10.4

a) $\dfrac{\sigma}{\omega\epsilon} = \dfrac{10^{-6}}{2\pi \times 10^{7} \times 5 \times \dfrac{10^{-9}}{36\pi}} = 3.6 \times 10^{-4} \Rightarrow$ low loss dielectric.

b) In the two special cases (lossless dielectric or low-loss dielectric), we have:

$$u = \frac{1}{\sqrt{\mu\epsilon}} \quad , \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\therefore \quad u = \frac{c}{\sqrt{750 \times 5}} = \frac{3 \times 10^{8}}{\sqrt{750 \times 5}} = 4.9 \times 10^{6} \text{ m/s}$$

$$= \frac{\omega}{\beta} = \frac{2\pi \times 10^{7}}{\beta} \Rightarrow \beta = 12.823 \ \frac{\text{rad}}{\text{m}}$$

$$\lambda = \frac{2\pi}{\beta} = 0.49 \text{ m}.$$

c) phase difference $= \beta \times$ distance $= (12.823 \frac{\text{rad}}{\text{m}}) \times (2\text{m})$

$= 25.646$ rad $\equiv 29.41^{\circ}$

d) $\eta \simeq 377 \sqrt{\dfrac{\mu_r}{\epsilon_r}} = 377 \sqrt{\dfrac{750}{5}} = 4617.3 \ \Omega$

10.5 $\mathcal{P}_{avg}(z) = \frac{1}{T}\int_0^T \mathcal{P}(z,t)\,dt$

$$= \frac{1}{T}\int_0^T \left[E_0 e^{-\alpha z}\cos(\omega t - \beta z + \theta_\eta)\vec{a}_x\right] \times \left[\frac{E_0}{|\eta|} e^{-\alpha z}\cos(\omega t - \beta z)\vec{a}_y\right] dt$$

$$= \vec{a}_z \frac{E_0^2}{|\eta|} e^{-2\alpha z} \times \frac{1}{T}\int_0^T \cos(\omega t - \beta z + \theta_\eta)\cos(\omega t - \beta z)\,dt$$

$$= \vec{a}_z \frac{E_0^2}{|\eta|} e^{-2\alpha z} \times \frac{1}{T}\int_0^T \left[\cos(\omega t - \beta z)\cos\theta_\eta - \sin(\omega t - \beta z)\sin\theta_\eta\right] \times \cos(\omega t - \beta z)\,dt$$

$$= \vec{a}_z \frac{E_0^2}{|\eta|} e^{-2\alpha z} \times \frac{1}{T}\int_0^T \left[\cos\theta_\eta \cos^2(\omega t - \beta z) - \sin\theta_\eta \sin(\omega t - \beta z)\cos(\omega t - \beta z)\right] dz$$

$$= \frac{E_0^2}{|\eta|} e^{-2\alpha z} \times \frac{1}{T}\left\{\frac{1}{2}\cos\theta_\eta T - 0 \times \sin\theta_\eta\right\}\vec{a}_z$$

$$= \frac{1}{2}\frac{E_0^2}{|\eta|} e^{-2\alpha z}\cos\theta_\eta\,\vec{a}_z$$

$$\vec{\mathcal{P}}_{avg} = \frac{1}{2}\mathrm{Re}\left\{\left(E_0 e^{-\alpha z} e^{-j\beta z - j\theta_\eta}\vec{a}_x\right) \times \left(\frac{E_0}{|\eta|} e^{-\alpha z} e^{-j\beta z}\vec{a}_y\right)^*\right\}$$

$$= \frac{1}{2}\mathrm{Re}\left\{\frac{E_0^2}{|\eta|} e^{-2\alpha z} e^{-j\theta_\eta}\vec{a}_z\right\} = \frac{1}{2}\frac{E_0^2}{|\eta|} e^{-2\alpha z}\cos\theta_\eta\,\vec{a}_z$$

because $e^{-j\theta_\eta} = \cos\theta_\eta - j\sin\theta_\eta$.

10.6

a) $\vec{E}_s = \dfrac{j30\beta I_0\, dl}{r} \sin\theta\, e^{-j\beta r}\, \vec{a}_\theta = E_{s\theta}\, a_\theta$

$\beta = \omega\sqrt{\mu_0\epsilon_0}$

$\nabla \times \vec{E}_s = -j\omega\mu_0 \vec{H}_s \Rightarrow \vec{H}_s = \dfrac{1}{-j\omega\mu_0} \nabla \times \vec{E}_s$

$\therefore \vec{H}_s = \dfrac{j}{\omega\mu_0} \dfrac{1}{r} \left[\dfrac{\partial(rE_{s\theta})}{\partial r}\right] \vec{a}_\phi$

$= \dfrac{j}{\omega\mu_0} \dfrac{1}{r} \left[\dfrac{\partial}{\partial r}\left(j30\beta I_0\, dl \sin\theta\, e^{-j\beta r}\right)\right] \vec{a}_\phi$

$= \dfrac{j}{\omega\mu_0} \times \dfrac{1}{r} \times (-j\beta)(j30\beta I_0\, dl \sin\theta\, e^{-j\beta r}) \vec{a}_\phi$

$\vec{H}_s = \dfrac{j30\beta^2 I_0\, dl \sin\theta\, e^{-j\beta r}}{\omega\mu_0 r} \vec{a}_\phi$

b) $\vec{\mathcal{P}}_{avg} = \frac{1}{2} Re\, (\vec{E}_s \times \vec{H}_s^{\,*})$

$= \frac{1}{2} Re \left[\vec{a}_r \left(\dfrac{j30\beta I_0\, dl \sin\theta\, e^{-j\beta r}}{r}\right)\left(\dfrac{-j30\beta^2 I_0\, dl \sin\theta\, e^{j\beta r}}{\omega\mu_0 r}\right)\right]$

$= \vec{a}_r \frac{1}{2} Re \left[\dfrac{900\beta^3 I_0^2\, dl^2 \sin^2\theta}{\omega\mu_0 r^2}\right]$

$= \vec{a}_r \dfrac{450\beta^3 I_0^2\, dl^2 \sin^2\theta}{\omega\mu_0 r^2}$