A Stable Time-Domain Beam Propagation Method for Modeling Ultrashort Optical Pulses

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***Abstract*—A new technique to model ultrashort optical pulses is proposed and verified. The technique uses Pade approximant to account for the fast pulse propagational variations. Numerical parameters of the technique have been tested and it was shown that the method is simple, very stable, and accurate in modeling ultrashort optical pulses in long propagation interaction.**

***Index Terms*—Finite-difference analysis, modeling, numerical analysis, optical waveguide theory, Pade approximant, partial differential equation, ultrashort pulse propagation beam propagation method (BPM).**

I. Introduction

MODERN optical communication circuits require efficient and accurate modeling techniques in order to better understand, design, and fabricate innovative devices. It is to be said that modeling optical circuits can be very challenging due to the complicated nature of such devices and the length of interaction involved. In most cases, the operation of these devices requires the propagation of optical beams for several thousands of wavelengths long and also demands time-domain (TD) analysis rather than continuous-wave (CW) operations. In principle, the well-known finite-difference TD (FDTD) technique is well suited for such problems [1]. However, the FDTD is a very computer-intensive technique and not suited for long longitudinal interaction. In the literature, there are a number of new techniques under the name of TD and they are very similar in their approach to the classical FDTD [2]. Some of these methods use slowly varying time stepping approximation (first-order time derivative) and some use additional techniques (i.e., Pade relations) to account for the neglected second derivative of time as an improvement process. Unfortunately, most of these techniques proved to be inferior to FDTD in terms of computer resources consumption [2]. The beam propagation method (BPM) was established over the years as a prime technique for long device interaction, but most of the efforts were developed for CW operation. Recently, we proposed very efficient TD-BPM techniques for modeling optical pulse propagation in long device interaction [3]. They involve writing the TD wave equation as a one-way paraxial equation for the propagation along the axial direction z while retaining the time variation as another element along with other spatial variables. This arrangement has the advantage of allowing the numerical time window to follow the evolution of the pulse and hence minimize computer resources. However, due to the paraxial approximation imposed, these techniques showed limitation in modeling ultrashort pulse propagation. In the literature, there are a few reported techniques proposed for TD solution of higher order parabolic equations in nonoptical fields, i.e., underwater acoustics and seismology [4]. Recently, a new wide-angle TD-BPM technique based on finite element has been reported for modeling short pulse propagation [5]. The method was applied under certain approximation by neglecting first-order derivatives of time and longitudinal terms. In this work, we continue the previous effort of using the one-way propagational wave equation approach for the purpose of modeling ultrashort pulses. The rational complex coefficient approximation based on the well-known Pade approximant has been used as an operator to march the pulse packet along the direction of propagation [6], [7].

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II. Numerical Method

Let us assume a two-dimensional optical structure (x and z) with the TD wave equation described as

$$\partial_z(r\partial_z\psi)+\partial_x(r\partial_x\psi)-\frac{s}{c_o^2}\partial_{tt}^2\psi=0 \qquad (1)$$

where for TE fields $r=1$, $s=n^2$, and $\psi=E_y$ representing the electric field, for TM fields $r=1/n^2$, $s=1$, and $\psi=H_y$ representing the magnetic field, c_o is the wave velocity in free space, and $n=n(\mathbf{x})$ is the position-dependent refractive index variation. A carrier frequency ω and a propagation coefficient $k=k_on_o$ in the direction of propagation are extracted from ψ, as $\psi=\Psi e^{-jkz}e^{j\omega t}+\Psi^*e^{jkz}e^{-j\omega t}$, where $k_o=\omega/c_o$, n_o is a reference refractive index, and * means the complex conjugate. Then (1) can be written in terms of Ψ as

$$r\partial_{zz}^2\Psi-2j\left(rk\partial_z\Psi+\frac{s}{c_o}k_o\partial_t\Psi\right)+\left(sk_o^2-rk^2\right)\Psi-\frac{s}{c_o^2}\partial_{tt}^2\Psi+\partial_x(r\partial_x\Psi)=0. \qquad (2)$$

The propagation of a compact pulse inside a limited time window will produce a condition that the pulse ultimately disappears after a certain number of propagation steps where it requires the computational window to be adjusted in time at each propagation step. The adjustment effectively moves at the group velocity of the pulse envelope. Hence, the substitution of a moving time coordinate $\tau=t-v_g^{-1}z$ with arbitrary v_g changes (2) to [3]

$$r\partial_{zz}^2\Psi-2j\left\{rk\partial_z\Psi+sk_o\left(\frac{1}{c_o}-\frac{1}{v_g}\right)\partial_\tau\Psi\right\}+\left(sk_o^2-rk^2\right)\Psi-\frac{s}{c_o^2}\partial_{\tau\tau}^2\Psi+\partial_x(r\partial_x\Psi)=0\ \ldots \qquad (3)$$

Equation (3) can be written in a product form as [6]

$$r\left\{\frac{\partial}{\partial z} - jk_o n_o[1-L]\right\}\left\{\frac{\partial}{\partial z} - jk_o n_o[1+L]\right\}\Psi = 0 \quad (4)$$

where the pseudodifferential operator is defined as shown in (5), at the bottom of the page. In this work, we concentrate on the forward propagation of Ψ and write the solution as $\Psi(z) = \exp(jk_o n_o(1-L)z)\,\Psi(0) = \exp\left(jk_o n_o(1-\sqrt{1+X})z\right)\Psi(0)$, where $\Psi(0)$ is the initial field. One of the most robust and well tested square root operator is the rational complex coefficient approximation based on the well-known Pade approximant given as [6], [7]

$$\sqrt{1+X} \approx \prod_{i=1}^{p} \frac{1+d_i^p X}{1+e_i^p X} \quad (6)$$

where d and e are called Pade coefficients with p being the Pade order. In problems involving scattering, it has been established that evanescent modes are not properly eliminated if real Pade coefficients are used where the associated error can be largely reduced by rotating the original real axis branch cut through an angle [7]. In this work, the finite-difference approach is used as a descritization scheme for both x and τ derivatives.

III. Results and Discussion

The propagation of a pulsed optical beam with a temporal Gaussian pulse of the form $\Psi(x, z = 0, \tau) = \Psi_0(x)\exp(-\tau^2/\sigma_\tau^2)$ is assumed as initial condition at $z = 0$ in all simulations, where σ_τ scales the duration of the initial pulse. In the numerical simulation, boundary conditions for the field are necessary in the transverse spatial variables and in the TD. At this stage, we have chosen simple zero spatial boundary conditions on the boundaries surrounding the structure. On the other hand, two simple methods of applying temporal boundary conditions can be used. The first method involves applying what is called the moving time window. The boundary conditions $\Psi = 0$ at $\tau = \pm M_\tau \Delta\tau$ are applied in the relative time window while moving in the absolute time frame with the velocity of the pulse, so that the relative motion of the pulse in the time window is cancelled. However, in some cases, the required group velocity v_g is not known prior to the simulation, and has to be generated dynamically from the propagation process. One can apply periodic boundary conditions at the ends of the relative time window, where a pulse leaves the window at one side and basically re-enters at the other side of the time window. Both of these techniques were tested numerically and proved to be very practical. In order to characterize the new technique, we apply it to model the behavior of known pulse propagation that has an approximate analytical formulation [8]. In the following simulation, a pulsed Gaussian beam is propagated in two dimensions (x and z) in a homogenous nondispersive medium, and the numerical results are compared with analytical results. With an initial spatial Gaussian waist w_o at $z = 0$, the evolution of each frequency component of the spectrum of the wavefunction in homogenous space is given in the frequency-domain as [3], [5], [8]

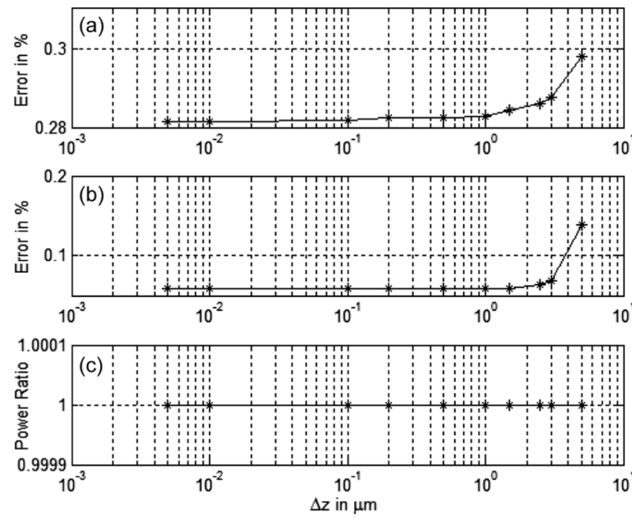

Fig. 1. Convergence of the technique with the longitudinal step size Δz. (a) The percentage maximum field error. (b) The percentage relative spatial waist error. (c) The power ratio of the pulsed beam.

$$\widetilde{\Psi}(x,z,\omega) = \Psi_o\sqrt{\frac{w_o}{w(z)}} \times \exp\left\{j\left[kz - \frac{\eta(z)}{2}\right] - x^2\left[\frac{1}{w^2(z)} - \frac{jk}{2R(z)}\right]\right\} \quad (7)$$

where the definition of the parameters in (7) are given in [8, eqs. (3)–(6)]. The evolution of a pulsed Gaussian beam in homogenous space can be found from (7) by taking the inverse Fourier transform of the product of $\widetilde{\Psi}$ and the Fourier transform of the initial pulse. Please see [8, eq. (9)] for more detail. The following results are for an initial spatial Gaussian waist $w_o = 2.5\ \mu$m, an initial pulsewidth of $\sigma_t = 50$ fs, the medium is assumed to be free space and $\lambda = 1.0\ \mu$m. The reference refractive index was chosen to be $n_o = 1.0$. The pulse is propagated to a distance of $Z = 30\ \mu$m. Fig. 1 shows the convergence of the technique with Δz when $\Delta x = 0.5\ \mu$m, $\Delta\tau = 0.5$ fs, and $p = 2$. The figure shows clearly that the method converges with a large longitudinal step size around $\Delta z = 1\ \mu$m. Also to be noticed from the same figure, the power part, that the operator is strongly unitary for the variation of Δz.

Fig. 2 shows the convergence of the technique with the spatial step size Δx when $\Delta\tau = 1.0$ fs, and $\Delta z = 0.1\ \mu$m and for two Pade order values of $p = 2$ and $p = 4$. Again the unitary of the operator is very clear in the figure with the change of Δx.

$$L = \sqrt{1+X} = \sqrt{\frac{1}{k_o^2 n_o^2 r}\left\{-2jsk_o\left(\frac{1}{c_o} - \frac{1}{v_g}\right)\partial_\tau + (sk_o^2 - rk^2) - \frac{s}{c_o^2}\partial_{\tau\tau}^2 + \partial_x(r\partial_x)\right\}} \quad (5)$$

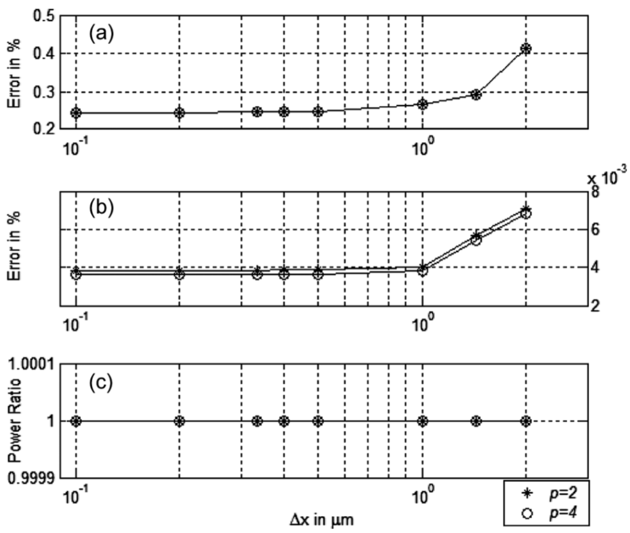

Fig. 2. Convergence of the technique with the spatial step size Δx. (a) The percentage maximum field error. (b) The percentage relative time waist error. (c) The power ratio of the pulsed beams.

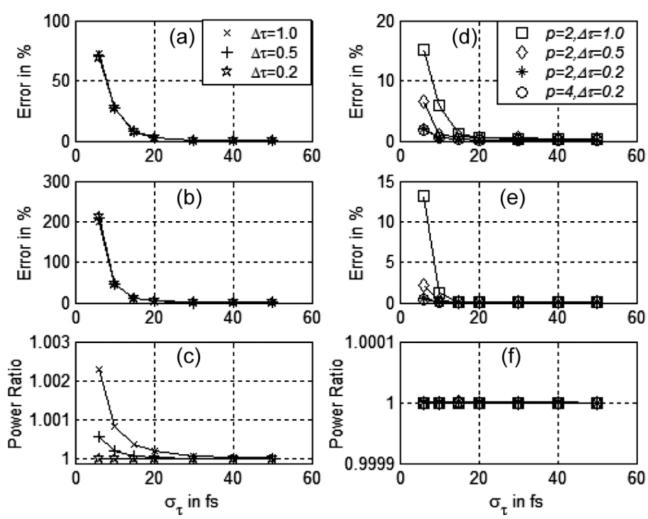

Fig. 3. Comparison between the new technique and the parabolic TD , [3] for different initial ultrashort pulse propagation. Left side figures [(a), (b), and (c)] for the parabolic method and right side figures [(d), (e), and (f)] are for the present technique. (a) and (d) The percentage maximum field error. (b) and (e) The percentage relative time waist error. (c) and (f) The power ratio of the pulsed beams.

Fig. 2 also shows that the difference between the two Pade order results is small.

Fig. 3 shows a comparison between the parabolic TD method [3] and the present technique for ultrashort initial pulsewidths. The results are for $\Delta x = 0.5$ μm and $\Delta z = 0.1$ μm, where Δz for the parabolic method is always adjusted between 0.1 and 0.01 μm to satisfy the stability condition. A comparison between the left-side figures that belong to the parabolic and the right-side figures that belong to the present technique shows the merit of using the present technique for modeling ultrashort optical pulses. The figure shows the convergence of the present technique with the decrease of $\Delta\tau$. Conversely, the error of the parabolic technique is not affected by the reduction of $\Delta\tau$ because the error here is mainly associated to the paraxial approximation; therefore, it is limited from modeling ultrashort pulses. It should be noted that in modeling ultrashort optical pulses, small time step sizes are needed to represent the rapid variation of the pulse envelope as seen by the results of the present technique.

Finally, the new TD-BPM technique showed that it has robust stability for the variation of numerical parameters with convergence values around a hundred times the values of the explicit FD. The present technique takes around 0.6 s/step when $p = 2$ and 60×150 mesh points of spatial and time discretizations respectively, when running on an ordinary laptop computer of 2.1-GHz-speed processor.

IV. Conclusion

A new stable technique to model ultrashort optical pulses has been proposed and tested. The technique is an extension to the TD-BPM, which solves the TD wave equation by marching the field along one direction. The technique uses the Pade approximant to account for fast envelope pulse propagational variations. The numerical parameters of the technique have been examined and it was shown that the method is efficient, very stable, and accurate in modeling ultrashort optical pulses.

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