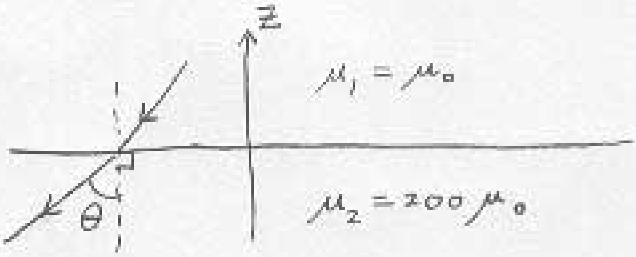
EE340 HW #9 Solution

3rd Edition



8.26

$$\vec{H}_{1t} = \vec{H}_{2t} = 10\vec{a}_x + 15\vec{a}_y$$

$$\vec{B}_{1n} = \vec{B}_{2n} = \left(-3\vec{a}_z\right) \times \mu_0$$

$$\vec{B}_{2t} = 200\mu_0 \vec{H}_{2t} = 200\mu_0\left(10\vec{a}_x + 15\vec{a}_y\right)$$

$$\therefore \vec{B}_2 = 2000\mu_0\vec{a}_x + 3000\mu_0\vec{a}_y - 3\mu_0\vec{a}_z$$

$$\tan\theta = \frac{B_{2t}}{B_{2n}} = \frac{\sqrt{(2000\mu_0)^2 + (3000\mu_0)^2}}{3\mu_0} = 1201.85$$

$\therefore \theta = 89.95°$ (i.e. $\vec{B}$ in iron is parallel to the boundary).

8.28 $\mu_r = 1 + \chi_m$ (eqn. 8.37).

$= 20$

$$W_m = \frac{1}{2}\mu H^2 = \frac{1}{2}(20\mu_0)\left[25x^4y^2z^2 + 100x^2y^4z^2 + 225x^2y^2z^4\right]$$

$$W_m = \int_{x=0}^{1}\int_{y=0}^{2}\int_{z=-1}^{2} 10\mu_0(25)\left(x^4y^2z^2 + 4x^2y^4z^2 + 9x^2y^2z^4\right)dx\,dy\,dz$$

$$= 250\mu_0\left[\left(\frac{1}{5}\right)\left(\frac{8}{3}\right)\left(\frac{z^3}{3}\Big|_{-1}^{2}\right) + 4\left(\frac{1}{3}\right)\left(\frac{32}{5}\right)\left(\frac{z^3}{3}\Big|_{-1}^{2}\right) + 9\left(\frac{1}{3}\right)\left(\frac{8}{3}\right)\left(\frac{z^5}{5}\Big|_{-1}^{2}\right)\right]$$

$$= 250\mu_0\left[\frac{8}{5} + \frac{128}{5} + \frac{264}{5}\right] = 20000\mu_0 = 25.1\ mJ$$

8.30

a) $H\,2\pi\rho = NI \Rightarrow \vec{H} = \dfrac{NI}{2\pi\rho}\,\vec{a}_\phi$

$$\vec{B} = \frac{\mu_0 NI}{2\pi\rho}\,\vec{a}_\phi$$

$$\psi = \int \vec{B}\cdot\vec{dS} = \int_{\rho=\rho_0-a/2}^{\rho_0+a/2}\int_{z=0}^{a} \frac{\mu_0 NI}{2\pi\rho}\,d\rho\,dz$$

$$= \frac{\mu_0 NIa}{2\pi}\,\ln\rho\,\Big|_{\rho_0-a/2}^{\rho_0+a/2} = \frac{\mu_0 NIa}{2\pi}\,\ln\frac{\rho_0+a/2}{\rho_0-a/2}$$

$$\Lambda = N\psi = \frac{\mu_0 N^2 Ia}{2\pi}\,\ln\frac{\rho_0+a/2}{\rho_0-a/2}$$

$$L = \frac{\Lambda}{I} = \frac{\mu_0 N^2 a}{2\pi}\,\ln\frac{2\rho_0+a}{2\rho_0-a}$$

b) If $\rho_0 >> a \Rightarrow \vec{B}$ over the cross-sectional area of the toroid can be assumed to be uniform.

$$\therefore\ \psi \simeq BS = \frac{\mu_0 NI}{2\pi\rho_0}(\pi a^2)$$

$$\Lambda = N\psi = \frac{\mu_0 N^2 I\,\pi a^2}{2\pi\rho_0}$$

$$L = \frac{\Lambda}{I} = \frac{\mu_0 N^2 a^2}{2\rho_0}$$

8.33

Assume the toroid has a square area. Then

$a^2 = 12\ cm^2 \implies a = \sqrt{12} = 3.46\ cm.$

Then $\rho_0 >> a$

$$\therefore \Psi = BS = \underbrace{\left(200\mu_0 \frac{NI}{2\pi\rho_0}\right)}_{B} \underbrace{(12\ cm^2)}_{S}$$

$$L = \frac{\Lambda}{I} = \frac{200\mu_0 N^2}{2\pi\rho_0}(12\ cm^2)$$

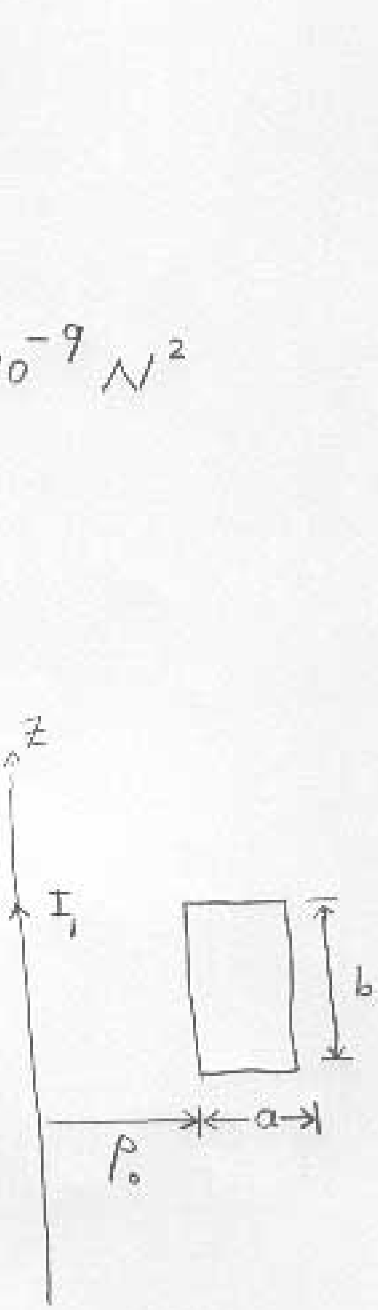
$$= \frac{200(4\pi 10^{-7})N^2(12\times 10^{-4})}{2\pi(50\times 10^{-2})} = 96\times 10^{-9} N^2$$

$$= 2.5$$

$$N^2 = \frac{2.5}{96\times 10^{-9}} = 26.042\times 10^6$$

$N = 5103$ turns

8.34



$$\vec{B}_1 = \frac{\mu I_1}{2\pi\rho}\vec{a}_\phi$$

$$\Psi_{21} = \int \vec{B}_1 \cdot \vec{ds} = \int_{\rho=\rho_0}^{(\rho_0+a)}\int_{z=0}^{b} \frac{\mu I_1}{2\pi\rho}(d\rho\, dz)$$

$$\therefore \Lambda_{21} = \Psi_{21} = \frac{\mu I_1 b}{2\pi} \ln\frac{\rho_0+a}{\rho_0}$$

$$\therefore M_{21} = M_{12} = \frac{\mu b}{2\pi}\ln\left(\frac{\rho_0+a}{\rho_0}\right)$$

for $a = b = \rho_0 = 1m \implies M_{12} = \frac{\mu_0}{2\pi}\ln 2 = 0.139[\mu H].$