KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

SUMMER SESSION 2007/2008

EE 340 (01) MAJOR EXAM II

TIME: 10:30 -11:45 A.M.

DATE: SATURDAY 16-AUGUST-2008

LOCATION: IN CLASS

Student's Name:............KEY............................

Student's I.D. Number: ....

	Maximum Score	Score
Problem 1	**25**	
Problem 2	**25**	
Problem 3	**25**	
Problem 4	**25**	
Total	**100**	

Problem 1 [25 points]

Consider an infinitely long coaxial transmission line. The cross-sectional area of the coaxial line is shown in the figure below. The solid inner conductor has radius a and the outer conductor has radius b with a negligible thickness. The inner conductor carries D.C. current I in the plus z direction, which returns through the outer conductor. **Derive** an expression for the magnetic field $\vec{H}$ in the region:

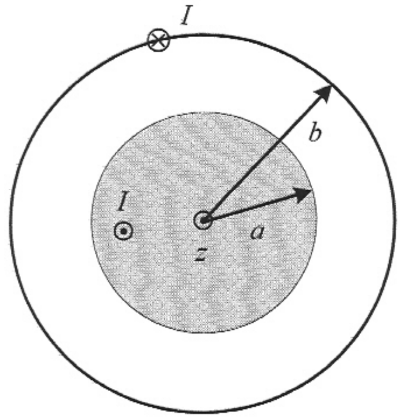
a) $\rho < a$.
b) $b > \rho > a$.
c) $\rho > b$.

$$\oint_l \vec{H} \cdot \vec{dl} = I$$

a) $H 2\pi\rho = I \dfrac{\pi\rho^2}{\pi a^2} \Rightarrow \vec{H} = \dfrac{I\rho}{2\pi a^2} \vec{a}_\phi$

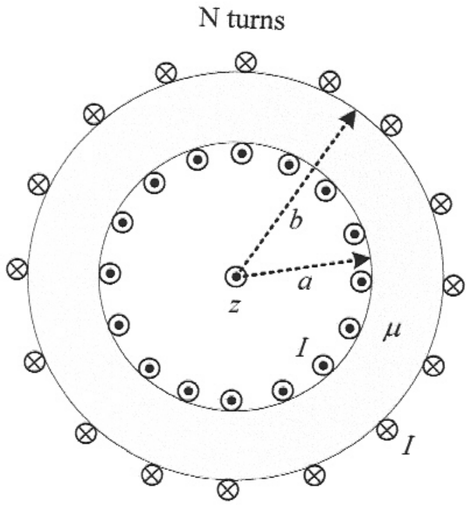
b) $H 2\pi\rho = I \Rightarrow \vec{H} = \dfrac{I}{2\pi\rho} \vec{a}_\phi$

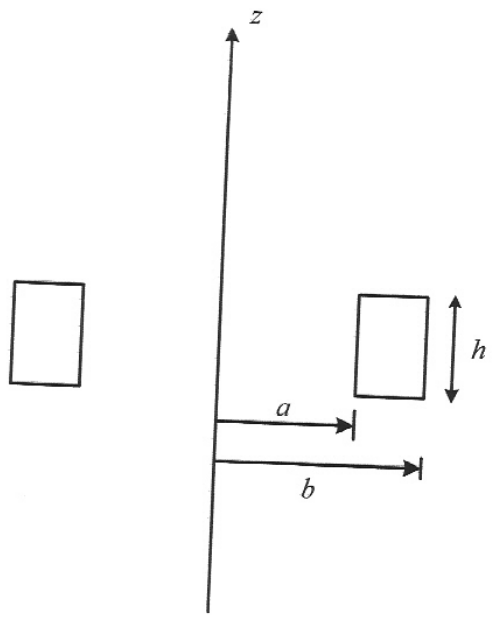
c) $H 2\pi\rho = 0 \Rightarrow \vec{H} = \vec{0}$



Problem 2 [25 points]

The N - turn toroidal coil shown in the diagram carries DC current I. The coil has a rectangular cross-sectional area of dimension $(b-a)\times h$. Where a and b are the inner and outer radii, respectively and h is the thickness. The core of the coil has permeability μ.





a) Derive an expression for the magnetostatic energy **density** in the core.

b) Use the expression found in part a) to develop an expression for the magnetostatic energy stored in the core.

a) $\oint_{l} \vec{H}\cdot d\vec{l} = I \Rightarrow H\,2\pi\rho = NI \Rightarrow \vec{H} = \frac{NI}{2\pi\rho}\vec{a}_\phi$

$$w_m = \frac{1}{2}\mu H^2 = \frac{\mu N^2 I^2}{8\pi^2\rho^2}$$

b) $W_m = \int_V w_m \, dv$

$$= \int_{z=0}^{h}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b} \frac{\mu N^2 I^2}{8\pi^2\rho^2}(\rho\, d\rho\, d\phi\, dz)$$

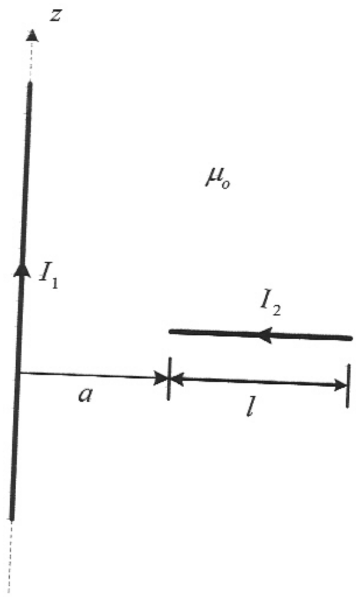
$$= \frac{\mu N^2 I^2 h}{4\pi}\ln\frac{b}{a}$$

Problem 3 [25 points]

The infinitely long straight filamentary conductor carries DC current I_1. The co-planar filamentary conductor of length l carries DC current I_2. The left end of the second conductor is placed at distance a from the first conductor. The two conductors are placed in air.

Derive an expression for the magnetic force $\vec{F}$ acting on the second conductor.

$$\vec{F} = I_2 \int_{l_2} \vec{dl} \times \vec{B}_1$$



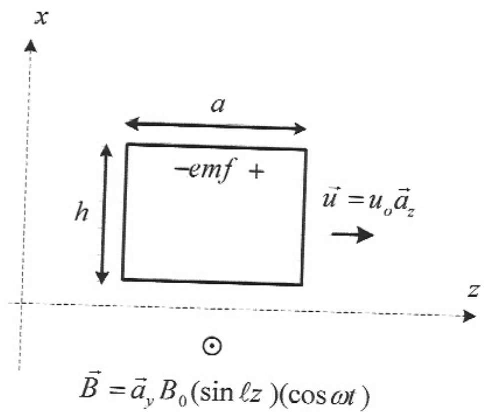
$$= I_2 \int_{a+l}^{l} (d\rho\, \vec{a}_\rho) \times \left(\frac{\mu_o I_1}{2\pi\rho} \vec{a}_\phi \right)$$

$$= I_2 \int_{a+l}^{a} \left(\frac{\mu_o I_1}{2\pi\rho} d\rho \right) \vec{a}_z$$

$$= -\vec{a}_z \frac{\mu_o I_1 I_2}{2\pi} \ln\left(\frac{a+l}{a}\right)$$

Problem 4 [25 points]

Consider the single-turn rectangular conducting circuit shown in the figure. The circuit moves with the constant velocity $\vec{u} = u_o \vec{a}_z$ in the magnetic field $\vec{B} = \vec{a}_y B_0 (\sin \ell z)(\cos \omega t)$, where B_0, ℓ, and ω are some constants. The left hand side of the loop coincides with the x-axis at $t = 0$. Develop an expression for the induced emf.



$$\Psi = \int \vec{B} \cdot d\vec{s}$$

$$= \int_{x=0}^{h} \int_{z=u_o t}^{u_o t + a} B_o \sin \ell z \cos \omega t \, dx \, dz$$

$$= -\frac{B_o h}{\ell} \cos \omega t \left[\cos \ell (u_o t + a) - \cos \ell u_o t \right]$$

$$\text{emf} = -\frac{d\Psi}{dt}$$

$$= -\frac{B_o h \omega}{\ell} \sin \omega t \left[\cos \ell (u_o t + a) - \cos \ell u_o t \right]$$

$$- u_o B_o h \cos \omega t \left[\sin \ell (u_o t + a) - \sin \ell u_o t \right]$$