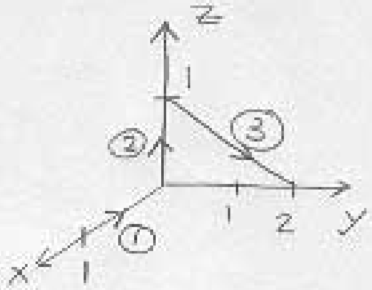


3.8

$$\int \vec{H} \cdot d\vec{l} = \int_{①} + \int_{②} + \int_{③}$$

$$\int_{①} \vec{H} \cdot d\vec{l} = \int_{x=1}^{0} (x-y)\,dx \Big|_{y=0} = \frac{x^2}{2}\Big|_1^0 = -1/2$$

$$\int_{②} \vec{H} \cdot d\vec{l} = \int_{z=0}^{1} 5yz\,dz \Big|_{y=0} = 0$$

$$\int_{③} \vec{H} \cdot d\vec{l} = \int (x^2+yz)\,dy + 5yz\,dz$$

$$= \int_{y=0}^{2} (x^2+yz)\,dy \Big|_{x=0,\, z=-\frac{1}{2}y+1} + \int_{z=1}^{0} 5yz\,dz \Big|_{y=-2z+2}$$

$$= \int_{y=0}^{2} y(-\tfrac{1}{2}y+1)\,dy + \int_{z=1}^{0} 10(1-z)\,z\,dz$$

$$= \left(-\frac{1}{2}\frac{y^3}{3} + \frac{y^2}{2}\right)\Big|_0^2 + \left(10\frac{z^2}{2} - \frac{10z^3}{3}\right)\Big|_1^0$$

$$= \left(-\tfrac{4}{3}+2\right) - \left(\tfrac{10}{2} - \tfrac{10}{3}\right) = -1$$

Then add the three parts. $= -\frac{1}{2} + 0 - 1 = -\frac{3}{2}$

3.12

a) $\nabla U = 4z^2 \vec{a}_x + 3z\vec{a}_y + (8xz + 3y)\vec{a}_z$

b) $\nabla W = (z^2+1)\cos\phi\, \vec{a}_\rho - 2(z^2+1)\sin\phi\, \vec{a}_\phi + 4\rho z\cos\phi\, \vec{a}_z$

c) $\nabla H = 2r\cos\theta\cos\phi\, \vec{a}_r - 2r\sin\theta\cos\phi\, \vec{a}_\theta - r\cos\theta\, \vec{a}_\phi$

3.16

a) $\nabla \vec{A} = ye^{xy} + x\cos xy - 2x\cos xz\sin xz$

$$\nabla\times\vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{xy} & \sin xy & \cos^2 xz \end{vmatrix}$$

$$= \vec{a}_x(0-0) + \vec{a}_y(0 + 2z\cos xz\sin xz) + \vec{a}_z(y\cos xy - xe^{xy})$$

b) $\nabla \vec{B} = \frac{1}{\rho}(3\rho^2 z) + \frac{1}{\rho}(0) + 6\rho z = 3\rho z + 6\rho z = 9\rho z$

$$\nabla\times\vec{B} = [0-0]\,\vec{a}_\rho + [\rho^2 - 3z^2]\,\vec{a}_\phi + \frac{1}{\rho}[4\rho^3 - 0]\,\vec{a}_z$$

$$= (\rho^2 - 3z^2)\vec{a}_\phi + 4\rho^2\vec{a}_z$$

c) $\nabla \vec{C} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^3\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(-\frac{1}{r}\sin^2\theta\right) + 0$

$$= 3\cos\theta - \frac{2}{r^2}\cos\theta$$

$$\nabla \times \vec{C} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(2r^2\sin^2\theta\right) - 0\right]\vec{a}_r$$

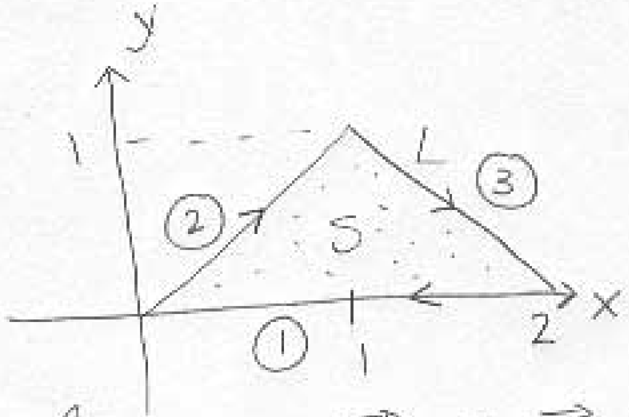
$$+ \frac{1}{r}\left[0 - \frac{\partial}{\partial r}\left(2r^3\sin\theta\right)\right]\vec{a}_\theta$$

$$+ \frac{1}{r}\left[0 + r\sin\theta\right]\vec{a}_\phi$$

$$= 4r\cos\theta\,\vec{a}_r - 6r\sin\theta\,\vec{a}_\theta + \sin\theta\,\vec{a}_\phi$$

3.17 Use the same procedure as **3.16**.

3.31



a) $\oint_L \vec{F}\cdot d\vec{l} = \int_{①} + \int_{②} + \int_{③}$, $\vec{F} = x^2y\,\vec{a}_x - y\,\vec{a}_y$

$$\int_{①} = \int_{x=2}^{0} x^2y\,dx\Big|_{y=0} = 0$$

$$\int_{②}(x^2y\,dx - y\,dy) = \int_{x=0}^{1} x^2y\,dx\Big|_{x=y} - \int_{y=0}^{1} y\,dy = \frac{x^4}{4}\Big|_0^1 - \frac{y^2}{2}\Big|_0^1$$

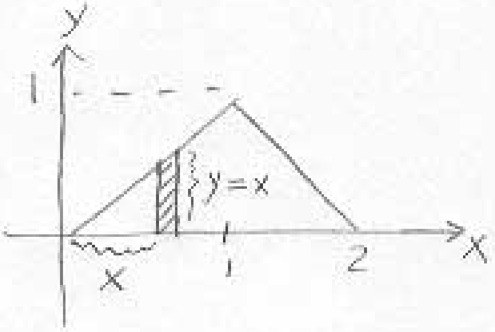
$$= \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

$$\int_{③} = \int (x^2 y\,dx - y\,dy) = \int_{\substack{x=1 \\ y=-x+2}}^{2} x^2 y\,dx - \int_{y=1}^{0} y\,dy$$

$$= \int_{x=1}^{2} (-x^3 + 2x^2)\,dx - \int_{y=1}^{0} y\,dy = -\frac{x^4}{4} + \frac{2x^3}{3}\Big|_1^2 - \frac{y^2}{2}\Big|_1^0$$

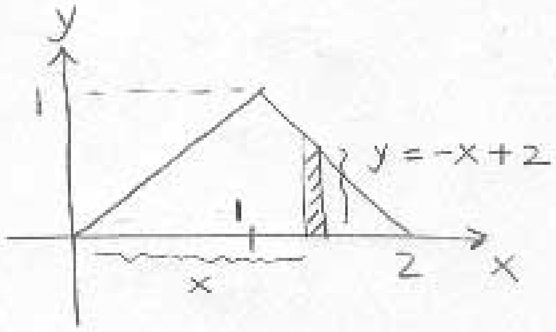
$$= \left(-\frac{16}{4} + \frac{16}{3}\right) - \left(-\frac{1}{4} + \frac{2}{3}\right) + \frac{1}{2} = \frac{14}{3} - \frac{13}{4}$$

$$\therefore \oint_L \vec{F} \cdot \vec{dl} = 0 - \frac{1}{4} + \frac{14}{3} - \frac{13}{4} = \frac{14}{3} - \frac{14}{4}$$



b) $\nabla \times \vec{F} = -x^2 \vec{a}_z$

$$\int (\nabla \times \vec{F}) \cdot \vec{ds} = \int_{①} + \int_{②}$$



① Left half of area.
② right " " ".

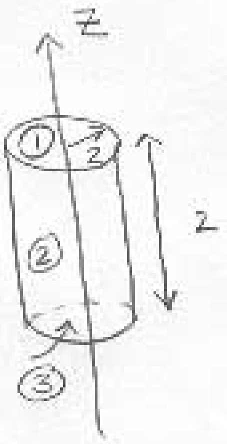
$$\int_{①} = \int_{x=0}^{1} \int_{y=0}^{x} +x^2\,dx\,dy = \int_{x=0}^{1} +x^2 y\Big|_{y=0}^{x} dx = \int_0^1 +x^3\,dx$$

$$= +\frac{x^4}{4}\Big|_0^1 = +\frac{1}{4}$$

$$\int_{②} = \int_{x=1}^{2} \int_{y=0}^{2-x} +x^2\,dx\,dy = \int_{x=1}^{2} +x^2(2-x)\,dx = \frac{14}{3} - \frac{15}{4}$$

$\therefore \int (\nabla \times \vec{F}) \cdot \vec{ds} = \frac{1}{4} + \frac{14}{3} - \frac{15}{4} = \frac{14}{3} - \frac{14}{4}$

c) Stock's Thm. is satisfied.



3.33 $\oint \vec{F} \cdot \vec{ds} = \int_{①} + \int_{②} + \int_{③}$

$$\int_{①} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{2} (z^2-1)\,\rho\, d\rho\, d\phi \Big|_{z=2}$$

$$= 3\left(\frac{\rho^2}{2}\Big|_0^2\right)(2\pi) = 12\pi$$

$$\int_{③} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{2} -(z^2-1)\,\rho\, d\rho\, d\phi \Big|_{z=0} = \left(\frac{\rho^2}{2}\Big|_0^2\right)(2\pi) = 4\pi$$

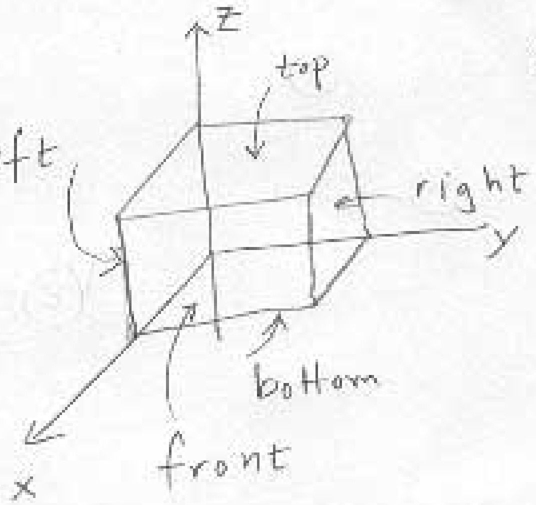
$$\int_{②} = \int \left[x^2\vec{a}_x + y^2\vec{a}_y\right] \cdot \rho\, d\phi\, dz\, \vec{a}_\rho \Big|_{\rho=2}$$

$$= \int 2x^2(\vec{a}_x \cdot \vec{a}_\rho)\, d\phi\, dz + \int 2y^2(\vec{a}_y \cdot \vec{a}_\rho)\, d\phi\, dz$$

$$= \int_{z=0}^{2}\int_{\phi=0}^{2\pi} 2(2\cos\phi)^2(\cos\phi)\, d\phi\, dz + \int_{z=0}^{2}\int_{\phi=0}^{2\pi} 2(2\sin\phi)^2(+\sin\phi)\, d\phi\, dz$$

$$= 8(2)\int_{\phi=0}^{2\pi} \cos^3\phi\, d\phi + 8(2)\int_{\phi=0}^{2\pi} \sin^3\phi\, d\phi = 0+0=0$$

3.35



$$\oint \vec{A}\cdot\vec{ds} = \int_{\text{front}} + \int_{\text{back}} + \int_{\text{left}} + \int_{\text{right}} + \int_{\text{top}} + \int_{\text{bottom}}$$

$$\int_{\text{front}} = \int x^2y^2\,dy\,dz\Big|_{x=1} = \int_{y=0}^{1}\int_{z=0}^{1} y^2\,dy\,dz = \frac{1}{3}$$

$$\int_{\text{back}} = -\int x^2y^2\,dy\,dz\Big|_{x=0} = 0$$

$$\int_{\text{left}} = \int -y^3\,dx\,dz\Big|_{y=0} = 0$$

$$\int_{\text{right}} = \int_{z=0}^{1}\int_{x=0}^{1} +y^3\,dx\,dz\Big|_{y=1} = 1$$

$$\int_{\text{top}} = \int_{y=0}^{1}\int_{x=0}^{1} y^2z\,dx\,dy\Big|_{z=1} = \frac{1}{3}$$

$$\int_{\text{bottom}} = -\int_{y=0}^{1}\int_{x=0}^{1} x^2z\,dx\,dy\Big|_{z=0} = 0$$

$\therefore \oint \vec{A} \cdot \vec{ds} = \frac{1}{3} + \frac{1}{3} + 1 = \frac{5}{3}$

$$\int (\nabla \cdot \vec{A})\, dV = \int (y^2 + 3y^2 + y^2)\, dx\, dy\, dz$$

$$= \int_{x=0}^{1} \int_{y=0}^{1} \int_{z=0}^{1} 5y^2\, dx\, dy\, dz$$

$$= \left(\frac{5y^3}{3} \Big|_0^1 \right)(1)(1) = \frac{5}{3}$$

$\therefore \oint \vec{A} \cdot \vec{ds} = \int (\nabla \vec{A})\, dv$