

8.8 a)

$$H_{1t} - H_{2t} = k \quad (\text{eqn. 8.43})$$

$$\vec{M} = \chi_m \vec{H} \Rightarrow M_t = \chi_m H_t$$

$$\therefore \frac{M_{1t}}{\chi_{m1}} - \frac{M_{2t}}{\chi_{m2}} = k$$

$$B_{1n} = B_{2n} \quad (8.41)$$

$$\therefore \mu_1 H_{1n} = \mu_2 H_{2n}$$

$$\therefore \frac{\mu_1}{\chi_{m1}} M_{1n} = \frac{\mu_2}{\chi_{m2}} M_{2n}$$

b) For a boundary with surface current density  $k$ :

$$H_{1t} - H_{2t} = k$$

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = k$$

$$\frac{B_1 \sin \theta_1}{\mu_1} - \frac{B_2 \sin \theta_2}{\mu_2} = k \quad (1)$$

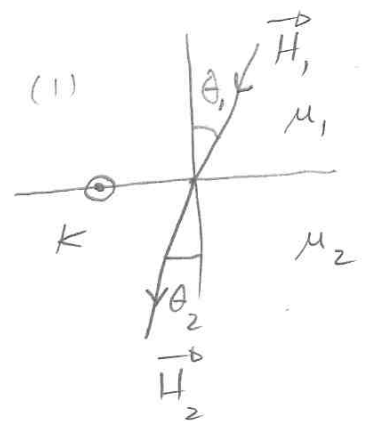
$$B_{1n} = B_{2n} \Rightarrow B_1 \cos \theta_1 = B_2 \cos \theta_2 \quad (2)$$

Divide (1) by (2)  $\Rightarrow$

$$\frac{\tan \theta_1}{\mu_1} - \frac{\tan \theta_2}{\mu_2} = \frac{k}{B_2 \cos \theta_2} \quad \left( \times \frac{\mu_1}{\tan \theta_2} \right)$$

$$\frac{\tan \theta_1}{\tan \theta_2} - \frac{\mu_1}{\mu_2} = \frac{\mu_1 k}{B_2 \tan \theta_2 \cos \theta_2} = \frac{\mu_1 k}{B_2 \sin \theta_2}$$

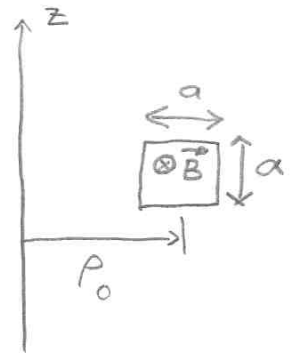
$$\therefore \frac{\tan \theta_1}{\tan \theta_2} = \frac{\mu_1}{\mu_2} \left[ 1 + \frac{\mu_2 k}{B_2 \sin \theta_2} \right]$$



8.9 a)  $\oint \vec{H} \cdot d\vec{\ell} = NI$

$$\therefore H 2\pi \rho = NI \Rightarrow \vec{H} = \frac{NI}{2\pi\rho} \vec{a}_\phi$$

$$\therefore \vec{B} = \frac{\mu_0 NI}{2\pi\rho} \vec{a}_\phi \quad (\text{inside the core})$$



$$\psi = \int_S \vec{B} \cdot d\vec{S} = \int_{z=0}^a \int_{\rho=\rho_0-\frac{a}{2}}^{\rho_0+\frac{a}{2}} \left( \frac{\mu_0 NI}{2\pi\rho} \vec{a}_\phi \right) \cdot (\vec{a}_\phi d\rho dz)$$

$$z=0 \quad \rho = \rho_0 - \frac{a}{2}$$

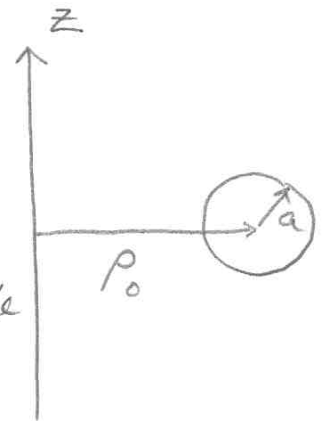
$$= \frac{\mu_0 NI}{2\pi} a \ln \frac{\rho_0 + \frac{a}{2}}{\rho_0 - \frac{a}{2}} = \frac{\mu_0 NI a}{2\pi} \ln \left( \frac{2\rho_0 + a}{2\rho_0 - a} \right)$$

$$L = \frac{\Lambda}{I} = \frac{N\psi}{I} = \frac{\mu_0 N^2 a}{2\pi} \ln \left( \frac{2\rho_0 + a}{2\rho_0 - a} \right)$$

b)  $\vec{B} = \frac{\mu_0 NI}{2\pi\rho} \vec{a}_\phi$  (inside the core).

since  $\rho_0 \gg a$ , then:

$$\vec{B} \approx \frac{\mu_0 NI}{2\pi\rho_0} \vec{a}_\phi \quad (\text{uniform inside the core}).$$



$$\therefore \psi = BS = \frac{\mu_0 NI}{2\pi\rho_0} \pi a^2 = \frac{\mu_0 NI a^2}{2\rho_0}$$

$$\therefore L = \frac{\Lambda}{I} = \frac{N\psi}{I} = \frac{\mu_0 N^2 a^2}{2\rho_0}, \quad (\rho_0 \gg a)$$

8.10

$I_1$  passed  $\Rightarrow H_1$

$$H_1 \approx n_1 I_1$$

$$= \frac{N_1}{l_1} I_1$$

$$\therefore B_1 = \frac{\mu N_1 I_1}{l_1}$$

$$\psi_{21} = B_1 S_2 = \frac{\mu N_1 I_1}{l_1} (\pi r_2^2)$$

$$= \frac{\mu N_1 I_1}{l_1} (\pi r_1^2)$$

$$\Lambda_{21} = N_2 \psi_{21} = \frac{\mu N_1 N_2 I_1 \pi r_1^2}{l_1}$$

$$M_{21} = \frac{\Lambda_{21}}{I_1} = \frac{\mu N_1 N_2 \pi r_1^2}{l_1}$$

