# EE 340 HW #14 Solution
## 3rd Edition

**10.40**

$$\sqrt{1}\ \sin 45^\circ = \sqrt{4.5}\ \sin\theta_{t1} \Longrightarrow \theta_{t1} = 28.13^\circ$$

$$\sqrt{4.5}\ \sin 28.13^\circ = \sqrt{2.25}\ \sin\theta_{t2} \Longrightarrow \theta_{t2} = 41.82^\circ$$

**10.42**

$$\Gamma_{\parallel} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}, \quad \text{for } \mu = \mu_0,\ \mu_r = 1$$

$$\therefore \Gamma_{\parallel} = \left(\frac{1}{\sqrt{\epsilon_{2r}}}\cos\theta_t - \frac{1}{\sqrt{\epsilon_{1r}}}\cos\theta_i\right) \Big/ \left(\frac{1}{\sqrt{\epsilon_{2r}}}\cos\theta_t + \frac{1}{\sqrt{\epsilon_{1r}}}\cos\theta_i\right)$$

$$\because \sqrt{\epsilon_{2r}}\ \sin\theta_t = \sqrt{\epsilon_{1r}}\ \sin\theta_i \quad (\text{Snell's law}).$$

$$\therefore \Gamma_{\parallel} = \frac{\frac{\sqrt{\epsilon_{1r}}}{\sqrt{\epsilon_{2r}}}\cos\theta_t - \cos\theta_i}{\frac{\sqrt{\epsilon_{1r}}}{\sqrt{\epsilon_{2r}}}\cos\theta_t + \cos\theta_i} = \frac{\frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t - \cos\theta_i}{\frac{\sin\theta_t}{\sin\theta_i}\cos\theta_t + \cos\theta_i}$$

$$= \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i} = \frac{\sin(\theta_t - \theta_i)\cos(\theta_t + \theta_i)}{\sin(\theta_t + \theta_i)\cos(\theta_t - \theta_i)}$$

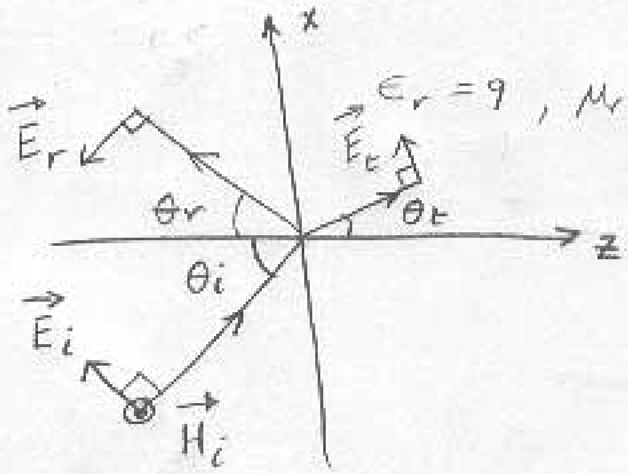
$$= \frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}$$

The other expressions can be derived in a similar way.

**10.44**

a) $k_z = \sqrt{8}\, k = \beta_1 \cos\theta_i$

$k_x = k = \beta_1 \sin\theta_i$




$\therefore \tan\theta_i = \frac{1}{\sqrt{8}} \Rightarrow \theta_i = 19.47^\circ$

$\therefore \theta_r = \theta_i = 19.47^\circ$

$\sqrt{1}\ \sin 19.47^\circ = \sqrt{9}\ \sin\theta_t \Rightarrow \theta_t = 6.38^\circ$

b) $k = \beta_1 \sin\theta_i = \frac{\omega}{c}\sqrt{1}\ \sin\theta_i = \frac{10^9}{3\times10^8} \sin 19.47^\circ = 1.11$ [rad/m]

c) $\beta_2 = \frac{\omega}{c}\sqrt{\mu_{r2}\epsilon_{r2}} = \frac{10^9}{3\times10^8}\sqrt{9} = 10$ [rad/m]

$\lambda_2 = \frac{2\pi}{\beta_2} = 0.2\pi$ [m]

$\beta_1 = \frac{\omega}{c}\sqrt{\mu_{r1}\epsilon_{r1}} = \frac{\omega}{c} = \frac{10^9}{3\times10^8} = \frac{10}{3}$ [rad/m]

$\lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{10/3} = \frac{6\pi}{10} = 0.6\pi$ [m]

d) $\vec{E}_i = (\cos\theta_i \vec{a}_x - \sin\theta_i \vec{a}_z)(0.2\eta_0)\cos(10^9 t - kx - k\sqrt{8}z)$

$= (0.943\vec{a}_x - 0.333\vec{a}_z)\, 75.4 \cos(10^9 t - 1.11x - 3.14z)$

$= (71.1\vec{a}_x - 25.11\vec{a}_z)\cos(10^9 t - 1.11x - 3.14z)$ [V/m]

e) This is a parallel incident case.

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = \frac{\frac{1}{3}\cos 6.38° - \cos 19.47°}{\frac{1}{3}\cos 6.38° - \cos 19.47°}$$

$$= \frac{0.331 - 0.943}{0.331 + 0.943} = -0.48$$

Notice $\tau_{\parallel} \neq 1 + \Gamma_{\parallel}$ (but $\tau_{\perp} = 1 + \Gamma_{\perp}$)

$$\tau_{\parallel} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} \quad \text{or} \quad 1 + \Gamma_{\parallel} = \tau_{\parallel}\left(\frac{\cos\theta_t}{\cos\theta_i}\right)$$

$$\tau_{\parallel} = \frac{\frac{2}{3}\cos 19.47°}{\frac{1}{3}\cos 6.38° + \cos 19.47°} = 0.493$$

$$\therefore \vec{E}_t = (\cos\theta_t \vec{a}_x - \sin\theta_t \vec{a}_z)(75.4)(0.493)\cos(10^9 t - \beta_2 \sin\theta_t x - \beta_2 \cos\theta_t z)$$

$$= (0.994\vec{a}_x - 0.111\vec{a}_z)\, 37.17 \cos(10^9 t - 1.11x - 9.94z)$$

$$= (36.95\vec{a}_x - 4.13\vec{a}_z)\cos(10^9 t - 1.11x - 9.94z) \quad [V/m]$$

$$\vec{E}_r = (-\cos\theta_r \vec{a}_x - \sin\theta_r \vec{a}_z)(75.4)(-0.48)\cos(10^9 t - \beta_1 \sin\theta_r x + \beta_1 \cos\theta_r z)$$

$$= (-0.943\vec{a}_x - 0.333\vec{a}_z)(-36.192)\cos(10^9 t - 1.11x + 3.14z)$$

$$= (34.129\vec{a}_x + 12.052\vec{a}_z)\cos(10^9 t - 1.11x + 3.14z) \quad [V/m]$$

f) $\theta_B = \tan^{-1}\left(\frac{3}{1}\right) = 71.57°$