Problem 4.45

A dipole located at the origin and aligned with the z -axis has the following electrostatic potential:

$$V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$V(0,1nm) = 9 = \frac{p\cos 0}{4\pi\varepsilon_0 (10^{-9})^2}$$

$$p = 9\times 10^{-18}\times 4\pi\varepsilon_0 = 10^{-27}\ C.m$$

$$\therefore V(1nm,1nm) = \frac{10^{-27}\cos 45^o}{\frac{10^{-9}}{9}(\sqrt{2}\times 10^{-9})^2} = \frac{9}{2\sqrt{2}} = 3.18V$$

<u>Problem 4.49</u>

A dipole located at the origin and aligned with the z -axis has the following electrostatic potential:

$$V = \frac{Qd\ \cos\theta}{4\pi\varepsilon_0 r^2}$$

Since –Q is at (0, -d/2, 0) and +Q is at (0, d/2, 0), the dipole has moment p=Qd and it is oriented along the + y – axis.

$$V = \frac{Qd\ \cos\theta}{4\pi\varepsilon_0 r^2} \quad \overset{r\to r}{\Rightarrow} \quad \& \quad \overset{\cos\theta=\frac{z}{r}\to\frac{y}{r}}{\Rightarrow} \quad V = \frac{Qd\ (\frac{y}{r})}{4\pi\varepsilon_0 r^2} = \frac{yQd}{4\pi\varepsilon_0 r^3} = \frac{(r\ \sin\theta\,\sin\phi)Qd}{4\pi\varepsilon_0 r^3} = \frac{Qd\ \sin\theta\,\sin\phi}{4\pi\varepsilon_0 r^2}$$

$$\therefore V = \frac{Qd\ \sin\theta\sin\phi}{4\pi\varepsilon_0 r^2}$$

Using $\vec{E} = -\nabla V$ in spherical coordinates:

$$\vec{E} = -\nabla V = -(\frac{\partial V}{\partial r}\vec{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\vec{a}_\theta + \frac{1}{r\ \sin\theta}\frac{\partial V}{\partial\phi}\vec{a}_\phi)$$

$$\vec{E} = \frac{Qd\ \sin\theta\sin\phi}{2\pi\varepsilon_0 r^3}\vec{a}_r - \frac{Qd\ \cos\theta\sin\phi}{4\pi\varepsilon_0 r^3}\vec{a}_\theta - \frac{Qd\ \cos\phi}{4\pi\varepsilon_0 r^3}\vec{a}_\phi$$

Problem 4.50

$V = \rho^2 z \sin\phi$. Using $\vec{E} = -\nabla V$ in cylindrical coordinates:

$$\vec{E} = -\nabla V = -(\frac{\partial V}{\partial \rho}\vec{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\vec{a}_\phi + \frac{\partial V}{\partial z}\vec{a}_z) = -(2\rho z \sin\phi \vec{a}_\rho + \rho z \cos\phi \vec{a}_\phi + \rho^2 \sin\phi \vec{a}_z)$$

The electrostatic energy density: $w_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi)$

The electrostatic energy W_e in the given volume:

$$W_e = \int_V w_e dv = \int_{z=-2}^{2}\int_{\phi=0}^{\pi/3}\int_{\rho=1}^{4} \frac{1}{2}\varepsilon_0 (4\rho^2 z^2 \sin^2\phi + \rho^2 z^2 \cos^2\phi + \rho^4 \sin^2\phi)\rho d\rho d\phi dz$$

$$W_e = \frac{1}{2}\varepsilon_0 \int_{\phi=0}^{\pi/3}\int_{\rho=1}^{4} (4\rho^2 \frac{z^3}{3}\sin^2\phi + \rho^2 \frac{z^3}{3}\cos^2\phi + z\rho^4 \sin^2\phi)\rho d\rho d\phi \Big|_{z=-2}^{2}$$

$$= \frac{1}{2}\varepsilon_0 \int_{\phi=0}^{\pi/3}\int_{\rho=1}^{4} (\frac{64}{3}\rho^2 \sin^2\phi + \frac{16}{3}\rho^2 \cos^2\phi + 4\rho^4 \sin^2\phi)\rho d\rho d\phi$$

$$= \frac{1}{2}\varepsilon_0 \int_{\phi=0}^{\pi/3} (\frac{64}{3}\frac{\rho^4}{4}\sin^2\phi + \frac{16}{3}\frac{\rho^4}{4}\cos^2\phi + 4\frac{\rho^6}{6}\sin^2\phi)d\phi \Big|_{\rho=1}^{4}$$

$$= \frac{1}{2}\varepsilon_0 \int_{\phi=0}^{\pi/3} (\frac{64}{3}\frac{255}{4}\sin^2\phi + \frac{16}{3}\frac{255}{4}\cos^2\phi + 4\frac{4095}{6}\sin^2\phi)d\phi$$

$$= \frac{1}{2}\varepsilon_0 \int_{\phi=0}^{\pi/3} (1360\sin^2\phi + 340\cos^2\phi + 2730\sin^2\phi)d\phi$$

$$= \frac{1}{2}\varepsilon_0 \int_{\phi=0}^{\pi/3} (4090\sin^2\phi + 340\cos^2\phi)d\phi$$

It is easy to show that $\int_{\phi=0}^{\pi/3} \sin^2\phi d\phi = 0.30709$ and $\int_{\phi=0}^{\pi/3} \cos^2\phi d\phi = 0.74011$

$$W_e = \frac{1}{2}\varepsilon_0 (4090 \times 0.30709 + 340 \times 0.74011)$$

$$W_e = 6.67 \text{ nJ}$$

Problem 5.4

$$I = \int_S \vec{J} . \overrightarrow{ds}$$

$$= \int_{\rho=0}^{1.6\times10^{-3}} \int_{\phi=0}^{2\pi} \left(\frac{500\vec{a}_z}{\rho}\right).(\vec{a}_z \rho d\rho d\phi)$$

$$= \int_{\rho=0}^{1.6\times10^{-3}} 1000\pi d\rho = 1000\pi \times 1.6\times10^{-3} = \pi \times 1.6 = 5.027\ A$$

Problem 5.7

$$R = \frac{l}{\sigma S}$$

$$10^6\Omega = \frac{0.02m}{\sigma[\pi(0.004m)^2]}$$

$$\therefore \quad \sigma = 3.98\times10^{-4}\,\mathrm{m}/\Omega = 3.98\times10^{-4}\ \mathrm{S}.m$$