E6.4 The closed-loop transfer function is

$$T(s) = \frac{-K(s-1)}{s^3 + 3s^2 + (2-K)s + K} .$$

Therefore, the characteristic equation is

$$s^3 + 3s^2 + (2-K)s + K = 0 .$$

The corresponding Routh array is given by

$$\begin{array}{c|cc} s^3 & 1 & (2-K) \\ s^2 & 3 & K \\ s^1 & b & 0 \\ s^o & K & \end{array}$$

where

$$b = \frac{3(2-K) - K}{3} = \frac{6-4K}{3} .$$

For stability we require $K > 0$ and $b > 0$. Thus, the range of K for stability is $0 < K < 1.5$.

E6.14 The Routh array is

$$\begin{array}{c|ccc} s^4 & 1 & 45 & 50 \\ s^3 & 9 & 87 & \\ s^2 & 35.33 & 50 & \\ s^1 & 74.26 & 0 & \\ s^o & 50 & & \end{array}$$

The system is stable. The roots of $q(s)$ are $s_{1,2} = -3 \pm j4$, $s_3 = -2$ and $s_4 = -1$.

P6.1

(e) Given

$$s^4 + s^3 + 3s^2 + 2s + K ,$$

we have the Routh array

$$\begin{array}{c|ccc} s^4 & 1 & 3 & K \\ s^3 & 1 & 2 & 0 \\ s^2 & 1 & K & \\ s^1 & 2-K & 0 & \\ s^o & K & & \end{array}$$

Examining the first column, we determine that the system is stable for $0 < K < 2$.

(g) Given

$$s^5 + s^4 + 2s^3 + s^2 + s + K ,$$

we have the Routh array

s^5	1	2	1
s^4	1	1	K
s^3	1	$1 - K$	
s^2	K	K	
s^1	$-K$	0	
s^o	K		

Examining the first column, we determine that for stability we need $K > 0$ and $K < 0$. Therefore the system is unstable for all K.

P6.3 (a) Given

$$G(s) = \frac{K}{(s+1)(s+2)(0.5s+1)} ,$$

and

$$H(s) = \frac{1}{0.005s + 1} ,$$

the closed-loop transfer function is

$$T(s) = \frac{K(0.005s+1)}{0.0025s^4 + 0.5125s^3 + 2.52s^2 + 4.01s + 2 + K} .$$

Therefore, the characteristic equation is

$$0.0025s^4 + 0.5125s^3 + 2.52s^2 + 4.01s + (2 + K) = 0 .$$

The Routh array is given by

s^4	0.0025	2.52	$2 + K$
s^3	0.5125	4.01	0
s^2	2.50	$2 + K$	
s^1	$3.6 - 0.205K$	0	
s^o	$2 + K$		

Examining the first column, we determine that for stability we require

$$-2 < K < 17.6 .$$

(b) Using $K = 9$, the roots of the characteristic equation are

$$s_1 = -200 , \quad s_{2,3} = -0.33 \pm 2.23j , \quad \text{and} \quad s_4 = -4.35 .$$

Assuming the complex roots are dominant, we compute the damping ratio $\zeta = 0.15$. Therefore, we estimate the percent overshoot as

$$P.O. = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 62\% .$$

The actual overshoot is 27%, so we see that assuming that the complex poles are dominant does not lead to accurate predictions of the system response.

P6.11 Given

$$s^3 + (1 + K)s^2 + 10s + (5 + 15K) = 0 ,$$

the Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 10 \\ s^2 & 1+K & 5+15K \\ s^1 & b & \\ s^o & 5+15K & \end{array}$$

where

$$b = \frac{(1+K)10 - (5+15K)}{1+K} = \frac{5-5K}{1+K} .$$

Given that $K > 0$, we determine that the system is stable when $5 - 5K > 0$ or

$$0 < K < 1 .$$

When $K = 1$, the s^2 row yields the auxilary equation

$$2s^2 + 20 = 0 .$$

The roots are $s = \pm j\sqrt{10}$. So, the system frequency of oscillation is $\sqrt{10}$ rads/sec.

AP6.3 (a) The steady-state tracking error to a step input is

$$e_{ss} = \lim_{s \to 0} s(1 - T(s))R(s) = 1 - T(0) = 1 - \alpha .$$

We want

$$|1 - \alpha| < 0.05 .$$

This yields the bounds for α

$$0.95 < \alpha < 1.05 .$$

(b) The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & \alpha \\ s^2 & 1+\alpha & 1 \\ s^1 & b & 0 \\ s^o & 1 & \end{array}$$

where

$$b = \frac{\alpha^2 + \alpha - 1}{1 + \alpha} .$$

Therefore, using the condition that $b > 0$, we obtain the stability range for α:

$$\alpha > 0.618 .$$

(c) Choosing $\alpha = 1$ satisfies both the steady-state tracking requirement and the stability requirement.

DP6.2 (a) The closed-loop characteristic equation is

$$1+\frac{10(Ks+1)}{s^2(s+10)}=0 ,$$

or

$$s^3+10s^2+10Ks+10=0 .$$

The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 10K \\ s^2 & 10 & 10 \\ s^1 & b & \\ s^o & 1 & \end{array}$$

where

$$b=\frac{10K-1}{1} .$$

For stability, we require $K > 0.1$.

(b) The desired characteristic polynomial is

$$(s^2+as+b)(s+5)=s^3+s^2(a+5)+s(5a+b)+5b=0 .$$

Equating coefficients with the actual characteristic equation we can solve for a, b and K, yielding $b=2$, $a=5$, and

$$K=\frac{5a+b}{10}=\frac{27}{10} .$$

(c) The remaining two poles are $s_1=-4.56$ and $s_2=-0.438$.

(d) The step response is shown in Figure DP6.2.

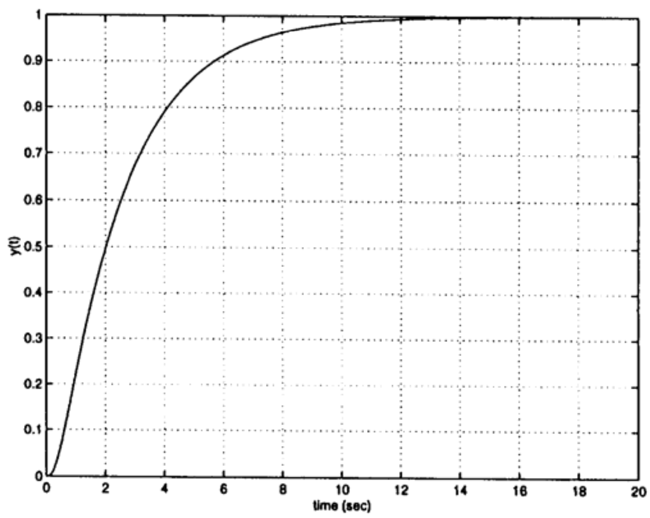

FIGURE DP6.2
Mars guided vehicle step response.

MP6.2 The MATLAB script is shown in Figure MP6.2.

```
K1=1;K2=2;K3=5; den=[1 2 1];
num1=K1*[1 -1 2];num2=K2*[1 -1 2];num3=K3*[1 -1 2];
[n1,d1]=feedback(num1,den,1,1);
[n2,d2]=feedback(num2,den,1,1);
[n3,d3]=feedback(num3,den,1,1);
roots(d1),roots(d2),roots(d3),
```

```
ans =
 -2.5000e-01 + 1.1990e+00i
 -2.5000e-01 - 1.1990e+00i
```

```
ans =
        0 + 1.2910e+00i
        0 - 1.2910e+00i
```

```
ans =
  2.5000e-01 + 1.3307e+00i
  2.5000e-01 - 1.3307e+00i
```

FIGURE MP6.2
$K = 1$ is stable; $K = 2$ is marginally stable; and $K = 5$ is unstable.