10.26

a) $|\vec{E}_s| = \sqrt{25+144}\; e^{-\alpha z} = 13\, e^{-0.2(4)} = 5.84\,[\text{V/m}]$.

b) The amplitude is reduced by a factor $e^{-0.2(3)}$
$= 0.5488$

The dB loss $= 20\log_{10}(0.5488) = -5.21$ dB

c) $\vec{E} = (5\vec{a}_x + 12\vec{a}_y)\, e^{-0.2z} \cos(10^8 t - j\,3.4z)$

$$\alpha = 0.2 = \frac{\omega}{c}\sqrt{\frac{\mu_r \epsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]}$$

$$\beta = 3.4 = \frac{\omega}{c}\sqrt{\frac{\mu_r \epsilon_r}{c}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]}$$

$$\therefore \frac{0.2}{3.4} = \frac{\sqrt{\sqrt{1+(\sigma/\omega\epsilon)^2} - 1}}{\sqrt{\sqrt{1+(\sigma/\omega\epsilon)^2} + 1}} = 0.0588$$

$$\frac{\sqrt{1+(\sigma/\omega\epsilon)^2} - 1}{\sqrt{1+(\sigma/\omega\epsilon)^2} + 1} \doteq 3.460\times 10^{-3} = \frac{x-1}{x+1} \Rightarrow x = 1.00694$$

since medium is nonmagnetic $\Rightarrow \mu_r = 1$

$$\therefore \alpha = 0.2 = \frac{10^8}{3\times 10^8}\sqrt{\frac{(1)(\epsilon_r)}{2}\left[1.00694 - 1\right]} = \frac{1}{3}\sqrt{\frac{\epsilon_r}{2}(0.00694)}$$

$$\therefore \epsilon_r = 2(0.6)^2/0.00694 = 103.75$$

$$\eta = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r\left(1 - j\frac{\sigma}{\omega\epsilon}\right)}} = 377\sqrt{\frac{1}{103.75}}\; \frac{e^{\frac{1}{2}j\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}}{\sqrt{\sqrt{1+(\sigma/\omega\epsilon)^2}}}$$

$$x = \sqrt{1+(\sigma/\omega\epsilon)^2} = 1.00694$$

$$\frac{\sigma}{\omega\epsilon} = 0.118$$

$$\therefore \eta = \frac{377}{\sqrt{103.75}} \frac{e^{\frac{1}{2} j \tan^{-1}(0.118)}}{\sqrt{1.00694}} = 36.88 \angle 3.365^\circ \ \Omega$$

$$\vec{H} = \frac{1}{36.88}\left(5\vec{a}_y - 12\vec{a}_x\right) e^{-0.2z} \cos(10^8 t - 3.4z - 3.365^\circ)$$

$$\vec{\mathcal{P}} = \vec{E} \times \vec{H} = \frac{25\vec{a}_z + 144\vec{a}_z}{36.88} e^{-0.4z} \cos(10^8 t - 3.4z)$$
$$\cos(10^8 t - 3.4z - 3.365^\circ)$$

$$= \vec{a}_z \ 4.58 \ e^{-0.4z} \cos(10^8 t - 3.4z) \cos(10^8 t - 3.4z - 3.365^\circ)$$

$$\vec{\mathcal{P}}(z=4, t=T/8) = \vec{a}_z \ 4.58 \ e^{-1.6} \cos\left(\frac{\pi}{2} - 3.4\times 4\right)$$
$$\cos\left(\frac{\pi}{2} - 3.4\times 4 - 3.365^\circ\right)$$

$$= \vec{a}_z \ 4.58 \ e^{-1.6} (0.859)(0.888)$$

$$= \vec{a}_z \ 0.705 \quad [W/m^2]$$

10.27

a) Medium is lossless.

$$u = \frac{\omega}{\beta} = \frac{c}{\sqrt{\mu_r \epsilon_r}} \implies \frac{2\pi \times 10^8}{6} = \frac{3\times 10^8}{\sqrt{\epsilon_r}}$$

$$\sqrt{\epsilon_r} = 18/2\pi = 2.865$$

$$\eta = 377\sqrt{\frac{1}{\epsilon_r}} = \frac{377}{2.865} = 131.59\,\Omega$$

b) $$\vec{E} = -\vec{a}_z\ 30\,(131.59)\cos(2\pi\times10^8 t - 6x)\ [\mathrm{mV/m}]$$

$$\vec{\mathcal{P}} = \vec{E}\times\vec{H} = \vec{a}_x\ 118.43\times10^{-3}\cos^2(2\pi\times10^8 t - 6x)$$

$[\mathrm{W/m^2}]$

c) $$\vec{\mathcal{P}}_{avg} = \vec{a}_x\ \frac{118.43\times10^{-3}}{2}$$

$$= 59.22\ \vec{a}_x\ \left[\frac{\mathrm{mW}}{\mathrm{m}^2}\right]$$

$$P_{avg} = (59.22\times10^{-3})(2)(3) = 355.32\,[\mathrm{mW}]$$

10.31

a) $\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{\sqrt{\dfrac{\mu_{2r}}{\epsilon_{2r}}} - \sqrt{\dfrac{\mu_{1r}}{\epsilon_{1r}}}}{\sqrt{\dfrac{\mu_{2r}}{\epsilon_{2r}}} + \sqrt{\dfrac{\mu_{1r}}{\epsilon_{1r}}}} = \dfrac{\sqrt{\dfrac{1}{4}} - \sqrt{\dfrac{1}{1}}}{\sqrt{\dfrac{1}{4}} + \sqrt{\dfrac{1}{1}}}$

$= \dfrac{\frac{1}{2} - 1}{\frac{1}{2} + 1} = \dfrac{-\frac{1}{2}}{\frac{3}{2}} = -\dfrac{1}{3}$

$\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1} = \dfrac{2\sqrt{\frac{1}{4}}}{\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{1}}} = \dfrac{2}{3}$

$S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{4/3}{2/3} = 2$

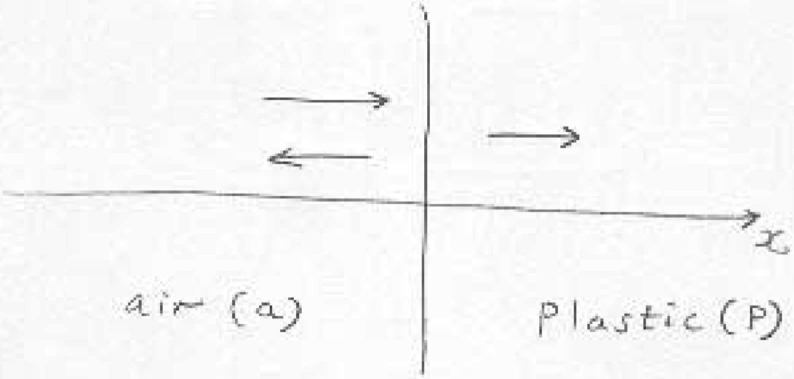
b) $\vec{E}_r = -10 \cos(\omega t + z)\, \vec{a}_x \;\; [V/m]$

$\vec{H}_r = -\dfrac{10}{377} \cos(\omega t + z)(-\vec{a}_y) \;\; [A/m]$

$= 0.027 \cos(\omega t + z)\, \vec{a}_y \;\; [A/m]$

10.32

a)



$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{1}{4}} - \sqrt{\frac{1}{1}}}{\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{1}}} = \frac{-1}{3}$$

$$\left(\vec{E}_a\right)_{\text{incident}} = -\vec{a}_z\ 4 \times 377 \sin(\omega t - 5x) \quad [V/m]$$

$$= -\vec{a}_z\ 1508 \sin(\omega t - 5x) \quad [V/m]$$

$$\left(E_a\right)_{\text{reflected}} = +\vec{a}_z\ \frac{1508}{3} \sin(\omega t + 5x) \quad [V/m]$$

$$\left(E_a\right)_{\text{total}} = \left(\vec{E}_a\right)_{\text{inc.}} + \left(\vec{E}_a\right)_{\text{ref}}$$

$$= -\vec{a}_z\ 1508 \left[\sin(\omega t - 5x) - \frac{1}{3}\sin(\omega t + 5x)\right]$$

b) $\tau = 1 + \Gamma = 1 - \frac{1}{3} = 2/3$

$$\vec{E}_p = -\vec{a}_z\ 1508\left(2/3\right) \sin(\omega t - \beta_p x)$$

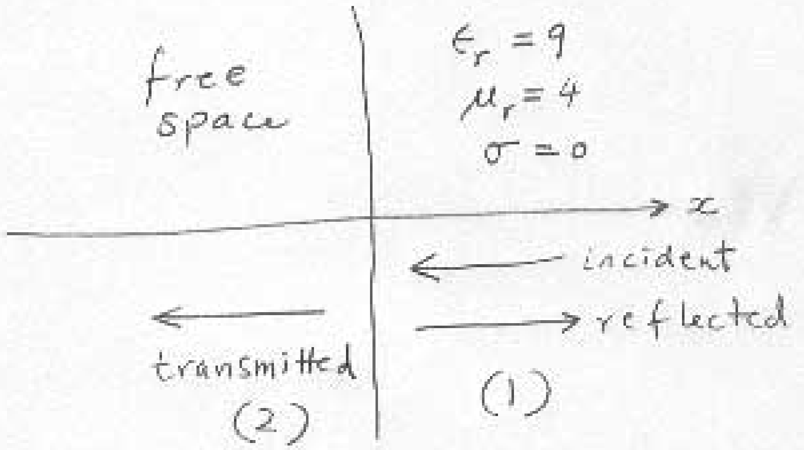
$$\eta_p = 377\sqrt{\frac{1}{4}} = 188.5\Omega$$

$$\vec{H}_p = \vec{a}_y\ \frac{1508\ (2/3)}{188.5} \sin(\omega t - \beta_p x)$$

$$\vec{\mathcal{P}}_{avg} = \vec{a}_x \left(\tfrac{1}{2}\right)\left[1508(2/3)\right]^2 / 188.5 = \vec{a}_x\, 2681 \quad [\mathrm{W/m^2}]$$

c) $$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{4/3}{2/3} = 2$$

<u>10·36</u>



a) Linearly polarized in the y-direction.

b) $$\beta_1 = \omega\sqrt{\mu_1\epsilon_1} = \frac{\omega}{c}\sqrt{\mu_{1r}\epsilon_{1r}}$$

$$= \frac{2\pi \times 30\times 10^6}{3\times 10^8}\sqrt{(4)(9)} = 3.77 \quad [\mathrm{rad/m}]$$

c) $$\frac{\partial \vec{D}_1}{\partial t} = \frac{\partial}{\partial t}\left[9\epsilon_0 \vec{E}_1\right]$$

$$\eta_1 = 377\sqrt{\frac{4}{9}} = 251.3\ \Omega$$

$$\left(\vec{E}_1\right)_{inc.} = -2.513\, \vec{a}_y \sin(\omega t + \beta x)$$

$$= -\vec{a}_y\, 2.513 \sin\left(60\pi\times 10^6 t + 3.77x\right)$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{1}{1}} - \sqrt{\frac{4}{9}}}{\sqrt{\frac{1}{1}} + \sqrt{\frac{4}{9}}} = \frac{1 - 2/3}{1 + 2/3} = \frac{+1/3}{5/3} = +1/5 = +0.2$$

$$\left(\vec{E}_1\right)_{ref.} = +\vec{a}_y\, 0.50 \sin\left(6\pi\times 10^6 t - 3.77x\right)$$

$$\vec{E}_1 = (\vec{E}_1)_{inc.} + (\vec{E}_1)_{ref.}$$

$$= -\vec{a}_y\ 2.513 \sin(60\pi \times 10^6 t + 3.77x)$$

$$- \vec{a}_y\ 0.50 \sin(60\pi \times 10^6 t - 3.77x)$$

$$\therefore \frac{\partial \vec{D}_1}{\partial t} = 9\epsilon_0 (60\pi \times 10^6)(+\vec{a}_y)\left[-2.513 \cos(60\pi \times 10^6 t + 3.77x) - 0.50 \cos(60\pi \times 10^6 t - 3.77x)\right]$$

$$= +\vec{a}_y\ 15 \times 10^{-3}\left[-2.513 \cos(60\pi \times 10^6 t + 3.77x) + 0.50 \cos(60\pi \times 10^6 t - 3.77x)\right]$$

d) $\vec{H}_{ref} = (\vec{H}_{1r})$

$$= -\vec{a}_z\ 2 \sin(60\pi \times 10^6 t - 3.77x)\ [mA/m].$$

$$\tau = 1 + \Gamma = 1 + 0.2 = 1.2$$

$$\vec{E}_{tran.} = -\vec{a}_y\ 2.513(1.2) \sin(66\pi \times 10^6 t + \beta_2 x)$$

$$\beta_2 = \omega/c = \frac{2\pi \times 30 \times 10^6}{3 \times 10^8} = 0.2\pi = 0.628\ [rad/m]$$

$$\therefore \vec{E}_{trans.} = -\vec{a}_y\ 3.016 \sin(60\pi \times 10^6 t + 0.628x)\ [V/m]$$

$$\vec{H}_{trans.} = \vec{a}_z \frac{3.016}{377} \sin(60\pi \times 10^6 t + 0.628x) \; [A/m]$$

$$= \vec{a}_z \; 8 \sin(60\pi \times 10^6 t + 0.628x) \; [mA/m]$$

e) The total "phasor" electric and magnetic fields in region (1) are:

$$\vec{E}_{1S} = -\vec{a}_y \, 2.513 \, e^{j3.77x} \quad \begin{bmatrix} \text{Using a} \\ \text{sine reference} \end{bmatrix}.$$

$$- \vec{a}_y \, 0.50 \, e^{-j3.77x} \quad [V/m]$$

$$\vec{H}_{1S} = \vec{a}_z \, 0.01 \, e^{j3.77x} - \vec{a}_z \, 0.002 \, e^{-j3.77x}$$

$$\vec{\mathcal{P}}_{avg} = \frac{1}{2} \mathrm{Re}\left[\vec{E}_{1S} \times \vec{H}_{1S}^{*}\right]$$

$$= \frac{1}{2} \mathrm{Re}\Big[-\vec{a}_x \, 25.13 \times 10^{-3} + \vec{a}_x \, 5.00 \times 10^{-3} e^{j7.54x}$$

$$- \vec{a}_x \, 5.00 \times 10^{-3} e^{-j7.54x} + \vec{a}_x \, 10^{-3}\Big]$$

$$= -\vec{a}_x \frac{1}{2}\left[24.13 \times 10^{-3}\right] + \frac{1}{2} \mathrm{Re}\begin{bmatrix} -\vec{a}_x \, 5.00 \times 10^{-3} e^{j7.54x} \\ +\vec{a}_x \, 5.00 \times 10^{-3} e^{-j7.54x} \end{bmatrix}$$

$$= -\vec{a}_x \, 12.06 \times 10^{-3}$$

$$= -12.06 \, \vec{a}_x \; [mW/m^2]$$

In region (2)

$$\vec{E}_{2s} = -\vec{a}_y\, 3.016\, e^{j0.628x}$$ [sine reference used].

$$\vec{H}_{2s} = \vec{a}_z\, 0.008\, e^{j0.628x}$$

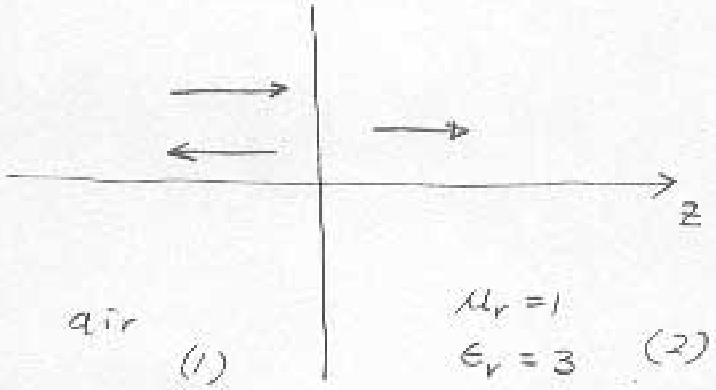
$$\vec{\mathcal{P}}_{avg} = \frac{1}{2} Re\left[\vec{E}_{2s} \times \vec{H}_{2s}^{*}\right]$$

$$= \frac{1}{2} Re\left[-\vec{a}_x\, 24.128 \times 10^{-3}\right]$$

$$= -12.06\,\vec{a}_x \quad [mW/m^2]$$

(i.e. the average Poynting's vector in both regions is the same).

10.37



a) $\beta_1 = 1$

$$\lambda_1 = \frac{2\pi}{\beta_1} = 2\pi \; [m]$$

$$\frac{\omega}{\beta_1} = c \Rightarrow \omega = c\,\beta_1 = 3\times10^8(1) = 3\times10^8 \; [rad/s]$$

$$\frac{\omega}{\beta_2} = \frac{c}{\sqrt{3}} \Rightarrow \beta_2 = \frac{\sqrt{3}\,\omega}{c} = \frac{\sqrt{3}\times 3\times10^8}{3\times10^8} = \sqrt{3}$$
$$= 1.732 \; [rad/m]$$

$$\lambda_2 = \frac{2\pi}{\beta_2} = 3.628 \; [m]$$

b) $$\vec{H}_i = \frac{10}{377}\cos\left(3\times10^8 t - z\right)(-\vec{a}_x) \quad [A/m]$$

c) $$\Gamma = \frac{\sqrt{\frac{1}{3}} - 1}{\sqrt{\frac{1}{3}} + 1} = -0.268$$

$$\tau = 1 + \Gamma = 0.732$$

d) For regim (2):

$$\vec{E}_s = 7.32\, e^{-j1.732z}\, \vec{a}_y \quad \text{(cosine reference used)}$$

$$\vec{H}_s = \frac{7.32}{\eta_2} e^{-j1.732z} (-\vec{a}_x)$$

$$\eta_2 = 377 \frac{1}{\sqrt{3}} = 217.66 \ \Omega$$

$$\vec{H}_s = 0.0336\, e^{-j1.732z} (-\vec{a}_x)$$

$$\vec{P}_{avg} = \vec{a}_z \ 123 \ [mW/m^2]$$

For region (1):

$$\vec{E}_s = 10\, e^{-jz}\, \vec{a}_y - 2.68\, e^{jz}\, \vec{a}_y \quad \left\{\begin{matrix}\text{incident}\\ +\text{reflected}\end{matrix}\right\}$$

$$\vec{H}_s = \frac{10}{377} e^{-jz} (-\vec{a}_x) - \frac{2.68}{377} e^{jz} (+\vec{a}_x) \quad \left\{\begin{matrix}\text{incident}\\ +\text{reflected}\end{matrix}\right\}$$

$$\vec{P}_{avg} = \frac{1}{2} Re \left[\vec{E}_s \times \vec{H}_s^{\,*} \right]$$

$$= \frac{1}{2} Re \left[\left(10 e^{-jz} \vec{a}_y - 2.68 e^{jz} \vec{a}_y\right) \times \left(-\vec{a}_x \frac{10}{377} e^{jz} - \vec{a}_x \frac{2.68}{377} e^{-jz}\right) \right]$$

$$= \frac{1}{2} Re \left[\vec{a}_z \frac{100}{377} + \vec{a}_z \frac{26.8}{377} e^{-2jz} - \vec{a}_z \frac{26.8}{377} e^{j2z} - \vec{a}_z \frac{(2.68)^2}{377} \right]$$

$$= \frac{1}{2} \left[\frac{100 - (2.68)^2}{377} \right] \vec{a}_z = 123 \ \vec{a}_z \ [mW/m^2]$$