E5.8 (a) The closed-loop transfer function is

$$T(s) = \frac{K}{s^2 + \sqrt{2K}s + K} .$$

The damping ratio is

$$\zeta = \frac{\sqrt{2}}{2}$$

and the natural frequency is $\omega_n = \sqrt{K}$. Therefore, we compute the percent overshoot to be

$$P.O. = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 4.3\%$$

for $\zeta = 0.707$. The settling time is estimated via

$$T_s = \frac{4}{\zeta\omega_n} = \frac{8}{\sqrt{2K}} .$$

(b) The settling time is less than 1 second whenever $K > 32$.

E5.9 The second-order closed-loop transfer function is given by

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} .$$

From the percent overshoot specification, we determine that

$$P.O. \leq 5\% \quad \text{implies} \quad \zeta \geq 0.69 .$$

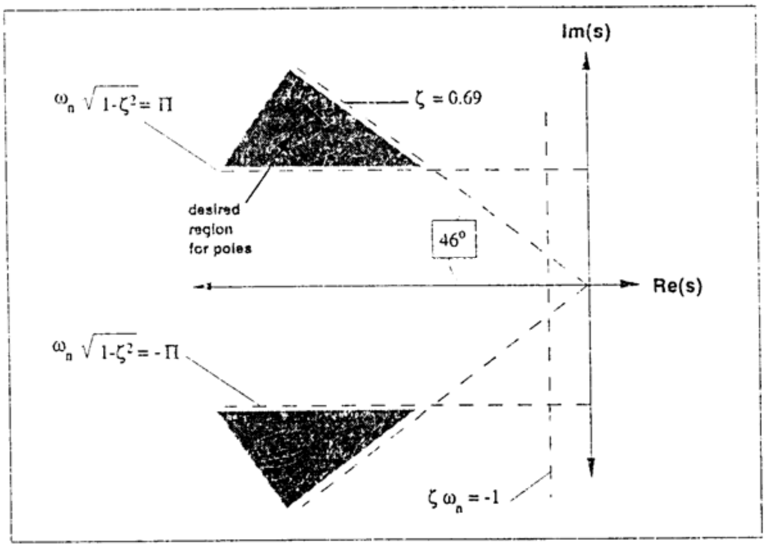
From the settling time specification, we find that

$$T_s < 4 \quad \text{implies} \quad \omega_n\zeta > 1 .$$

And finally, from the peak time specification we have

$$T_p < 1 \quad \text{implies} \quad \omega_n\sqrt{1-\zeta^2} > \pi .$$

The constraints imposed on ζ and ω_n by the performance specifications define the permissible area for the poles of $T(s)$, as shown in Figure E5.9.



E5.10 The system is a **type** 1. The error constants are

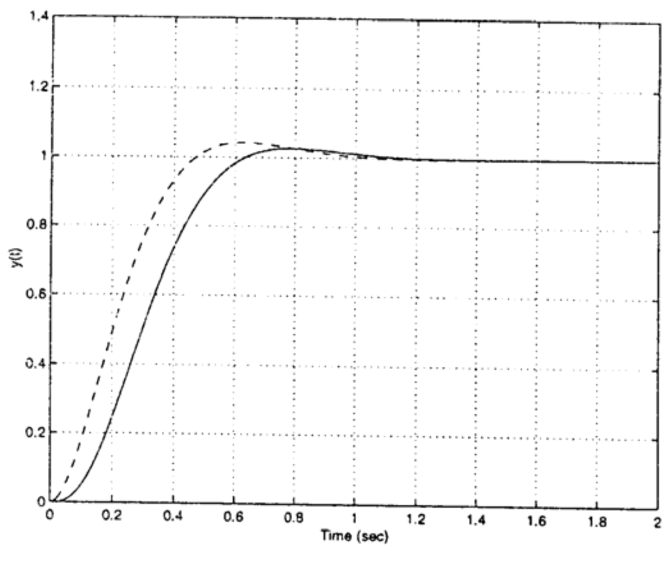
$$K_p = \infty \quad \text{and} \quad K_v = 4\,.$$

Therefore, the steady-state error to a step input is 0; the steady-state error to a ramp input is 0.25.

E5.12 The system is a **type 0**. The error constants are $K_p = 0.2$ and $K_v = 0$. The steady-state error to a ramp input is ∞. The steady-state error to a step input is

$$e_{ss} = \frac{1}{1+K_p} = 0.833.$$

E5.14 The plot of $y(t)$ is shown in Figure E5.14.



Using the dominant poles, the approximate closed-loop transfer function is

$$T_a(s) = \frac{50}{s^2 + 10s + 50}\,.$$

The actual transfer function is

$$T(s) = \frac{500}{(s+10)(s^2+10s+50)}\,.$$

E5.16 The desired pole locations for the 5 different cases are shown in Figure E5.16.

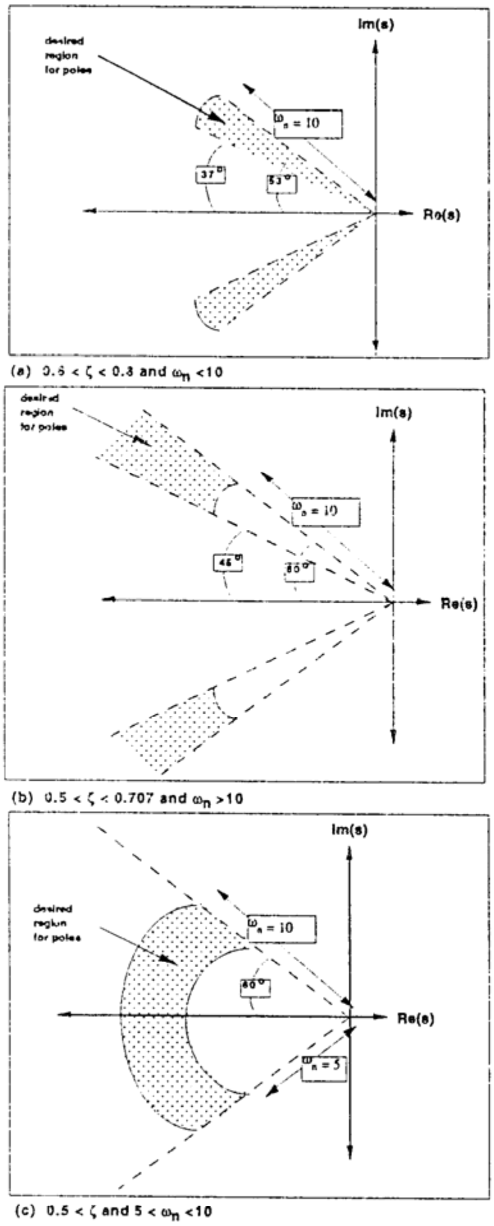


FIGURE E5.16
Desired pole locations.

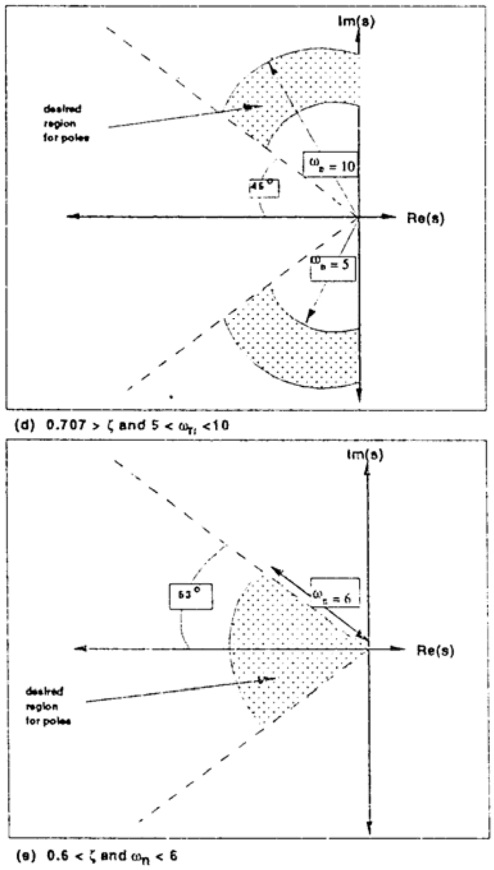


FIGURE E5.16
CONTINUED: Desired pole locations.

P5.19 The steady-state error is

$$e_{ss} = \lim_{s \to 0} \frac{(s+10)(s+12) + K(1-K_1)}{(s+10)(s+12)+K} = \frac{120 + K(1-K_1)}{120+K} .$$

To achieve a zero steady-state tracking error, select K_1 as follows:

$$K_1 = 1 + \frac{120}{K} .$$

AP5.6 (a) The closed-loop transfer function is

$$T(s) = \frac{KK_m}{KK_m + s(s + K_mK_b + 0.01)} .$$

The steady-state tracking error for a ramp input $R(s) = 1/s^2$ is

$$\begin{aligned} e_{ss} &= \lim_{s\to 0} s(1 - T(s))R(s) \\ &= \lim_{s\to 0} \frac{s + K_mK_b + 0.01}{KK_m + s(s + K_mK_b + 0.01)} \\ &= \frac{K_mK_b + 0.01}{KK_m} . \end{aligned}$$

(b) With

$$K_m = 10$$

and

$$K_b = 0.05 .$$

we have

$$\frac{K_mK_b + 0.01}{KK_m} = \frac{10(0.05) + 0.01}{10K} = 1 .$$

Solving for K yields

$$K = 0.051 .$$

(c) The plot of the step and ramp responses are shown in Figure AP5.6. The responses are acceptable.

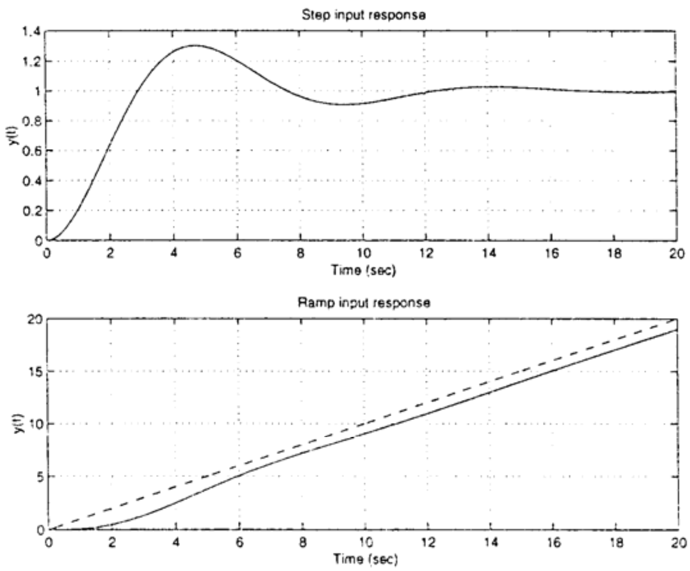


FIGURE AP5.6
Closed-loop system step and ramp responses.

MP5.2 The ramp response is shown in Figure MP5.2. The error as $t \to \infty$ is $e_{ss} \to \infty$.

```
n=[1 5]; d=[1 10 0 0];
t=[0:0.1:25];
u=t;
lsim(n,d,u,t);
```

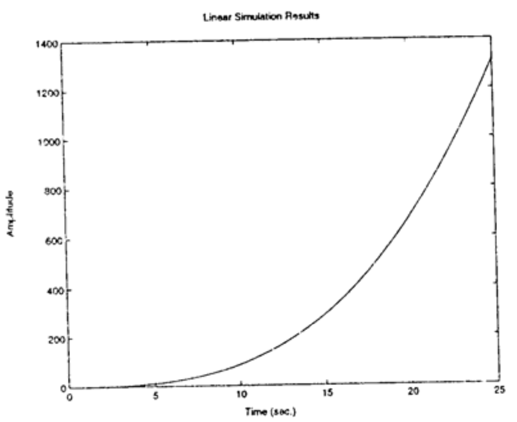


FIGURE MP5.2
Ramp responses.

MP5.5 The unit step response is shown in Figure MP5.5. The performance numbers are as follows:

$$M_p = 1.3 \quad \text{and} \quad T_p = 0.47 \quad \text{and} \quad T_s = 1.55 \, .$$

```
numg=[50]; deng=[1 5 0];
[num,den]=cloop(numg,deng);
t=[0:0.01:2];
[y,x]=step(num,den,t);
plot(t,y,[0 2],[0.98 0.98],[0 2],[1.02 1.02]),grid
xlabel('Time [sec]')
ylabel('y(t)')
```