EE 380 HW #3

E3.2 We know that the velocity is the derivative of the position, therefore we have

$$\frac{dy}{dt} = v \,,$$

and from the problem statement

$$\frac{dv}{dt} = -k_1 v(t) - k_2 y(t) + k_3 i(t) \,.$$

This can be written in matrix form as

$$\frac{d}{dt}\begin{pmatrix} y \\ v \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -k_2 & -k_1 \end{bmatrix}\begin{pmatrix} y \\ v \end{pmatrix} + \begin{bmatrix} 0 \\ k_3 \end{bmatrix} i \,.$$

Define $u = i$, and let $k_1 = k_2 = 1$. Then,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ k_3 \end{bmatrix}, \text{ and } \quad \mathbf{x} = \begin{pmatrix} y \\ v \end{pmatrix} \,.$$

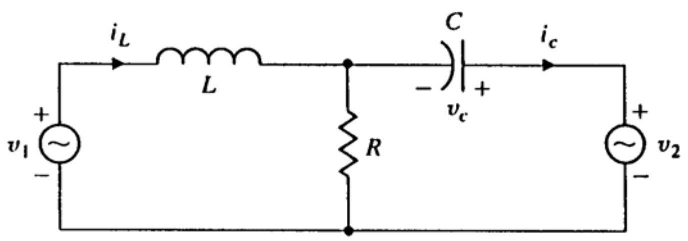


FIGURE P3.3 *RLC* circuit.

P3.3 The signal flow graph is shown in Figure P3.3. Using Kirchoff's voltage law

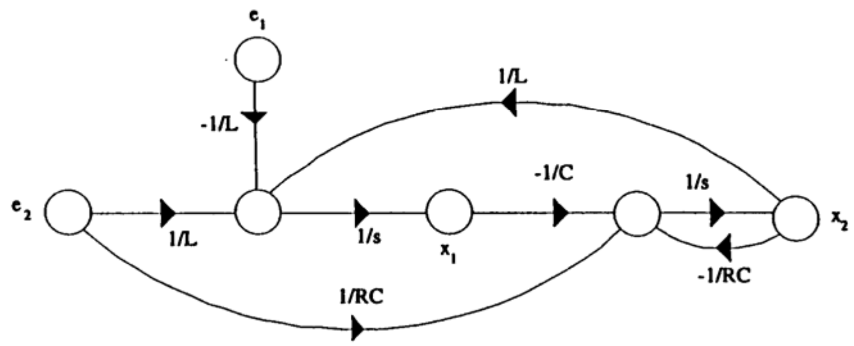


FIGURE P3.3
Signal flow graph.

around the outer loop, we have

$$L\frac{di_L}{dt} - v_c + e_2 - e_1 = 0\,.$$

Then, using Kirchoff's current law at the node, we determine that

$$C\frac{dv_c}{dt} = -i_L + i_R\,,$$

where i_R is the current through the resistor R. Considering the right loop we have

$$i_R R - e_2 + v_c = 0 \quad \text{or} \quad i_R = -\frac{v_c}{R} + \frac{e_2}{R}\,.$$

Thus,

$$\frac{dv_c}{dt} = -\frac{v_c}{RC} - \frac{i_L}{C} + \frac{e_2}{RC} \quad \text{and} \quad \frac{di_L}{dt} = \frac{v_c}{dt} + \frac{e_1}{L} - \frac{e_2}{L}\,.$$

In matrix form, the state equations are

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} 0 & 1/L \\ -1/C & -1/RC \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} 1/L & -1/L \\ 0 & 1/RC \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix},$$

where $x_1 = i_L$ and $x_2 = v_c$.

P3.5 (a) The closed-loop transfer function is

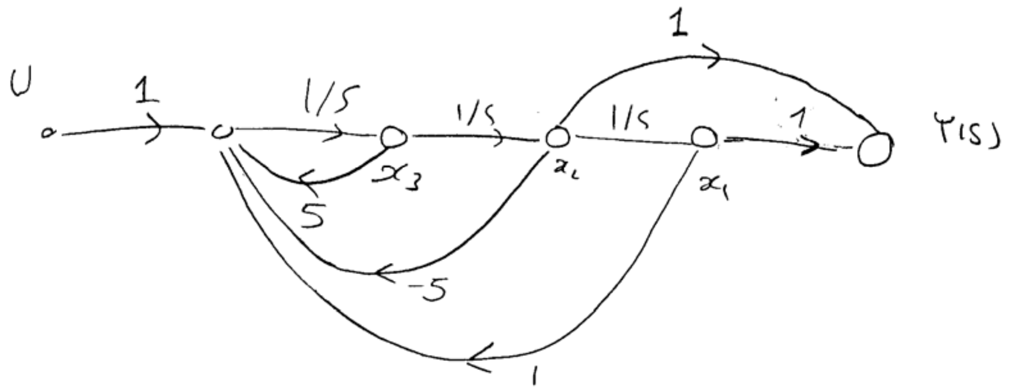
$$T(s) = \frac{s+1}{s^3 + 5s^2 - 5s + 1}.$$

(b) The matrix differential equation in phase variable form is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 5 & -5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}.$$



P3.25 The matrix representation of the state equations is

$$\dot{\mathbf{x}} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d.$$

When $u_1 = 0$ and $u_2 = d = 1$, we have

$$\dot{x}_1 = 3x_1 + u_2$$

$$\dot{x}_2 = 2x_2 + 2u_2$$

So we see that we have two independent equations for x_1 and x_2. With $U_2(s) = 1/s$ and zero initial conditions, the solution for x_1 is found to be

$$x_1(t) = \mathcal{L}^{-1}\{X_1(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s-3)}\right\}$$
$$= \mathcal{L}^{-1}\left\{-\frac{1}{3s} + \frac{1}{3}\frac{1}{s-3}\right\} = -\frac{1}{3}\left(1 - e^{3t}\right)$$

and the solution for x_2 is

$$x_2(t) = \mathcal{L}^{-1}\{X_2(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s(s-2)}\right\} = \mathcal{L}^{-1}\left\{-\frac{1}{s} + \frac{1}{s-2}\right\} = -1 + e^{2t}.$$

DP3.2 The desired transfer function is

$$\frac{Y(s)}{U(s)} = \frac{10}{s^2 + 4s + 3} .$$

The transfer function derived from the phase variable representation is

$$\frac{Y(s)}{U(s)} = \frac{d}{s^2 + bs + a} .$$

Therefore, we select $d = 10$, $a = 3$ and $b = 4$.

MP3.1 The MATLAB script to compute the state-space models using the **tf2ss** function is shown in Figure MP3.1.

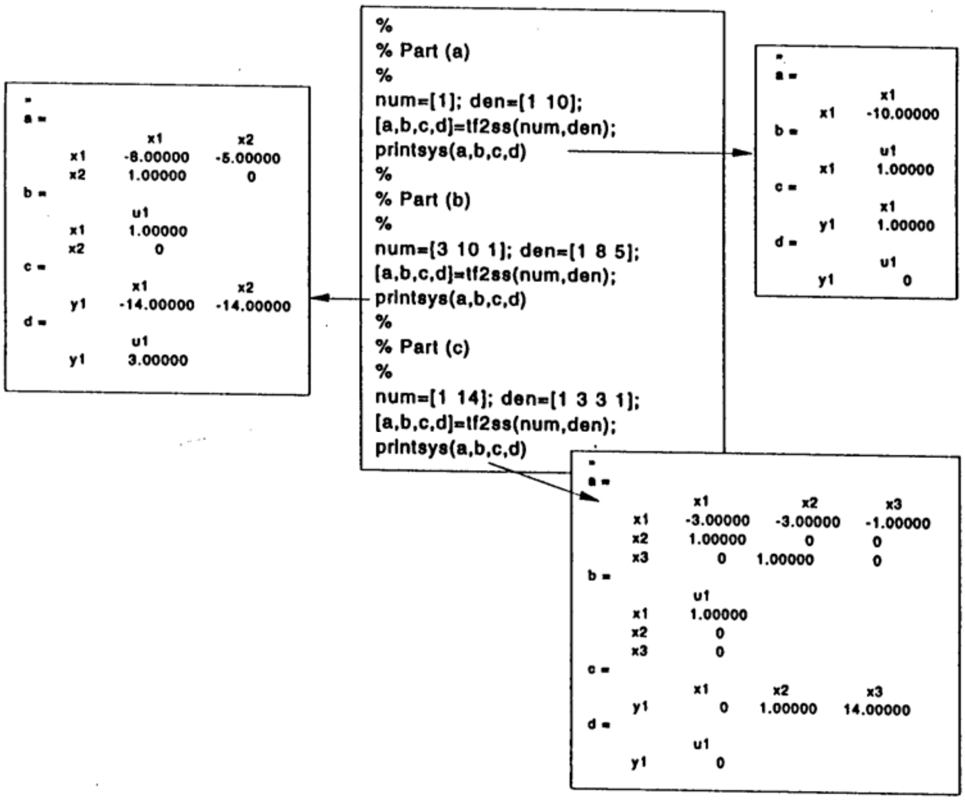


FIGURE MP3.1
Script to compute state-space models from transfer functions.

For example, in part (c) the state-space model is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u ,$$

where $\mathbf{D} = [0]$ and

$$\mathbf{A} = \begin{bmatrix} -3 & -3 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} , \quad \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} , \quad \mathbf{C} = \begin{bmatrix} 0 & 1 & 14 \end{bmatrix} .$$

MP3.4 The MATLAB script and state history is shown in Figure MP3.4.

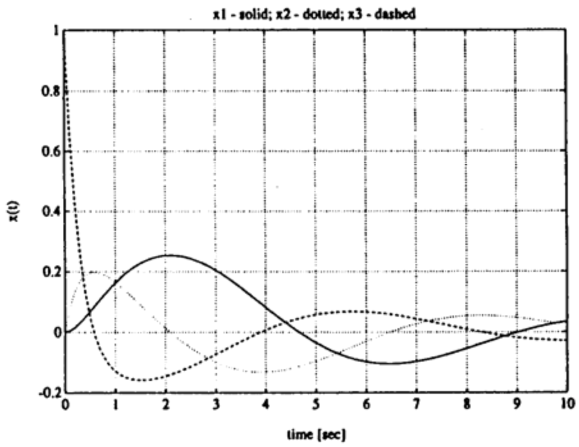


FIGURE MP3.4
(a) The system step response.

```
a=[0 1 0;0 0 1;-2 -2 -4];
b=[0;0;1];
c=[1 0 0];
d=[0];
%
% Part (a)
%
[num,den]=ss2tf(a,b,c,d);
printsys(num,den)
%
% Part (b)
%
x0=[0 0 1];
t=[0:0.1:10];
u=0.0*t;
[y,x]=lsim(a,b,c,d,u,t,x0);
plot(t,x(:,1),t,x(:,2),':',t,x(:,3),'--'), grid
xlabel('time [sec]')
ylabel('x(t)')
title('x1 - solid; x2 - dotted; x3 - dashed')
xf_sim=x(length(t),:)'
%
% Part (c)
%
dt=10;
Phi=expm(a*dt);
xf_phi=Phi*x0'
```

```
num/den =
                 1
      --------------------
      s^3 + 4 s^2 + 2 s + 2
```

```
xf_sim =

    0.0343
    0.0230
   -0.0283
```

```
xf_phi =

    0.0343
    0.0230
   -0.0283
```

FIGURE MP3.4
CONTINUED: (b) The MATLAB script using the **lsim** function.

The transfer function equivalent is

$$G(s) = \frac{1}{s^3 + 4s^2 + 2s + 2} .$$

The computed state vector at $t = 10$ is the same using the simulation and the state transition matrix.