms is

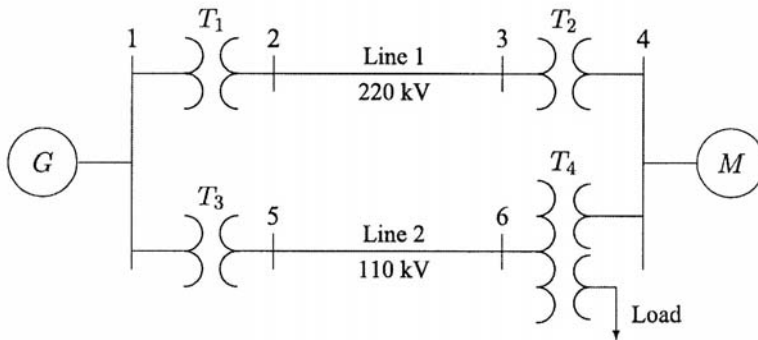
$$Z_L = \frac{(V_{L-L})^2}{S_{L(3\phi)}^*} = \frac{(200)^2}{48 - j64} = 300 + j400 \ \Omega$$

The load impedance in per unit is

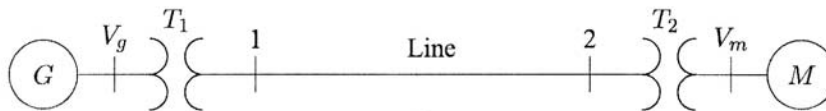
$$Z_{L(pu)} = \frac{300 + j400}{400} = 0.75 + j1.0 \text{ pu}$$

The per unit equivalent circuit is shown in Figure 27.





3.15. The three-phase power and line-line ratings of the electric power system shown in Figure 30 are given below.



**FIGURE 30**  
One-line diagram for Problem 3.15

$G_1$ :	60 MVA	20 kV	$X = 9\%$
$T_1$ :	50 MVA	20/200 kV	$X = 10\%$
$T_2$ :	50 MVA	200/20 kV	$X = 10\%$
$M$ :	43.2 MVA	18 kV	$X = 8\%$
Line:		200 kV	$Z = 120 + j200 \Omega$

(a) Draw an impedance diagram showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator.

(b) The motor is drawing 45 MVA, 0.80 power factor lagging at a line-to-line terminal voltage of 18 kV. Determine the terminal voltage and the internal emf of the generator in per unit and in kV.

The base voltage  $V_{BG1}$  on the LV side of  $T_1$  is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20 \left( \frac{200}{20} \right) = 200 \text{ kV}$$

This fixes the base on the HV side of  $T_2$  at  $V_{B2} = 200$  kV, and on its LV side at

$$V_{Bm} = 200 \left( \frac{20}{200} \right) = 20 \text{ kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, from (3.69) and (3.70) are

$$\begin{aligned} G: \quad X &= 0.09 \left( \frac{100}{60} \right) = 0.15 \text{ pu} \\ T_1: \quad X &= 0.10 \left( \frac{100}{50} \right) = 0.20 \text{ pu} \\ T_2: \quad X &= 0.10 \left( \frac{100}{50} \right) = 0.20 \text{ pu} \\ M: \quad X &= 0.08 \left( \frac{100}{43.2} \right) \left( \frac{18}{20} \right)^2 = 0.15 \text{ pu} \end{aligned}$$

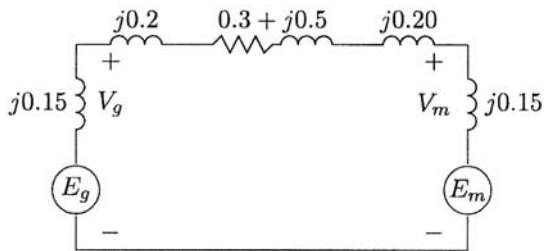
The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \Omega$$

The per unit line impedance is

$$\text{Line:} \quad Z_{line} = \left( \frac{120 + j200}{400} \right) = 0.30 + j0.5 \text{ pu}$$

The per unit equivalent circuit is shown in Figure 31.



**FIGURE 31**  
Per unit impedance diagram for Problem 3.15.

(b) The motor complex power in per unit is

$$S_m = \frac{45 \angle 36.87^\circ}{100} = 0.45 \angle 36.87^\circ \text{ pu}$$

and the motor terminal voltage is

$$V_m = \frac{18 \angle 0^\circ}{20} = 0.90 \angle 0^\circ \text{ pu}$$

$$I = \frac{0.45\angle-36.87^\circ}{0.90\angle 0^\circ} = 0.5\angle-36.87^\circ \text{ pu}$$

$$V_g = 0.90\angle 0^\circ + (0.3 + j0.9)(0.5\angle-36.87^\circ) = 1.31795\angle 11.82^\circ \text{ pu}$$

Thus, the generator line-to-line terminal voltage is

$$V_g = (1.31795)(20) = 26.359 \text{ kV}$$

$$E_g = 0.90\angle 0^\circ + (0.3 + j1.05)(0.5\angle-36.87^\circ) = 1.375\angle 13.88^\circ \text{ pu}$$

Thus, the generator line-to-line internal emf is

$$E_g = (1.375)(20) = 27.5 \text{ kV}$$

