Problem 9.1

$\psi = B_o S \cos\omega t$

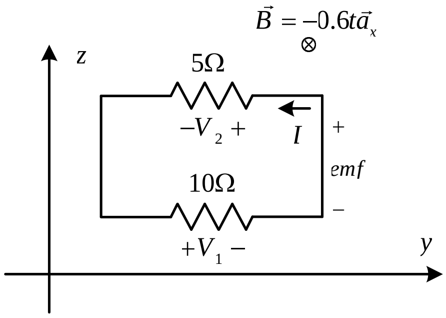
$$emf = -\frac{N d\psi}{dt}$$

$$emf = -\frac{N d(B_o S \cos\omega t)}{dt} = \omega N B_o S \sin\omega t$$

The induced voltage (emf) is $emf = 130 \times 50 \times 0.06 \times (0.3 \times 0.4) \sin 130t = 46.8 \sin 130t$

Thus amplitude of the induced voltage equals $46.8V$.

Problem 9.3



Since $\vec{B}$ is not a function of space, then it is uniform.

$$\psi = \int_S \vec{B}.\vec{ds} = \int_S (-0.6t\vec{a}_x).(-\vec{a}_x dydz) = 0.6t \times S$$

(since $\vec{B}$ is uniform and normal to the area, then the flux equals B times the area. The minus sign in $\vec{ds}$ is needed because the normal to the area is taken in the negative x direction. This is due to the choice of the emf polarity shown in the figure).

$$\psi = -0.6t \times (-10 \times 10^{-4}) = 0.6t \times 10^{-3}$$

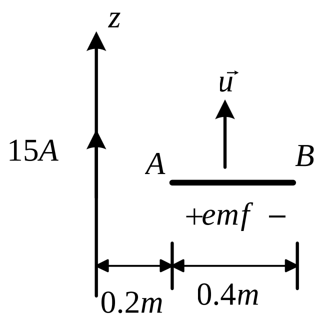
$$emf = -N \frac{d\psi}{dt} = -1 \times \frac{d(0.6t \times 10^{-3})}{dt} = -0.6\, mV$$

The current in the circuit $I = \dfrac{emf}{R} = \dfrac{-0.6 \times 10^{-3}}{(5+10)} = -40 \times 10^{-6}\, A$

$$\therefore \quad V_1 = IR_1 = -40 \times 10^{-6} \times 5 = -0.200 \times 10^{-3} V = -0.2\, mV$$

$$\therefore \quad V_2 = IR_2 = -40 \times 10^{-6} \times 10 = -0.400 \times 10^{-3} V = -0.4\, mV$$

Problem 9.5



$$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\vec{a}_\phi = \frac{4\pi\times10^{-7}\times15}{2\pi\rho}\vec{a}_\phi = \frac{3\times10^{-6}}{\rho}\vec{a}_\phi$$

Only motional emf exists, because the current and thus the magnetic field is time-invariant.

$$emf = \oint_l (\vec{u}\times\vec{B}).\vec{dl} \quad \Rightarrow \quad emf = \int_{\rho=0.6}^{0.2} (3\vec{a}_z \times \frac{3\times10^{-6}}{\rho}\vec{a}_\phi).(d\rho\vec{a}_\rho)$$

$$emf = \int_{\rho=0.6}^{0.2} (-3\frac{9\times10^{-6}}{\rho}\vec{a}_\rho).(d\rho\vec{a}_\rho) = -27\times10^{-6}\ln\frac{0.2}{0.6} = 2.966\times10^{-5} = 29.66\ \mu V$$

[The integration limits were taken from $\rho = 0.6$ to $\rho = 0.2$ (not the other way around). The reason for this is the choice of emf polarity shown in the figure. If the emf polarity is defined opposite to the one shown in the figure, the limit should be taken form $\rho = 0.2$ to $\rho = 0.6$ and the emf will be negative].

Because the emf is positive, then A is a higher potential than B.

Problem 9.6

$$emf = uBL = (80\times 10^{3})\times(70\times 10^{-6})\times 2.5 = 14\ \text{V}$$

Problem 9.8

$\psi = BS$ (because $\vec{B}$ is uniform and normal to the area).

The area S keeps changing with time. Therefore, we need to first find an expression for the area S.

$$u = \frac{dy}{dt} = 2\cos 10t \quad \Rightarrow \quad y = \frac{2}{10}\sin 10t + c = 0.2\sin 10t + c$$

Assuming $y(0) = 0 \quad \Rightarrow \quad c = 0$

$$\therefore \quad y = 0.2\sin 10t$$

The area equals width time length, therefore:

$$S = 5\times(10 + y) = 5(10 + 0.2\sin 10t) = 50 + \sin 10t$$

$$\psi = BS \quad \Rightarrow \quad \psi = 6\times10^{-3}\cos(10t)\times(50 + \sin 10t)$$

$$emf = -\frac{d\psi}{dt} = -[-60\times10^{-3}\sin(10t)]\times(50 + \sin 10t) - [6\times10^{-3}\cos(10t)]\times(10\cos 10t)$$

$$emf = 60\times10^{-3}\sin(10t)\times(50 + \sin 10t) - 6\times10^{-3}\cos(10t)\times(10\cos 10t)$$

$$emf = 3\sin(10t) + 60\times10^{-3}\sin^2 10t - 60\times10^{-3}\cos^2(10t)$$

$$emf = 3\sin(10t) - 60\times10^{-3}\cos 20t \quad \text{V}$$

(note that the polarity of the emf is defined such that the plus sign is at the bottom of the moving bar and therefore the normal to the area is taken in the plus x direction).