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# An Explicit Finite-Difference Scheme for Wave Propagation in Nonlinear Optical Structures

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**Abstract**—In this paper, we present an algorithm that solves a time-domain nonlinear coupled system arising in nonlinear optics. The algorithm is an explicit nonlinear finite-difference method (NFDM) based on the exact solution of the nonlinear discrete equations. It enables simulations that preserve the characteristics of nonlinearity as well as coupling, and can be extended to arbitrary input waveform conditions. 

**Keywords**—Nonlinear optics, Finite difference, Nonlinear wave propagation, Time domain.

## 1. INTRODUCTION

Time-domain analysis of nonlinear effects in modern optical devices provides an invaluable insight into the understanding of device behavior and wave-device interactions. With such type of analysis, several important phenomena can be investigated and studied. It also enables the construction of accurate models incorporating material characteristics as well as wave propagation effects. The material relaxation effects and time-domain transients can, thus, be included.

All previously reported models for the second-harmonic generation (SHG) in second-order nonlinear optical devices have implicitly assumed continuous-wave (CW) operations and have relied on frequency-domain representations of the propagating fields. See, for example, [1]. Recently, Alsunaidi *et al.* have proposed a time-domain model that is capable, in principle, of characterizing second-order nonlinearities for all input signal conditions [2]. However, they have restricted

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the solution of their model to CW operations and validated the results against well-documented literature.

The model proposed in [2] is given by

$$\begin{aligned} c^2 \triangle f &= n_f^2 f_{tt} + 2\chi^{(2)} (fs)_{tt}, \\ c^2 \triangle s &= n_s^2 s_{tt} + \chi^{(2)} (f^2)_{tt}. \end{aligned} \tag{1}$$

The unknowns $f = f(x, y, z, t)$ and $s = s(x, y, z, t)$ are the fundamental and second-harmonic field strength, respectively. The parameter $c$ is the speed of light, and $\chi^{(2)}$ is the second-order nonlinear optical susceptibility. The coefficients $n_f$ and $n_s$ are the material refractive indices with respect to the fundamental and second-harmonic fields. The operator $\triangle$ is the spatial Laplacian.

This model describes the generation of a second-harmonic field in response to an applied fundamental field that propagates along the optical device. It also describes the nonlinear energy coupling and energy exchange between the fundamental and second-harmonic fields.

In this paper, we present an algorithm that avoids the simplifying assumptions on the time derivatives imposed in [2]. Doing so, the time-domain numerical solution preserves the characteristics of nonlinearity, as well as coupling, and can be extended to arbitrary input waveform conditions such as pulsed optical beams.

The algorithm is an explicit nonlinear finite-difference method (NFDM) based on the exact solution of the nonlinear discrete equation for the fundamental field strength $f$. It preserves the second-order accuracy and does not increase the computation intensity.

In the following, we describe the algorithm and apply it to an optical waveguide problem. Then, we analyze and compare some numerical simulations.

## 2. NUMERICAL ALGORITHM

To discretize (1), we expand the time derivatives and approximate them by finite central differences at a time level $n$. The discrete equations can be decoupled by substituting for $s^{n+1}$ from the second equation into the first one. The resulting equation is a cubic polynomial for $f^{n+1}$,

$$\left(f^{n+1}\right)^3 + a_n \left(f^{n+1}\right)^2 + b_n f^{n+1} + c_n = 0. \tag{2}$$

The coefficients are given by

$$a_n = 3\phi_n, \tag{3}$$

$$b_n = f_{n-1} \left(3f_{n-1} - 4f_n\right) - \frac{2n_f^2 n_s^2}{\left(\chi^{(2)}\right)^2} + \frac{2\left(2c^2 n_s^2 \left(s_{n-1} - 2s_n\right) - c^4\, dt^2 \Delta s_n\right)}{c^2 \chi^{(2)}}, \tag{4}$$

$$\begin{aligned} c_n = \phi_n \left(f_{n-1}^2 - 4\phi_n f_n\right) &+ \frac{2n_s^2 \left(\phi_n n_f^2 + c^2\, dt^2 \Delta f_n\right)}{\left(\chi^{(2)}\right)^2} \\ &+ \frac{4n_s^2 \left(\phi_n s_n + f_{n-1} \left(s_n - s_{n-1}\right)\right) - 2c^2\, dt^2 \phi_n \Delta s_n}{\chi^{(2)}}, \end{aligned} \tag{5}$$

where $\phi_n = 2f_n - f_{n-1}$.

For the range of values used in optical waveguides, the polynomial (2) has three real solutions (cf. [3, pp. 178–180]). Out of the three solutions, only one solution is found to be physical and given by

$$f^{n+1} = -2\sqrt{Q_n} \cos\left(\frac{\theta_n - 2\pi}{3}\right) - \frac{a_n}{3},$$

where

$$Q_n = \frac{a_n^2 - 3b_n}{9}, \qquad R_n = \frac{2a_n^3 - 9a_n b_n + 27c_n}{54}, \qquad \text{and} \qquad \theta_n = \cos^{-1}\left(\frac{R_n}{\sqrt{Q_n^3}}\right).$$

Once $f^{n+1}$ is calculated, $s^{n+1}$ is calculated explicitly from the second equation.

The time step can be chosen based on the CFL condition. For example, one can use a uniform time step corresponding to the maximum propagation speed

$$\frac{c}{\sqrt{n_m^2 - 2\chi^{(2)} A}},$$

where $n_m = \min\{n_f, n_s\}$ and $A$ is the max amplitude of the fields strength.

Note that $b_n$ and $c_n$ in (4) and (5) are singular for $\chi^{(2)} = 0$. In this case, however, the wave equation (1) is linear, decoupled, and can be solved directly [4].

## 3. NUMERICAL SIMULATION OF SHG IN WAVEGUIDES

We use the scheme presented in Section 2 to simulate SHG in the two-dimensional symmetric dielectric slab waveguide shown in Figure 1. For the wavelengths $\lambda_f = 1.0\,\mu$m and $\lambda_s = 0.5\,\mu$m, the parameters of the waveguide shown in Figure 1 are set such that only the first guided modes for both the fundamental and second-harmonic fields are allowed to propagate with effective refractive index of $n_{eff} = 1.15295$ (phase-matched condition [5]). For the theory of dielectric waveguides and modes of propagation, see, for example [4,6].

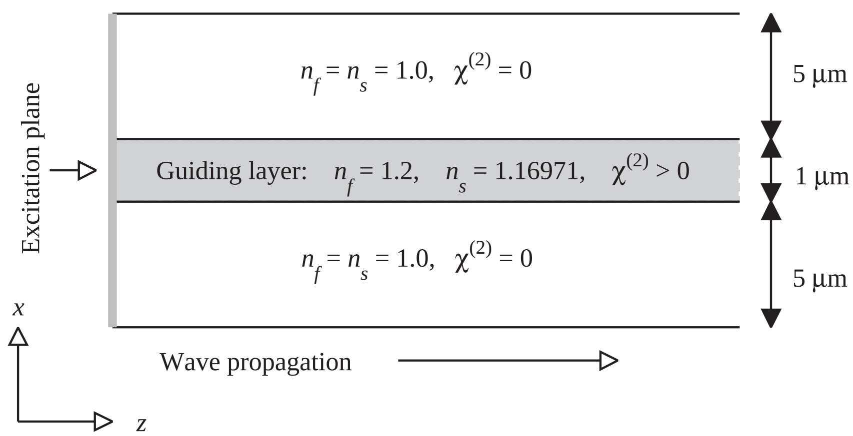



Figure 1. Dielectric slab waveguide.

As an excitation (input) field, we apply a CW beam with amplitude of $1.0x10^7$ V/m. The transverse profile of the excitation corresponds to the first guided mode at the fundamental operating frequency.

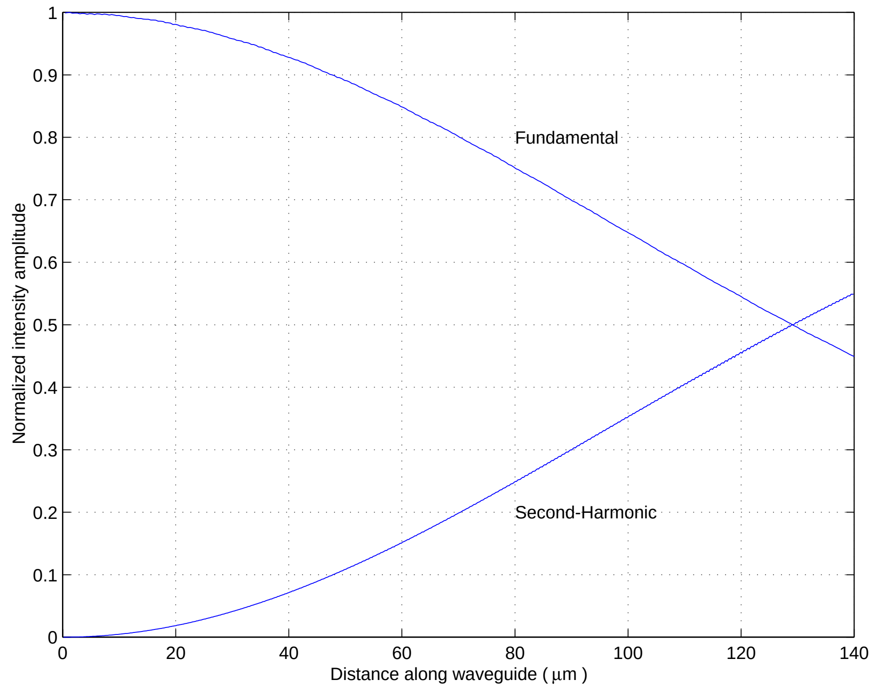
We use the spatial resolution $dz = \lambda_f/100 = .01\,\mu$m and $dx = 0.1\,\mu$m. This resolution is fine enough to resolve the rapid signal variations.

## 4. NUMERICAL RESULTS

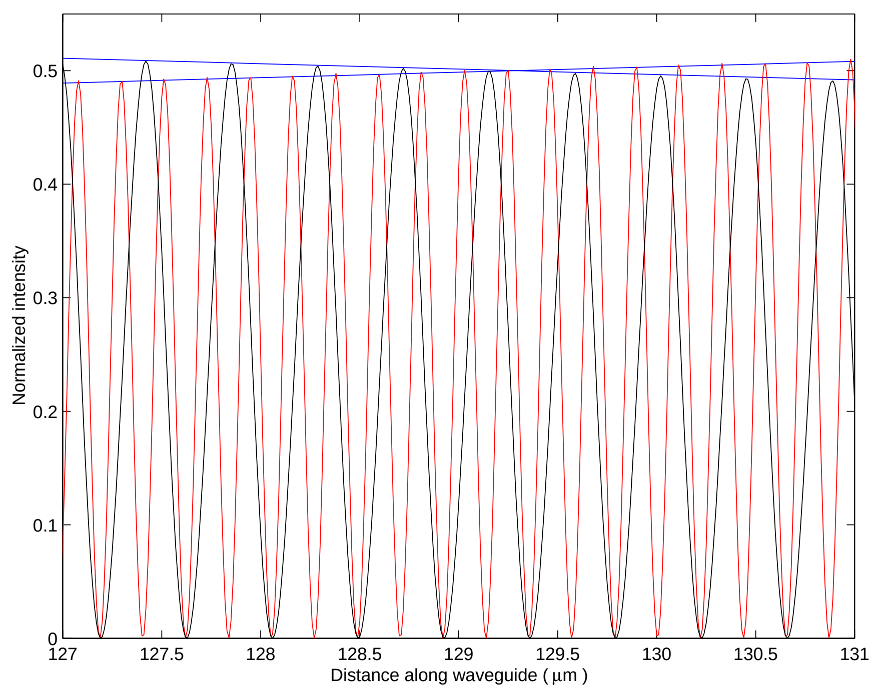
In the absence of analytical solutions, we validate our method by performing two types of comparisons on the calculated fields strength and intensity. It is to be mentioned that the intensity is proportional to the sum of the squares of the field strength over $x$.

First, we compare the NFDM simulation results of the intensity exchange between the fundamental and second-harmonic fields along the device with those generated by the beam propagation method (BPM) [7]. This comparison is shown in Figure 2 for $\chi^{(2)} = 3 \times 10^{-10}$ m/V. The two results coincide [1]. Also shown in the figure is an extended view of the intensity curves around the 50% coupling between the fundamental and second-harmonic fields for detailed comparison.

Second, the convergence of the NFDM solution to the linear solution as $\chi^{(2)}$ vanishes is examined. Figure 3 shows that as $\chi^{(2)}$ decreases the depletion of the fundamental beam decreases and approaches the no-coupling case (linear solution).




Amplitudes of the two methods coincide.

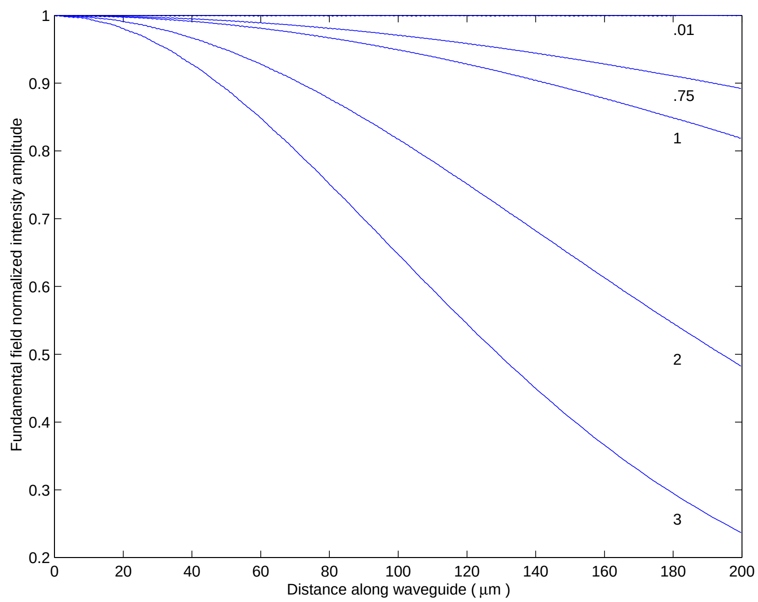



Normalized intensity in the vicinity of the 50% intensity exchange as calculated by NFDM.

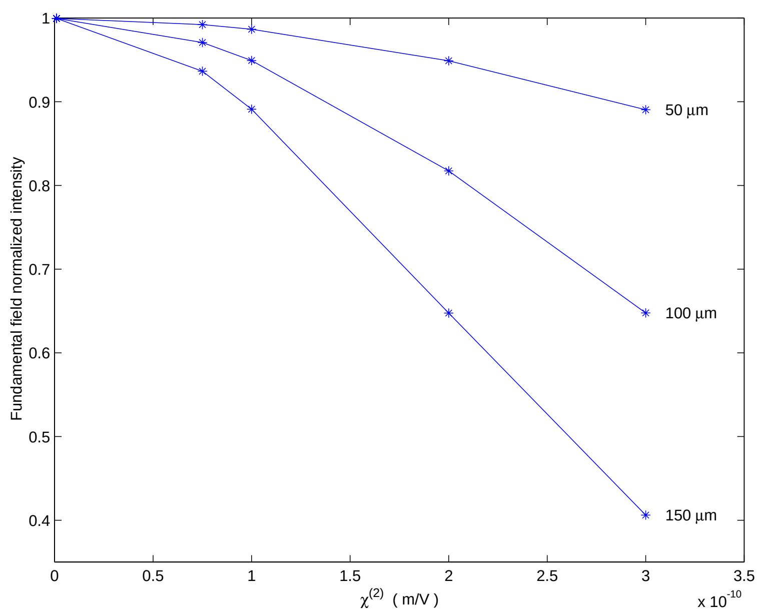
Figure 2. Normalized intensity using BPM and NFDM for $\chi^{(2)} = 3 \times 10^{-10}\,\mathrm{m/V}$. Solid oscillations: fundamental; dotted oscillations: second-harmonic.

## 5. CONCLUSION

We presented and validated a nonlinear explicit finite difference algorithm that solves a time-domain nonlinear coupled system. The results obtained match with the BPM and the linear




Convergence of the fundamental field intensity envelops. The curves correspond to $10^{10}\chi^{(2)} = \{3, 2, 1, .75, .01\}$.




Convergence of the fundamental field intensity amplitude at $z = 50, 100, 150\,\mu$m.

Figure 3. Comparison with the linear response as $\chi^{(2)}$ vanishes.

response. Maintaining all the terms, the scheme reflects more accurately the nonlinearity effects and can simulate a broader class of problems. Such issues are under investigation.

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