6.11

$V(z) = c_1 z + c_2$, $V(0) = 0 = c_2$ $\therefore$ $V(z) = c_1 z$

$V(2\text{mm}) = 50 = c_1 \times 2\times 10^{-3} \Rightarrow c_1 = 25000$

$\therefore$ $\boxed{V(z) = 25000 z}$ Volts

$$\boxed{\vec{E} = -\nabla V = -25000\,\vec{a}_z \ \ \text{V/m}}$$

$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$, $\therefore$ $\vec{D} = 1.5\epsilon_0(-25000\,\vec{a}_z)$

$$\therefore \boxed{\vec{D} = -331.6\,\vec{a}_z \ \frac{nC}{m^2}}$$

$\boxed{\rho_s = D_n = 331.6 \frac{nC}{m^2}}$ (Positive on the upper plate).

6.12

$$\nabla^2 V = 0 \Rightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \cancelto{0}{\frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2}} + \cancelto{0}{\frac{\partial^2 V}{\partial z^2}} = 0$$

$\therefore \rho\frac{dV}{d\rho} = c_1 \Rightarrow V = c_1 \ln\rho + c_2$

$V(5\times10^{-3}) = 100 = c_1 \ln 0.005 + c_2$ ①

$V(1.5\times10^{-3}) = 0 = c_1 \ln 0.015 + c_2$ ②

From ① & ② $\Rightarrow 100 = c_1[\ln 0.005 - \ln 0.015]$

$\therefore 100 = -c_1 \ln 3 \Rightarrow c_1 = -91.02$

$\therefore c_2 = -c_1 \ln 0.015 \Rightarrow c_2 = -382.26$

$$\therefore \boxed{V(\rho) = -91.02 \ln\rho - 382.26} \ (V)$$

$$\vec{E} = -\nabla V = -\frac{dV}{d\rho}\,\vec{a}_\rho$$

$$\therefore \boxed{\vec{E} = \frac{91.02}{\rho}\,\vec{a}_\rho} \;(\mathrm{V/m})$$

$$\therefore \boxed{\vec{D} = 2\epsilon_0 \vec{E} = \frac{1.61}{\rho}\,\vec{a}_\rho \left(\frac{nC}{m^2}\right)}$$

$$V(10\,mm) = 36.9\ V$$

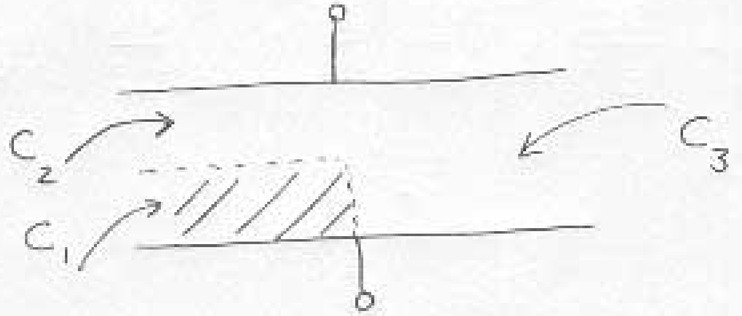
$$\vec{E}(10\,mm) = 9.102\,\vec{a}_\rho\ \mathrm{KV/m}$$

$$\vec{D}(10\,mm) = 161\,\vec{a}_\rho\ \frac{nC}{m^2}$$

$$\rho_s = -\frac{1.61}{0.015}\,\frac{nC}{m^2} = -107.33\,\frac{nC}{m^2} \quad \text{(outer conductor).}$$

$$\rho_s = +\frac{1.61}{0.005}\,\frac{nC}{m^2} = +322\,\frac{nC}{m^2} \quad \text{(inner conductor).}$$

6.28



$$C = \frac{\epsilon S}{d}$$

$$C_1 = \frac{6\epsilon_0 (5\times 10^{-4} m^2)}{10^{-3} m} = 26.53\ \mathrm{pf}$$

$$C_2 = C_1/6 = 4.42\ \mathrm{pf}$$

$$C_3 = C_2/2 = 2.21\ \mathrm{pf}$$ (because the plate separation of C_3 is twice that of C_2).

$$C = \frac{C_1 C_2}{C_1 + C_2} + C_3 = 3.789 + 2.21 = 6\ \mathrm{pf}$$

$$\therefore \boxed{C = 6\,\mathrm{pf}}$$

6.30

a) $C = \frac{\epsilon_0 S}{d} = \frac{\epsilon_0 (200 \times 10^{-4})}{3 \times 10^{-3}}$

$\boxed{C = 58.95 \text{ pf}}$

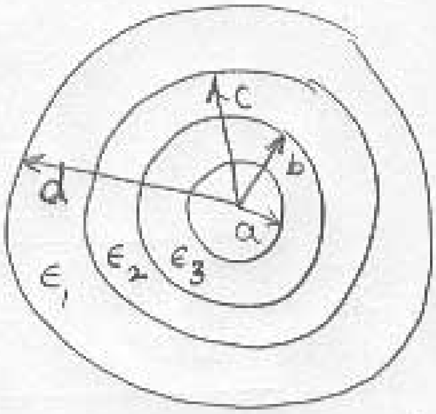
b) $V = \frac{Q}{C} = \frac{10^{-6} (200 \times 10^{-4})}{58.95 \times 10^{-12}} = \frac{2 \times 10^{-8}}{58.95 \times 10^{-12}}$

$\boxed{V = 0.339 \text{ KV}}$

c) $F = QE = Q\left(\frac{V}{d}\right) = \frac{2 \times 10^{-8} (16.96 \times 10^{3})}{3 \times 10^{-3}}$

$\boxed{F = 113 \text{ mN}}$

6.35



Note error in this problem.

$C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}$ eqn. 6.32

$\therefore C_3 = \frac{4\pi\epsilon_3}{\frac{1}{a} - \frac{1}{b}}$

$C_2 = \frac{4\pi\epsilon_2}{\frac{1}{b} - \frac{1}{c}}$

$C_1 = \frac{4\pi\epsilon_1}{\frac{1}{c} - \frac{1}{d}}$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ [Three capacitors in series].