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A Robust Controller For DC-DC Switched Mode Power Converters

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***Abstract* - A Linear Quadratic Gaussian with Loop Transfer Recovery (LQG/LTR) controller is proposed for the efficient control of the output voltage of a Buck-Boost DC-DC power converter. The proposed control scheme guarantees excellent regulation of the output voltage. This is true even in the presence of large variations of the duty ratio and circuit parameters.**

I. INTRODUCTION

Power converters often consist of linear circuits that are switched between two configurations. Even though each of the configurations is linear, the overall behavior of the circuit is nonlinear. DC-DC switched mode power converters constitute simple but yet efficient means of DC power regulation [1,2]. The analysis and control of such a circuit is fairly complicated. Different methods have been employed for the modeling and control of DC-DC power converters [3-12]. The state-space averaging (SSA) method is utilized for the analysis and control design of such circuits. This method assumes that the switching frequency is much higher than the bandwidth of the linear circuit configuration.

In this paper, the SSA is adopted to describe the circuit by its space equations. An average value of the state over an entire switching cycle is first obtained. This is linearized around some nominal operating point to obtain a description of the circuit. A linear quadratic gaussian with loop transfer recovery (LQG/LTR) controller is proposed. It guarantee the regulation of the output voltage to a nominal value in spite of parameter variation and duty cycle perturbations.

II. SYSTEM MODEL

Consider a linear circuit that has a DC excitation voltage and is switched periodically with a time period of T, (Fig. 1). The switch remains closed for dT and open for (1-d)T in each cycle, where d ($0 \le d \le 1$) is called the duty ratio of the switching. The circuit output voltage V_0 can be modulated by varying the value d. It is desirable to obtain an input-output description of the circuit where the input is the duty ratio or any of its variations.

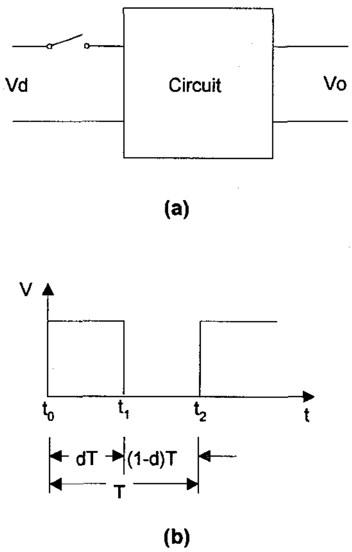

Fig. 1: Switchable DC-DC Circuit

Let the dynamics of the circuit be represented by its state space equations. With only two switch states being examined, it is only meaningful to consider a single input circuit. When the switch is closed $(t_0 \le t \le t_1)$ and the state space equation is given by

$$\dot{x} = A_1 x + B_1 V_d \qquad (1)$$

When the switch is open $(t_1 \le t \le t_2)$, the state space equation becomes

$$\dot{x} = A_2 x + B_2 V_d \qquad (2)$$

The output equation in either case is given by

$$y = V_0 = C\,x \qquad (3)$$

The method of state space averaging has been used to find the transfer functions of regulated DC power supply. The weighted averages of the circuit output and state values are obtained over one cycle by combining the two configurations. To obtain this model, (1) is multiplied by d and (2) by (1-*d*). Both are then added to obtain :

$$\dot{x} = \left[A_1 d + A_2(1-d)\right]x + \left[B_1 d + B_2(1-d)\right]V_d \quad (4)$$

This equation is based on the assumptions that the switching frequency is much higher than the circuit resonant frequency. Let

$$x = x_0 + \tilde{x} \quad (5a)$$

$$d = d_0 + \tilde{d} \quad (5b)$$

where the subscript 0 indicates a steady state value and the over-tilde indicates a perturbation around the steady state value. The following perturbed state model is obtained after substituting (5) in (4) and neglecting the product of x and $\tilde{d}$ terms :

$$\dot{\tilde{x}} = \left[A_1 d_0 + A_2(1-d_0)\right]\tilde{x} + \left[(A_1 - A_2)x_0 + (B_1 - B_2)V_d\right]\tilde{d} \quad (6)$$

Referring to the circuit of Fig. 2 for the notation and description of a buck-boost DC-DC converter with resistive load, the $A_1, A_2, B_1, B_2,$ *and* C matrices are obtained as:

$$A_1 = \begin{bmatrix} -1/RC & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1/RC & 1/C \\ -1/L & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

III. LQG/LTR DESIGN METHOD

In this section, the LQG/LTR design method will be summarized briefly. Detailed discussion can be found in [13-15]. The feedback loop considered by the LQG/LTR design method is shown in Fig. 3. It consists of the plant transfer function *G(s)*, the controller transfer function *K(s)*, *r(t)* is a command signal, *d(t)* is the disturbance and *n(t)* is the measurement noise.

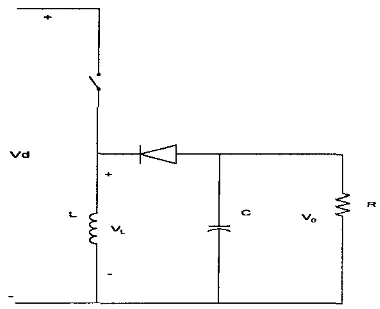

Fig. 2: Schematic of a Buck-Boost converter

Some important quantities which will be used in the design specifications are defined as

Loop Transfer Function $T(s) = G(s)K(s)$

Sensitivity Transfer Function $S(s) = [I + T(s)]^{-1}$

Closed Loop Transfer Function

$M(s) = [I + T(s)]^{-1}T(s)$

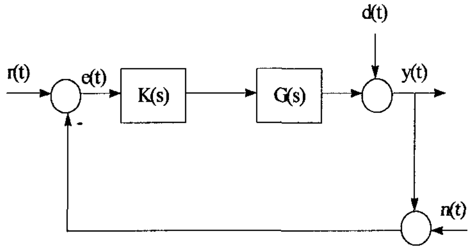

Fig. 3: Feedback loop considered by the LQG/LTR method

The objective is to find a controller *K(s)* to satisfy certain specifications related to the nominal stability, the robustness to modeling errors and good performance. The controller *K(s)* must ensure that the poles of the closed loop transfer function *M(s)* are located in the left-half of the *s*-plane. The condition for stability robustness is expressed as

$$\bar{\sigma}[M(j\omega)] < \frac{1}{e_m(\omega)} \quad (7)$$

Where $\bar{\sigma}(.)$ is the maximum singular value and $e_m(\omega)$ is the modeling error of the plant.

Good performance can be achieved if the following condition is satisfied

$$\bar{\sigma}[S(j\omega)] < l(\omega) \quad (8)$$

Where $\bar{\sigma}(.)$ is the maximum singular value and $l(\omega)$ is a large positive function in the appropriate frequency range. The two requirements are transformed into conditions on the loop transfer function T(s). These conditions are depicted in Fig. 4. The above design specifications can be satisfied by a two-step design procedure. Given the plant model G(s) represented in state space form

$$\dot{x}(t) = A_P x(t) + B_P u(t)$$
$$y(t) = C_P x(t) \quad (9)$$

Where x(t) is $n \times 1$ state vector, u(t) is $m \times 1$ input vector, y(t) is $m \times 1$ output vector. A_P, B_P, and C_P are constant matrices of appropriate dimensions.

The controller *K(s)*, shown in Fig. 5, can be written as

$$\dot{x}_c(t) = (A_P - B_P K_c - K_f C_P)x_c(t) + K_f e(t)$$
$$u(t) = K_c x_c(t) \tag{10}$$

The full state linear feedback gain K_c operates on an estimate of the plant states provided by a linear filter with gain K_f. The design is accomplished by determining the free parameters K_c and K_f. K_f can be obtained by solving the following Riccati equation:

$$A_P R_f + R_f A_P{}^T + L_f L_f{}^T - \left(\frac{1}{\mu}\right) R_f C_P{}^T C_P R_f = 0$$

$$K_f = \left(\frac{1}{\mu}\right) R_f C_P{}^T \tag{11}$$

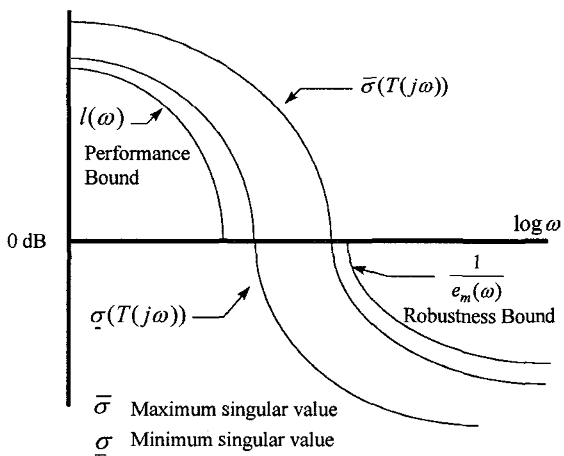

Fig. 4: Performance and robustness specifications

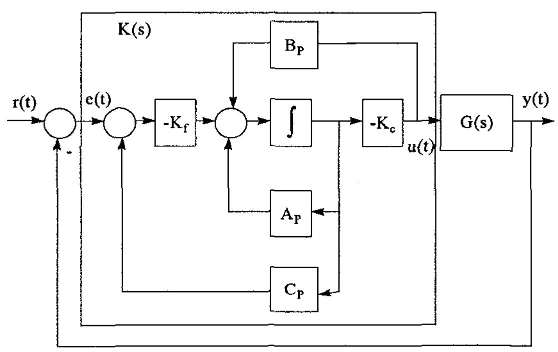

Fig. 5: LQG/LTR controller structure

The design parameters R_f and L_f are selected such that targe ttransfer function ($G_{TF}(j\omega) = C_P(j\omega I - A_P)^{-1}K_f$) satisfies the following conditions

$$\bar{\sigma}[(I + G_{TF}(j\omega))^{-1}G_{TF}(j\omega)] < \frac{1}{e_m(\omega)} \tag{12a}$$

$$\bar{\sigma}[(I + G_{TF}(j\omega))^{-1}] < \frac{1}{l(\omega)} \tag{12b}$$

On the other hand, K_c which represents the full-state feedback gain, is obtained by solving the following Riccati equation:

$$A_P{}^T R_c + R_c A_P + C_P{}^T C_P + \left(\frac{1}{\rho}\right) R_c B_P B_P{}^T R_c = 0$$

$$K_c = \left(\frac{1}{\rho}\right) B_P{}^T R_c \tag{13}$$

The design parameters R_c and ρ should be selected such that the target feedback loop transfer function is recovered; that is

$$G(j\omega)K(j\omega) \approx G_{TF}(j\omega) \tag{14}$$

IV. LQG/LTR CONTROLLER DESIGN AND SIMULTION RESULTS

This section describes the design of a buck-boost DC-DC controller using the proposed LQG/LTR. The converter is supplied by a DC source and a negative polarity output can be obtained with respect to the common terminal of the input voltage. A schematic of the buck-boost converter is shown in Fig. 2. The system parameters are:

$L = 4$ mH; $C = 25\mu$F; $R = 10\ \Omega$; $V_d = 100$ V

These parameters were selected such that the inductor current remains positive even for a duty ratio of 0.05 to avoid any discontinuous mode of conduction. The nominal duty ratio (d_0) is taken as 0.3.

The poles and zeros of the open loop system are given in Table 1. These poles and zeros indicate that the plant is stable and non-minimum phase.

Table 1: Poles and zeros of the converter

Poles	Zeros
-2000+948.7j -2000-948.7j	8747.8

To achieve zero steady state error, an integrator is required at the input [15]. Using the LQG/LTR design method described in Section III, the Kalman filter gain is found to be

$$k_f = [15.43 \quad 3162.30 \quad 515.02]^T$$

On the other hand, the full state feedback gain vector is

$$k_c = [1.88 \times 10^4 \; 1.86 \times 10^7 \quad 2.49 \times 10^8]$$

Fig. 6 shows that the singular values of the loop transfer function (G(s)K(s)) clearly approximate the desired loop shapes of the target transfer function ($C_P(sI - A_P)^{-1} K_f$). The exact recovery of the desired loop shapes in Fig. 6 was not possible due to the non-minimum phase characteristics .

To test the robustness of the proposed controller, the effects of varying the duty ratio and circuit parameters from their nominal values are investigated. This variation is applied after 0.01 sec of operation with the nominal values. Fig. 7 shows the converter output voltage as a result of changing the duty ratio (d_0). The nominal value of d_0 is 0.3. It was changed to 0.15, 0.45 and 0.6. The results show the robustness of the proposed controller even for 100% change in the duty ratio. The robustness of the proposed controller has also been investigated when varying the converter element values (*R*, *L*, and *C*). It has been found that the converter output voltage is insensitive to *L* and *C*. The converter output voltage as a result of varying the resistance *R* is shown in Fig. 8. The simulation results show that the LQG/LTR controller is insensitive to the changes in the duty ratio and the circuit parameters.

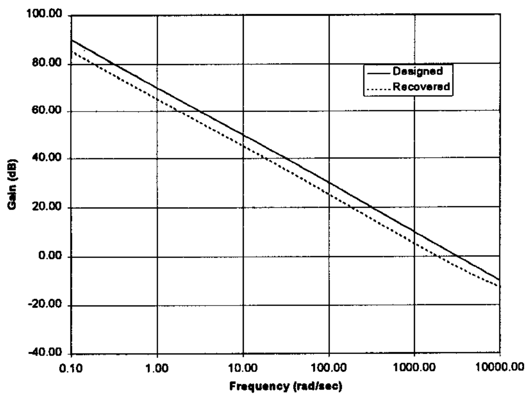

Fig. 6: Designed and recovered loop gains

V. CONCLUSION

An LQG/LTR controller is presented for the control of a Buck-Boost converter. Although the converter tested is a non minimum phase, the LQG/LTR controls the output to the desired value. Also, the controller is found to be robust against variations of the duty ratio and the converter parameters.

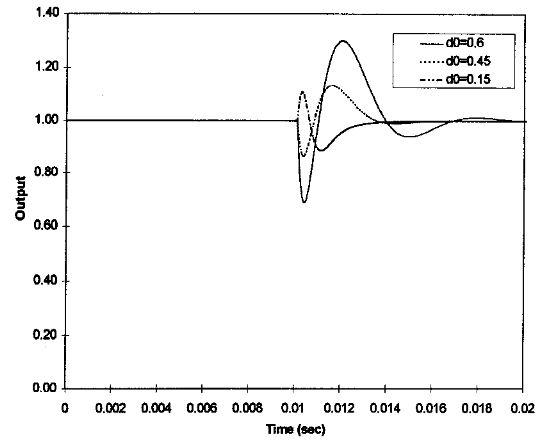

Fig.7: Per-unit converter output voltage

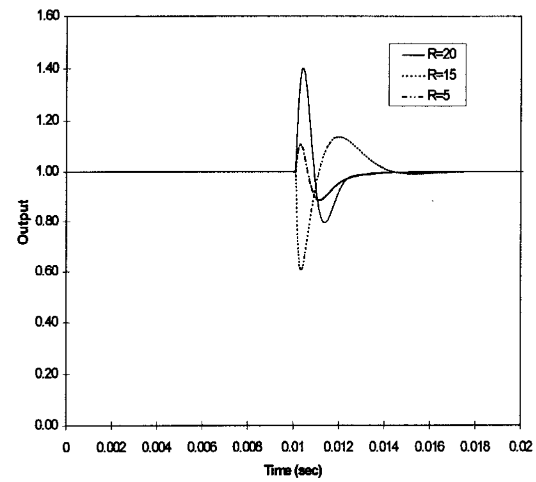

Fig. 8: Per-unit converter output voltage

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