Identification of Hammerstein Model with Known Nonlinearity Structure Using Particle Swarm Optimization

Hussain N. Al-Duwaish
Department of Electrical Engineering
King Fahd University of Petroleum and Minerals
Dhahran 31261
Saudi Arabia.
E-mail: hduwaish@dpc.kfupm.edu.sa

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Abstract

This paper investigates the use of particle swarm optimization in the identification of Hammerstein model with known nonlinearity structure. The parameters of the Hammerstein model are estimated using particle swarm optimization from the input-output data by minimizing the error between the true model output and the identified model output. Using particle swarm optimization, Hammerstein models with known nonlinearity structure and unknown parameters can be identified. Moreover, systems with non-minimum phase characteristics can be identified. Extensive simulations have been used to study the convergence properties of the proposed scheme. Simulation examples are included to demonstrate the effectiveness and robustness of the proposed identification scheme.

Introduction

The dynamic behavior of many chemical processes can be approximated by a static nonlinearity in series with a linear dynamic part as shown in Figure 1. This model structure is known as the Hammerstein model which has been successfully used to model a large class of nonlinear systems. The examples in which this model was applied are nonlinear filters [1], nonlinear network [2], detection of signals in non-Gaussian noise [3], nonlinear prediction [4], nonlinear data transmission channels [5], control systems [6], noise cancellation [7], heat exchangers [8], identification of biological systems [9], and many others. The above examples show the need for algorithms able to recover nonlinearities in systems of various kinds.

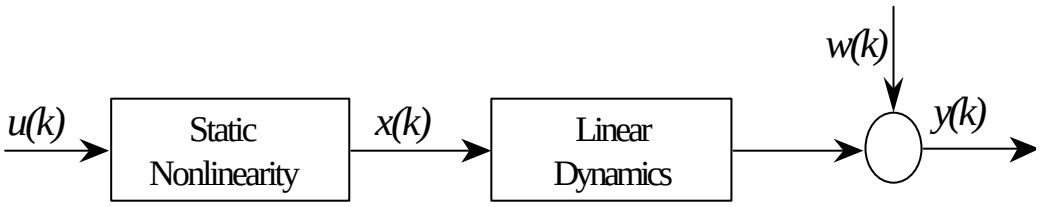

Figure 1: Hammerstein Model Structure

The literature on Hammerstein model is extensive and reflects considerable current activity [10-18]. The Hammerstein model is completely identified if the parameters describing the linear part and the static nonlinearity can be precisely estimated from input-output data. This approach will preserve the system structure and provide valuable information for many engineering applications. Many identification methods have been developed for identifying Hammerstein models [19-24]. These methods can be divided into parametric and nonparametric methods. For the parametric methods the Hammerstein model is represented by the following equations[21][23]:

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} x(k) + w(k) \qquad (1)$$

with

$$B(q^{-1}) = b_o + b_1 q^{-1} + \cdots\cdots + b_m q^{-m} \qquad (2)$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots\cdots + a_n q^{-n} \qquad (3)$$

The non measured intermediate variable *x(k)* is the output of the static nonlinearity given by

$$x(k) = c_1 u(k) + c_2 u^2(k) + \cdots\cdots + c_L u^L(k) \qquad (4)$$

Where q^{-1} is the unit delay operator, *u(k)* is the input, *y(k)* is the output, *w(k)* is the measurement noise, (m, n) represent the order of the linear part, and L is the degree of the polynomial. Thus, the problem of Hammerstein model identification is to estimate the unknown parameters $a_1, \ldots\ldots, a_n$, $b_o \ldots\ldots b_m, and\ c_1, \ldots\ldots, c_L$ from input-output data. The unknown parameters are estimated by transforming the problem into multi-input single-output (MISO) linear identification problem. The inputs to the MISO linear system are $u(t)$, $u^2(t), \ldots.$, $u^L(t)$. The main drawbacks of this approach are the assumption that the nonlinearity is of polynomial form and the increase in the number of inputs in the linear identification problem. Moreover, it has been shown in [24] that if the nonlinearity is not a polynomial and the input is not Gaussian, these algorithms do not converge.

Different nonparametric approaches [25] have been used for the Hammerstein model identification. Most of the nonparametric methods use kernels regression estimates to identify the nonlinearity. In these methods, the identification of the nonlinearity is performed separately from the linear part.

Recently, neural networks have been used to model the nonlinearity in the Hammerstein model for single-input single-output (SISO) systems [19], and multi-input multi-output (MIMO) systems [20].

None of the previous methods have considered the case when the structure of the nonlinearity is known with unknown parameters. This case can arise in many practical application including nonlinear actuators in control systems. Examples of nonlinearities in the actuators include saturation, dead zone, backlash, coulomb friction, etc. Usually, the mathematical model structure of these nonlinearities is known, however, some or all model parameters may not be known. Moreover, previous methods can not be used to identify Hammerstein model with non-minimum phase linear part. This problem has been addressed by the author using genetic algorithms in his work [31], but better results in terms of speed of convergence and accuracy are obtained in this paper using particle swarm optimization.

Recently, particle swarm optimization (PSO) have been used extensively in solving many optimization searching problems [26-29]. Compared to conventional optimization methods, PSO does not assume that the search space is differentiable or continuous. Also PSO does not require linearity in the parameters which is needed in iterative searching optimization techniques. This property of PSO makes it suitable to the identification of Hammerstein model with nonlinearities of known structure and unknown parameters which represents a nonlinear in the parameters identification problem.

In this paper, we propose a PSO-based scheme for the identification of the nonlinear Hammerstein model with known nonlinearity structure. This is accomplished by formulating the identification of the Hammerstein model as an optimization problem

and the PSO is used in the optimization process. In the PSO method, the input-output measurement data are used to estimate the model parameters such that a certain objective function is minimized. The advantages of using PSO include the possibility to identify Hammerstein models with non-minimum phase linear part and any nonlinearity with known structure and unknown parameters.

2. Particle Swarm Optimization

Particle swarm optimization (PSO) was developed through simulation of a simplified social system. In the simulation model of a social system, each particle position can be thought of being a state of mind of a particular setting of abstract variables that represent the individual's beliefs and attitudes. The movement of the particles thus correspond to the change of these concepts. Swarms or social groups adjust their beliefs and attitudes through the evaluation of stimuli from the environment and compare it to their existing knowledge. If such stimuli or values are found to be more fit, they replace their existing values. These three important properties of human or animal social behavior i.e., evaluation, comparison, and imitation, are the inspiration for the particle swarm optimization algorithm, and the particle swarm uses these concepts in adapting to the environmental changes and solving complex minimization problems.

Besides being a model of human or animal behavior, the particle swarm is closely related to swarm intelligence. Using PSO, there is no central control and no one gives orders. Each particle is a simple agent acting upon local information. But the swarm, as a whole, is able to perform tasks whose degree of complexity is well beyond the capabilities of an

individual particle. This is due to self-organization. The interactions among the particles (low-level components) result in complex structures at

the swarm (high-level or global) level making is possible for it to perform optimization of complex functions. These basic principles for PSO are:

- The proximity principle: The swarm should be able to carry out simple time and space calculations.
- The quality principle: The swarm should be able to respond to quality factors in the environment.
- The principle of diverse response: The swarm should not commit its activities along excessively narrow channels.
- The principle of stability: The swarm should not change its behavior every time the environment changes.
- The principle of adaptability: The swarm must be able to change its behavioral mode when its worth the computational price.

Thus, the PSO system is thought of as an intelligent system. This is because it is based upon artificial life and has roots in Evolutionary Computation (EC).

Like in other Evolutionary Computation methods, the particle swarm consists of a population of individuals that represent solutions to the optimization problems we need to be solved. An optimal solution is selected through an iterative and probabilistic modification of these solutions. There not much difference in PSO and other Evolutionary Algorithms (EAs) in EC-terms. However, the difference lies in how we change the population/swarm from one iteration to the next. In EAs, genetic operators like selection, mutation and crossover are used

whereas in PSO, the particles are modified according to two formulas after each iteration. Also, conceptually, in PSO, the particles stay *alive* and inhibit the search space during the whole run, where as in EA, the individuals are replaced in each generation. Another fundamental conceptual

difference is that in EA the objective is reach through *competitive search* whereas in PSO, it is reached through *cooperative search*.

Thus, PSO differs from other EAs [26-29] in terms of performance. The EA techniques have been successfully applied in many areas. However, PSO is a more robust and fast algorithm that can solve nonlinear, non-differentiable, multi-modal problems. Such problems involve the minimization of a static objective function i.e., the main goal of a global minimizer that does not change. The PSO technique can generate a high-quality solution within shorter calculation time and stable convergence characteristic than other stochastic methods. Due to this ability, it is effective in solving problems in a wide variety of scientific fields.

As in other EAs, a population of individuals exist in PSO. However, here instead of using genetic operators, these individuals are "evolved" by cooperation and competition among themselves through generations. Each particle adjusts its "flying" according to its own experience as well as its companions' experience. Each individual, called a "particle" in fact, represents a potential solution to the problem.

Each particle is treated as a point in D-dimensional space. The *i*th particle is represented as

$$X_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \tag{5}$$

The best previous position (the position giving the best fitness value) of any particle is recorded and represented as

$$P_i = (p_{i1}, p_{i2}, \ldots, p_{iD}) \tag{6}$$

Similarly, the position change (velocity) of each particle is

$$V_i = (v_{i1}, v_{i2}, \ldots, v_{iD}) \tag{7}$$

The particles are manipulated according to the following equations

$$\begin{aligned} V_i^{n+1} &= w * V_i^n + c_1 * r_{i1}^n * (P_i^n - X_i^n) + c_2 * r_{i2}^n * (P_g^n - X_i^n) \\ X_i^{n+1} &= X_i^n + x * V_i^{n+1} \end{aligned} \tag{8}$$

3. Proposed PSO-Based Identification Method

The Hammerstein model is represented by the following equations:

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} x(k) + w(k) \tag{9}$$

with

$$B(q^{-1}) = b_o + b_1 q^{-1} + \cdots\cdots + b_m q^{-m} \tag{10}$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots\cdots + a_n q^{-n} \tag{11}$$

The non measured intermediate variable *x(k)* is the output of the nonlinearity given by

$$x(k) = f(q, u(k)) \tag{12}$$

Where q^{-1} is the unit delay operator, *u(k)* is the input, *y(k)* is the output, *w(k)* is the measurement noise, (m, n) represent the order of the linear part, and *f(.)* is any nonlinear function, q is a set of parameters describing the nonlinearity. Thus, the problem of Hammerstein model identification is to estimate the unknown parameters

$a_1, \ldots\ldots, a_n$, $b_o, \ldots\ldots, b_m$, and q from input-output data. For nonlinearities with unknown structure, the polynomial approximation as in Eqn. 4 can be used.

The use of PSO in the estimation of the unknown parameters requires a fitness function to be defined to determine how well the estimates fit the system, and the domain of the unknown parameters to be specified. The following fitness function can be defined:

$$F = \sum_{k=1}^{M} (y(k) - \hat{y}(k))^2 \qquad (13)$$

where

$$\hat{y}(k) = \frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})} \hat{x}(k) \qquad (14)$$

with

$$\hat{B}(q^{-1}) = \hat{b}_o + \hat{b}_1 q^{-1} + \ldots \hat{b}_m q^{-m} \qquad (15)$$

$$\hat{A}(q^{-1}) = 1 + \hat{a}_1 q^{-1} + \ldots \hat{a}_n q^{-n} \qquad (16)$$

$$\hat{x}(k) = \hat{c}_1 u(k) + \hat{c}_2 u^2(k) + \ldots\ldots + \hat{c}_L u^L(k) \qquad (17)$$

Where M is the number of input-output data points used in the identification, and the parameters estimates $\hat{a}_1, \ldots\ldots, \hat{a}_n$, $\hat{b}_o, \ldots\ldots, \hat{b}_m$ and $\hat{q}$ are found by minimizing the fitness function given by Eqn. 9. To determine the domain of the parameters of the linear part, Eqn. 6 and Eqn. 7 can be transformed into pole-zero form given by

$$B(q^{-1}) = b_o(1 - z_1 q^{-1})(1 - z_2 q^{-1}) \ldots b_o(1 - z_m q^{-1}) \qquad (18)$$

$$A(q^{-1}) = (1 - p_1 q^{-1})(1 - p_2 q^{-1}) \ldots (1 - p_n q^{-1}) \qquad (19)$$

Where $p_i (i = 1, \ldots, n), and\ z_j\ (j = 1, \ldots, m)$ are in general complex numbers. For stable systems, the real and imaginary parts of p_i must lie in the interval from -1 to 1. Also, for

minimum phase systems, the real and imaginary parts of z_i must lie in the interval form -1 to 1. If the system is non-minimum phase, one has to decide how big the search space is depending on prior knowledge of the system. Also, one has to decide how big the search space is for the parameters describing the nonlinearity. Note that the

optimization is nonlinear in the parameters and gradient based optimization techniques can, at best, produce local minima. The proposed PSO-based identification algorithm can be summarized in the following steps:

1) Generate *M* input-output data points form the system to be identified.
2) Generate random initial solutions for zeros and poles of the linear part, and the parameters of the nonlinearity in the appropriate range.
3) Evaluate the fitness function for all possible solutions generated in step 2.
4) Use Eqn. 5-8 to generate new generation of solutions.
5) Evaluate the fitness function for the new generation.
6) Repeat step 4 until predetermined number of generation has been produced.

4. Convergence Analysis

Extensive simulation studies for the proposed identification scheme show that the accuracy of estimation of the parameters of the proposed method is greatly affected by the choice of the input signals. This is because any model developed from input-output data set is just a representation of the information contained in that data set. Among the most important issues that must be considered when generating input signals for system identification are:

- System noise: if the system is noisy it may be necessary to increase the amplitude of the input signals such that the signal to noise ratio is large enough and estimates of acceptable accuracy can be obtained.
- Nonlinearities: The input signals must be suitable for obtaining a characterization of the nonlinearities behavior. The amplitude of the input signals must be increased to enable the nonlinearities of the system. The selected maximum input amplitude
- influences the model parameter estimates. For example, a small input amplitude which does not enable the nonlinearity will result in a linear model for the nonlinear system.
- Identifiability: certain conditions must be imposed on the input signals such that the underlying properties of the linear part can be identified [20]. This means that the input signals should be persistently exciting which requires that the signal should adequately span the bandwidth of the system being identified. In [21] uniform or Gaussian random inputs are suggested for the use in the identification of nonlinear systems.

5. Simulation Results

In this section, the proposed identification approach is applied to different examples with practical static nonlinearities including saturation nonlinearity, coloumb friction nonlinearity, saturation with dead zone, with different linear parts. The performance is evaluated under various signal to noise ratios (SNR). Consider the following process with the linear part given by:

$$\frac{y(k)}{x(k)} = \frac{(1 - z_1 q^{-1})(1 - z_2 q^{-1})}{(1 - p_1 q^{-1})(1 - p_2 q^{-1})(1 - p_3 q^{-1})}$$

Different pole-zero configurations and different nonlinearities have been used as shown in Table 1. Using uniformly distributed random inputs in the range [-1,1], the process outputs are generated using the process model. The added measurement noise is zero mean white Gaussian with different variances to achieve different SNR The proposed identification approach is used to identify the poles, zeros and parameters of the different nonlinearities as shown in Table 1. The results shown in Table 1 indicate that

accurate and consistent results can be obtained even for low SNR. The behavior of the cost function for one of the examples given in Table 1 is shown in Figure 2 which indicates a rapid convergence to the optimal solution.

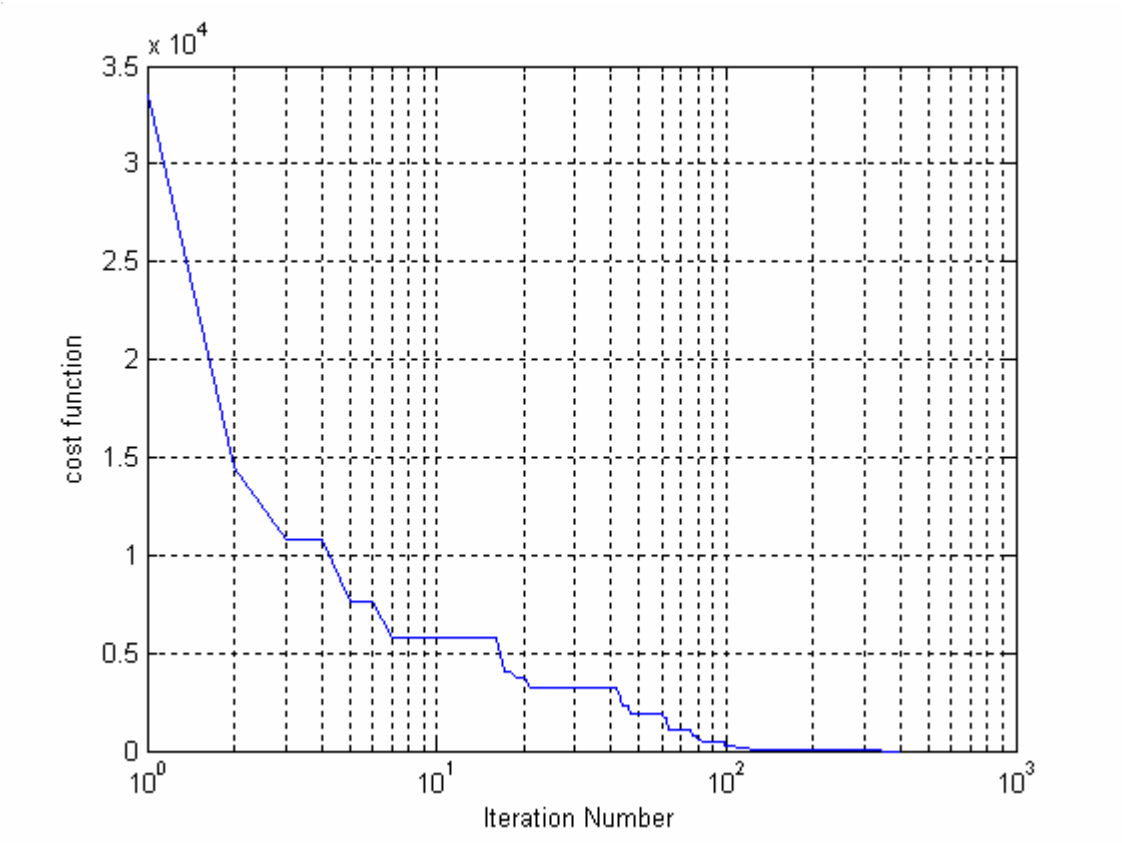

Figure 2: Convergence of the Cost Function

	Zeros and poles of the linear part	**Type of the nonlinearity**	**Parameters of the nonlinearity**
True values	$z_1 = 4.1,\ p_1 = 0.8,$ $p_2 = -0.2,\ p_3 = -0.5$	Saturation $x(k) = \begin{cases} a & for\ u(k) \geq \frac{a}{2} \\ b\,u(k) & for -\frac{a}{2} < u(k) < \frac{a}{2} \\ -a & for\ u(k) \leq -\frac{a}{2} \end{cases}$	$a = 0.5,$ $b = 2.2$
Estimated values: SNR=30 dB	$z_1 = 4.0973,$ $p_1 = 0.7999,$ $p_2 = -0.1995,$ $p_3 = -0.5002$		$a = 0.5000,$ $b = 2.2013$
Estimated values: SNR=10 dB	$z_1 = 4.0895,$ $p_1 = 0.7995,$ $p_2 = -0.1987,$ $p_3 = -0.5004$		$a = 0.5000,$ $b = 2.2048$
True values	$z_1 = 1.2,$ $p_1 = 0.1 - j0.5,$ $p_2 = 0.1 + j0.5,$ $p_3 = 0.9$	Columbic friction $x(k) = sign(u(k))\{a\lvert u(k)\rvert + b\}$	$a = 4.0,$ $b = 0.5$

Estimated values: SNR=30 dB	$z_1 = 1.1999$, $p_1 = 0.1 - j0.4999$, $p_2 = 0.1 + j0.4999$, $p_3 = 0.9$		$a = 4.0$, $b = 0.5001$
Estimated values: SNR=10 dB	$z_1 = 1.1996$, $p_1 = 0.0999 - j0.4994$, $p_2 = 0.0999 + j0.4994$, $p_3 = 0.8998$		$a = 4.0$, $b = 0.5005$
True values	$z_1 = 0.2$, $z_2 = 2.8$, $p_1 = 0.8$, $p_2 = -0.2$, $p_3 = -0.5$	Relay with dead zone $x(k) = \begin{cases} a & for\ u(k) \geq b \\ 0 & for\ -b < u(k) < b \\ -a & for\ u(k) \leq -b \end{cases}$	$a = 4.1$, $b = 0.5$
Estimated values: SNR=30 dB	$z_1 = 0.2074$, $z_2 = 2.8032$, $p_1 = 0.8010$, $p_2 = -0.1899$, $p_3 = -0.5032$		$a = 4.0985$, $b = 0.5002$

Estimated values: SNR=10 dB	$z_1 = 0.1889$, $z_2 = 2.8069$, $p_1 = 0.7989$, $p_2 = -0.2173$, $p_3 = -0.4929$		$a = 4.0869$, $b = 0.4995$

Table 1: Simulation Results

5. Conclusion

The use of PSO in the identification of the nonlinear Hammerstein model with known nonlinearity structure has been investigated in this paper. The Hammerstein identification problem has been formulated as an optimization problem and PSO is used to estimate the unknown parameters from input-output data. The advantages of using PSO include the possibility to identify systems with known nonlinearity structure and unknown parameters, and systems with non-minimum phase linear part. Extensive simulations have been used to study the convergence properties of the proposed identification scheme. Simulation results reveal that accurate and consistent results can be obtained even for low signal to noise ratios. The proposed approach can be extended to other types of linear systems with static nonlinearities.

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