E7.2 (a) The root locus is shown in Figure E7.2. When $K = 6.5$, the roots of the characteristic equation are

$$s_{1,2} = -2.65 \pm j1.23 \quad \text{and} \quad s_{3,4} = -0.35 \pm j0.8 \ .$$

The real part of the dominant root is 8 times smaller than the other two roots.

(b) The dominant roots are

$$(s + 0.35 + j0.8)(s + 0.35 - j0.8) = s^2 + 0.7s + 0.7625 \ .$$

From this we determine that

$$\omega_n = 0.873 \quad \text{and} \quad \zeta = \frac{0.7}{2(0.873)} = 0.40 \ .$$

Thus, the settling time is

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.35} = 11.43 \text{ sec} \ .$$

The percent overshoot is

$$P.O. = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 25.4\% \ .$$

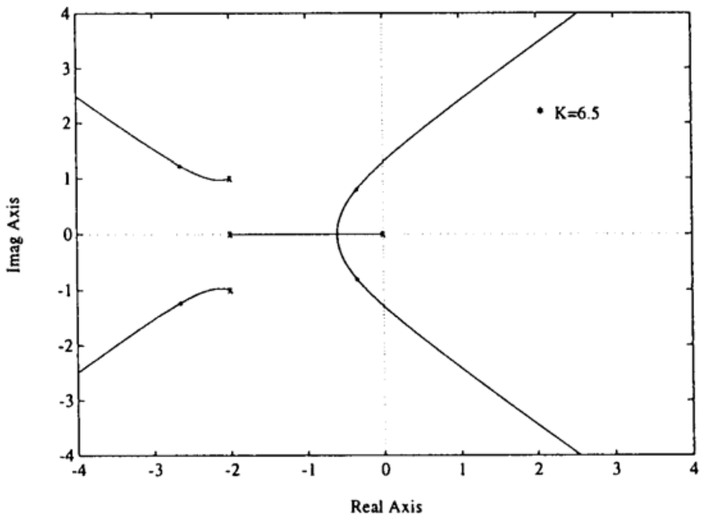

FIGURE E7.2
Root locus for $1 + K\frac{1}{s(s+2)(s^2+4s+5)} = 0$.

E7.7 The root locus is shown in Figure E7.7. The characteristic equation has 4 poles

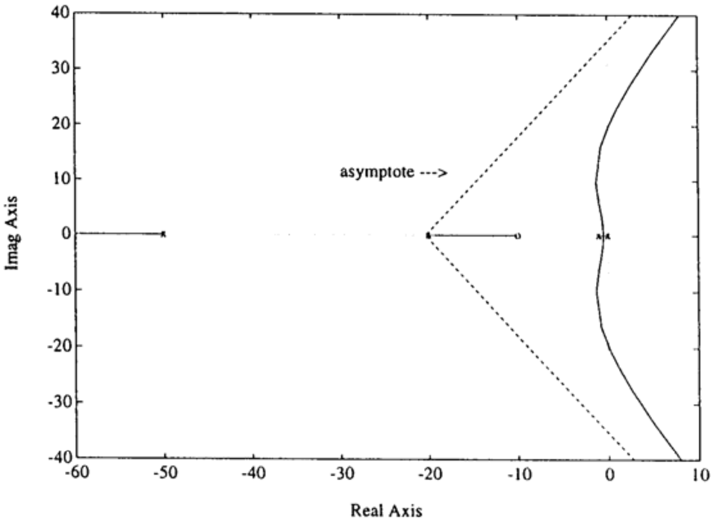

FIGURE E7.7
Root locus for $1 + K\frac{s+10}{s(s+1)(s+20)(s+50)} = 0$.

and 1 zero. The asymptote angles are $\phi = +60^o, -60^o, -180^o$ centered at $\sigma_{cent} = -20.3$. It is difficult to see from the root locus plot (very near the origin), but when $K \approx 40$ then $\zeta = 0.8$ for the complex roots.

E7.9 The characteristic equation is

$$1 + K\frac{1}{s(s^2 + 2s + 5)} = 0$$

or

$$s^3 + 2s^2 + 5s + K = 0 \, .$$

(a) The system has three poles at $s = 0$ and $-1 \pm j2$. The number of asymptotes is $n_p - n_z = 3$ centered at $\sigma_{cent} = -2/3$, and the angles are ϕ_{asymp} at $\pm 60^o$, 180^o.

(b) The angle of departure, θ_d, is $90^o + \theta_d + 116.6^o = 180^o$, so $\theta_d = -26.6^o$.

(c) The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 5 \\ s^2 & 2 & K \\ s^1 & b & \\ s^o & K & \end{array}$$

where $b = 5 - K/2$. So, when $K = 10$ the roots lie on the imaginary axis.

The auxilary equation is

$$2s^2 + 10 = 0 \quad \text{which implies} \quad s_{1,2} = \pm j\sqrt{5} \, .$$

(d) The root locus is shown in Figure E7.9.

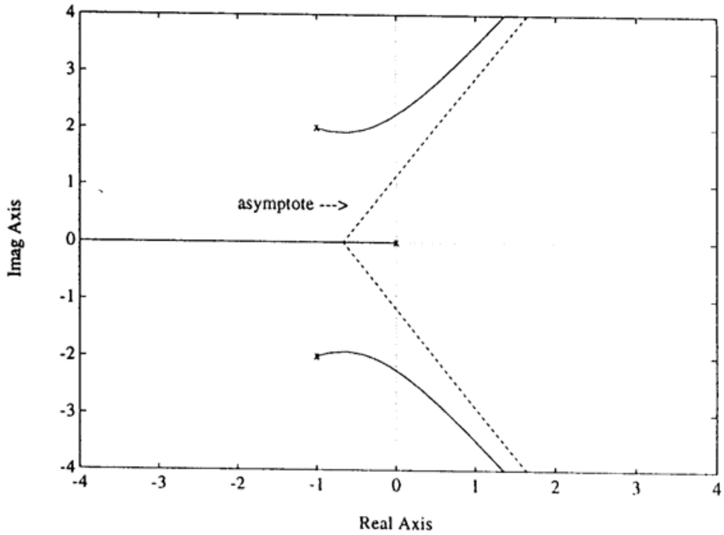

FIGURE E7.9
Root locus for $1 + K\frac{1}{s(s^2+2s+5)} = 0$.

E7.13 (a) The characteristic equation is

$$s(s+1)(s+3) + 4s + 4z = 0 \, .$$

Rewriting with z as the parameter of interest yields

$$1 + z\frac{4}{s(s+1)(s+3) + 4s} = 0 \, .$$

The root locus is shown in Figure E7.13a.

(b) The root locations for

$$z = 0.6 \, , \quad 2.0 \, , \quad \text{and} \ 4.0$$

are shown in Figure E7.13a. When $z = 0.6$, we have $\zeta = 0.76$ and $\omega_n = 2.33$. Therefore, the predicted step response is

$$P.O. = 2.4\% \quad \text{and} \quad T_s = 2.3 \text{ sec} \quad (\zeta = 0.6) \, .$$

When $z = 2.0$, we have $\zeta = 0.42$ and $\omega_n = 1.79$. Therefore, the predicted step response is

$$P.O. = 23\% \quad \text{and} \quad T_s = 5.3 \text{ sec} \quad (\zeta = 2.0) \, .$$

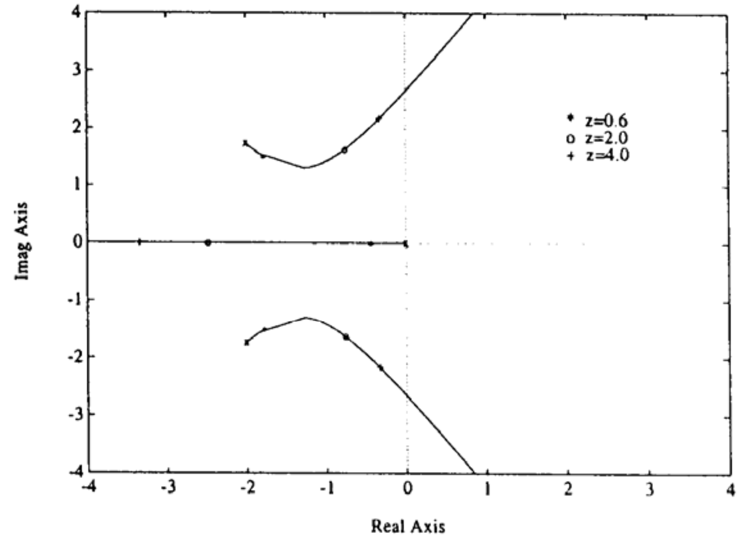

FIGURE E7.13
(a) Root locus for $1 + z\frac{4}{s(s+1)(s+3)+4s} = 0$.

Finally, when $z = 4.0$, we have $\zeta = 0.15$ and $\omega_n = 2.19$. Therefore, the predicted step response is $P.O. = 62\%$ and $T_s = 12$ sec.

(c) The actual step responses are shown in Figure E7.13b.

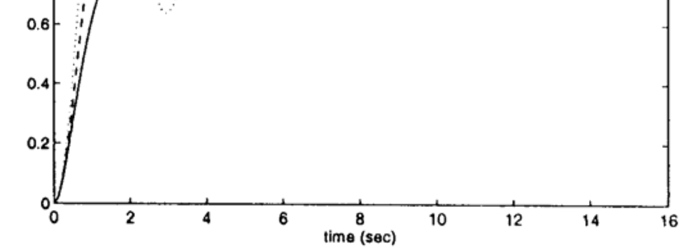

FIGURE E7.13
CONTINUED: (b) Step Responses for $z = 0.6, 2.0$, and 4.0.

E7.15 (a) The characteristic equation

$$1 + K\frac{(s+1)(s+3)}{s^3} = 0$$

has the root locus in Figure E7.15.

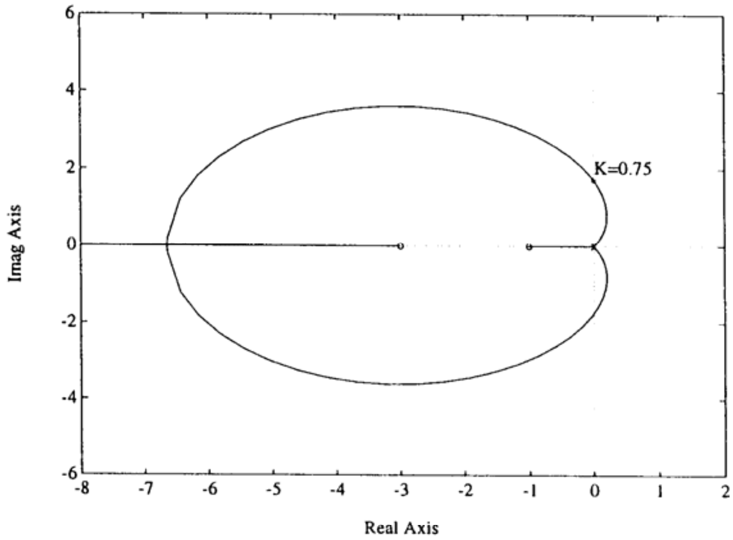

FIGURE E7.15
Root locus for $1 + \frac{K(s+1)(s+3)}{s^3} = 0$.

(b) The Routh array is

s^3	1	$4K$
s^2	K	$3K$
s^1	b	
s^o	$3K$	

when $b = 4K - 3$. For stability, we require all elements in the first column to be positive. Therefore,

$$K > 3/4\,.$$

(c) When $K > 3/4$, we have

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\frac{1}{1+GH(s)}\cdot\frac{1}{s^2} = \lim_{s\to 0}\frac{s^2}{s^3 + K(s+1)(s+3)} = 0\,.$$

E7.20 The characteristic equation is

$$1 + \frac{K(s+1)}{s(s-1)(s+4)} = 0 ,$$

and the root locus is shown in Figure E7.20. The system is stable for

$$K > 6 .$$

The maximum damping ratio of the stable complex roots is

$$\zeta = 0.2 .$$

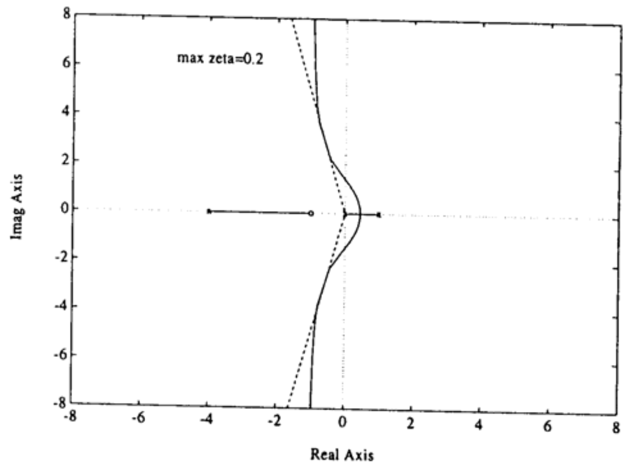

FIGURE E7.20
Root locus for $1 + \frac{K(s+1)}{s(s-1)(s+4)} = 0$.

E7.21 The root locus is shown in Figure E7.21.

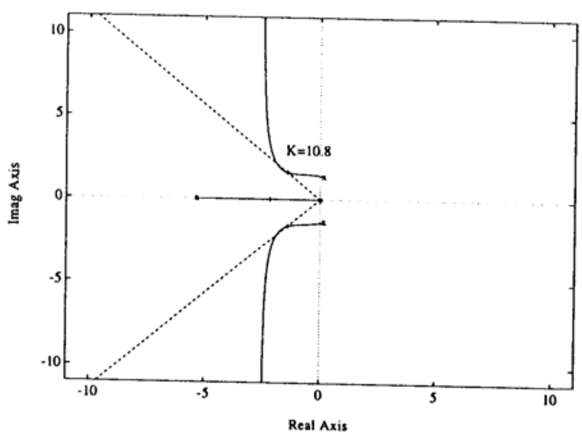

FIGURE E7.21
Root locus for $1 + \frac{Ks}{s^3+5s^2+10} = 0$.

P7.15 The characteristic equation is

$$1 + \frac{K(s^2 + 30s + 625)}{s(s+20)(s^2 + 20s + 200)(s^2 + 60s + 3400)} .$$

The root locus is shown in Figure P7.15. When

$$K = 30000 ,$$

the roots are

$$s_1 = -18.5$$
$$s_2 = -1.69$$
$$s_{3,4} = -9.8 \pm j8.9$$

and

$$s_{5,6} = -30.1 \pm j49.9 .$$

The real root near the origin dominates, and the step response is overdamped.

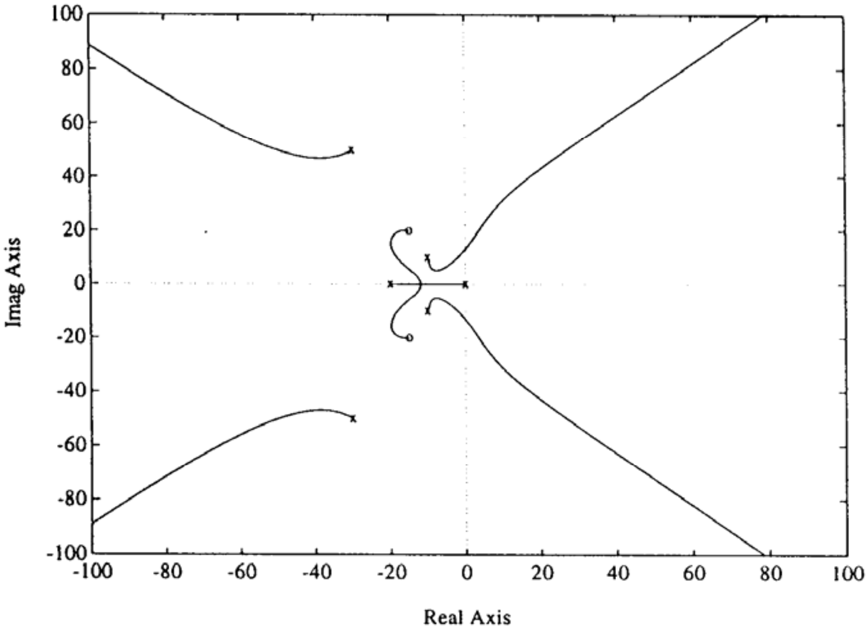

FIGURE P7.15
Root locus for $1 + K\frac{s^2+30s+625}{s(s+20)(s^2+20s+200)(s^2+60s+3400)} = 0$.

MP7.2 The maximum value of the gain for stability is

$$k = 0.791 .$$

The MATLAB script and root locus is shown in Figure MP7.2.

```
num=[1 -2 2]; den=[1 3 2 0];
rlocus(num,den)
rlocfind(num,den)
```

```
»Select a point in the graphics window

selected_point =
 -0.0115 - 0.6337i


ans =

   0.7515
```

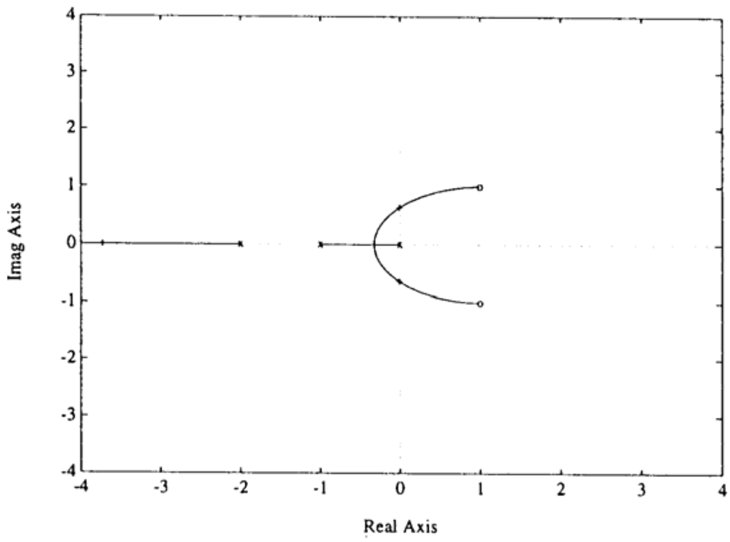

FIGURE MP7.2
Using the **rlocfind** function.

The value of $k = 0.7515$ selected by the **rlocfind** function is not exact since you cannot select the $j\omega$-axis crossing precisely. The actual value is determined using Routh-Hurwitz analysis.