King Fahd University of Petroleum & Minerals
Department of Electrical Engineering

Communications Engineering I
EE 370

Course Notes
Chapter 6

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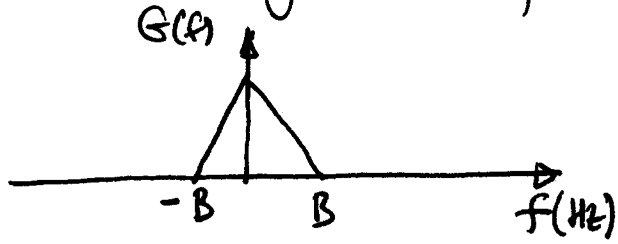
Chapter 6 — Sampling and pulse Code modulation

Analog Signal → [A/D] → Digital Signal Digital Signal → [D/A] → Analog Signal.

1. Sampling theorem:

Consider a signal $g(t)$ whose spectrum is bandlimited to B Hz,



or $[\, G(\omega) = 0$ for $|\omega| > 2\pi B \,]$.

This signal can be reconstructed exactly from its samples taken uniformly at a rate $R > 2B$ Hz. In other words, the minimum sampling frequency is $f_s = 2B$ Hz.

<u>Proof:</u> Let $\bar{g}(t)$ be the sampled signal of $g(t)$. The sampled signal consists of impulses spaced every T_s seconds (The sampling interval).

$$\bar{g}(t) = g(t)\, \delta_{T_s}(t)$$

$$\delta_{T_s}(t) = \sum_n \delta(t - nT_s)$$

Thus, $$\bar{g}(t) = \sum_n g(nT_s)\, \delta(t - nT_s)$$

$$\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + 2\cos 3\omega_s t + \cdots\right], \quad \omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$

(Example 2.9 Equation 2.77 : $\delta_{T_s}(t) = \frac{1}{T_s}\left[1 + 2\sum_{n=1}^{\infty} \cos n\omega_s t\right]$)

Hence, $$\bar{g}(t) = g(t)\, \delta_{T_s}(t)$$

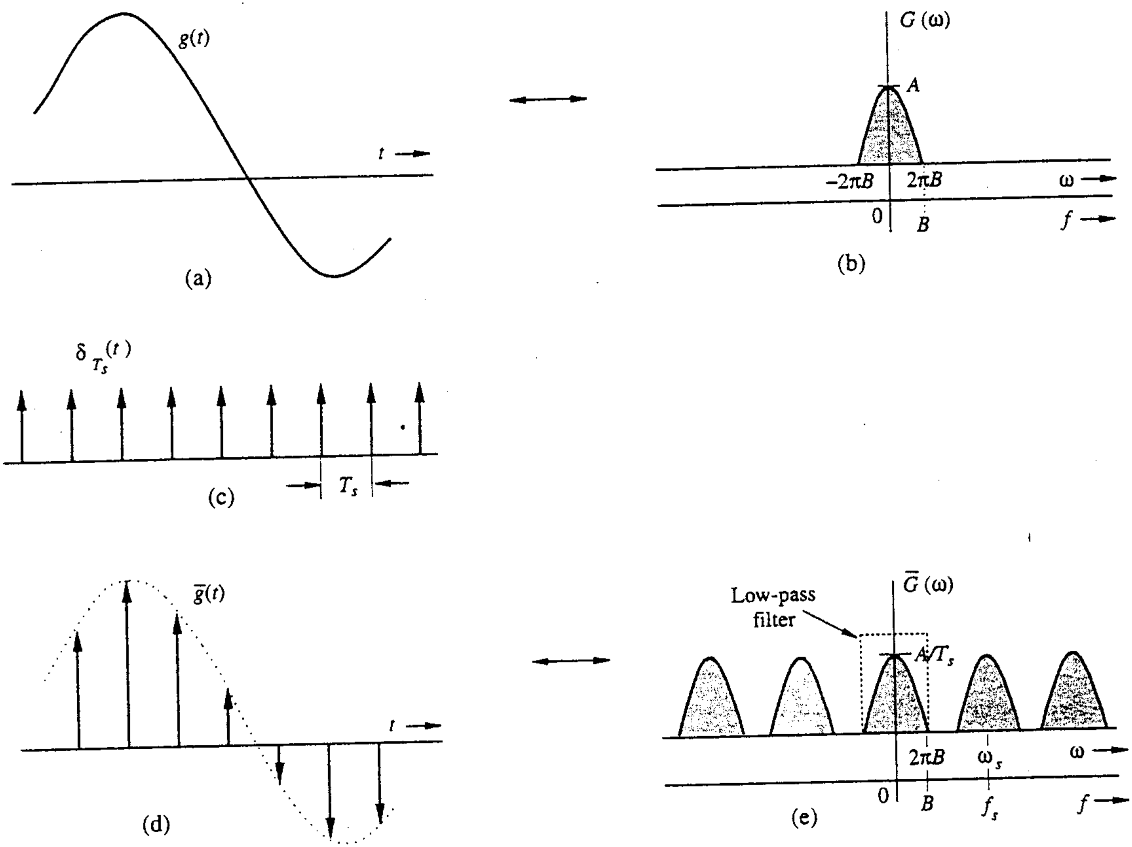


Figure 6.1 Sampled signal and its Fourier spectrum.

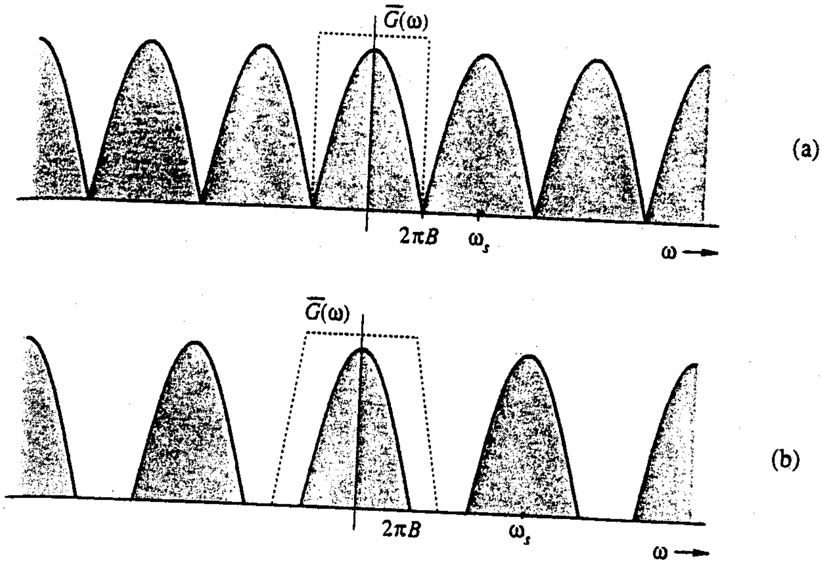
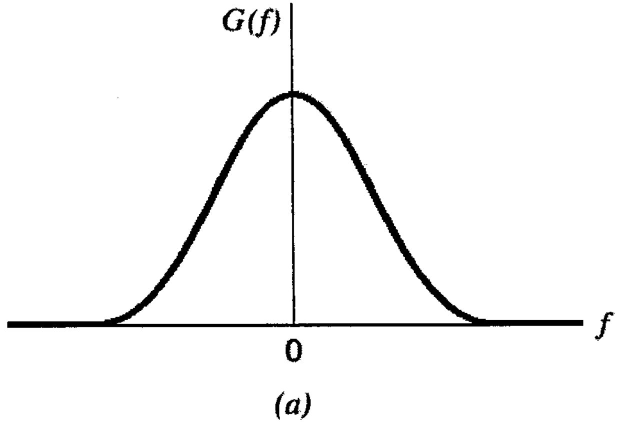
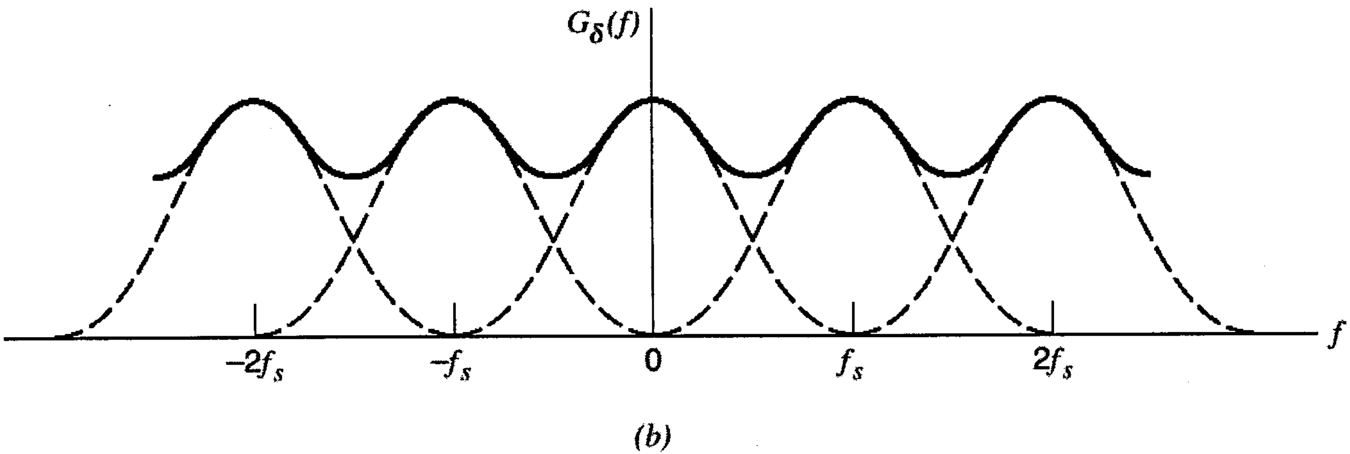


Figure 6.5 Spectra of a sampled signal. (a) At the Nyquist rate. (b) Above the Nyquist rate.

(*a*) Spectrum of a signal. (*b*) Spectrum of an undersampled version of the signal exhibiting the aliasing phenomenon.





$$\bar{g}(t) = \frac{1}{T_s}\left[g(t) + 2g(t)\cos\omega_s t + 2g(t)\cos 2\omega_s t + \cdots\right]$$

$$\Rightarrow \quad \bar{G}(\omega) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} G(\omega - n\omega_s)$$

If we have to reconstruct $g(t)$ from $\bar{g}(t)$, we should recover $G(\omega)$ from $\bar{G}(\omega)$. This is possible if there is no overlap between successive cycles of $\bar{G}(\omega)$, this requires:

$$f_s > 2B \qquad \text{(sampling rate).}$$

or

$$T_s < \frac{1}{2B} \qquad \left(\text{since } T_s = \frac{1}{f_s}\right).$$

Thus, as long as the sampling frequency f_s is greater than twice the signal bandwidth B, $\bar{G}(\omega)$ will consist of nonoverlapping repetitions of $G(\omega)$. When this is true, $g(t)$ can be recovered from its samples $\bar{g}(t)$ by passing the sampled $\bar{g}(t)$ through an ideal low-pass filter of bandwidth B. The minimum sampling rate $f_s = 2B$ required to recover $g(t)$ from $\bar{g}(t)$ is called the Nyquist rate for $g(t)$, and $T_s = \frac{1}{2B}$ is called the Nyquist interval for $g(t)$.

1.1 Signal reconstruction: The interpolation formula

$$\bar{g}(t) \longrightarrow \boxed{h(t)} \longrightarrow g(t)$$

Interpolation filter.

The process of reconstructing a continuous-time signal $g(t)$ from its samples is also known as interpolation.
In the last section, we saw that a signal $g(t)$ band-limited to B Hz can be reconstructed (interpolated) exactly from its samples. This is done

Interpolation filters:

1. Zero-order hold filter  $\Rightarrow$ poor approximation of the ideal low-pass filter. Staircase Interpolation.
2. First-order hold filter  $\Rightarrow \Delta\left(\frac{t}{2T_s}\right)$ (Linear approximation instead of the staircase)
3. Low-pass filter  $\Rightarrow$ Ideal Interpolation.

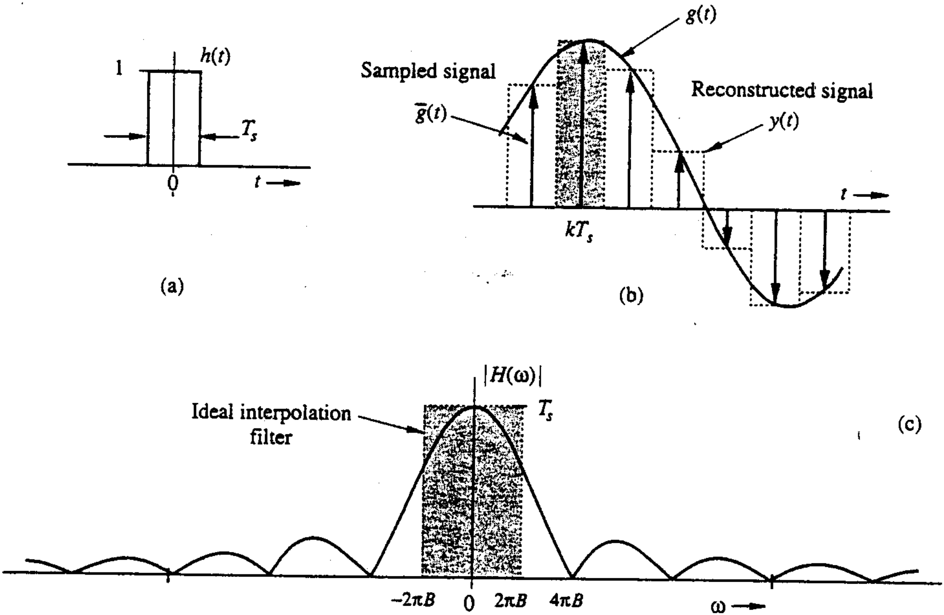


Figure 6.2 Simple interpolation using zero-order hold circuit.

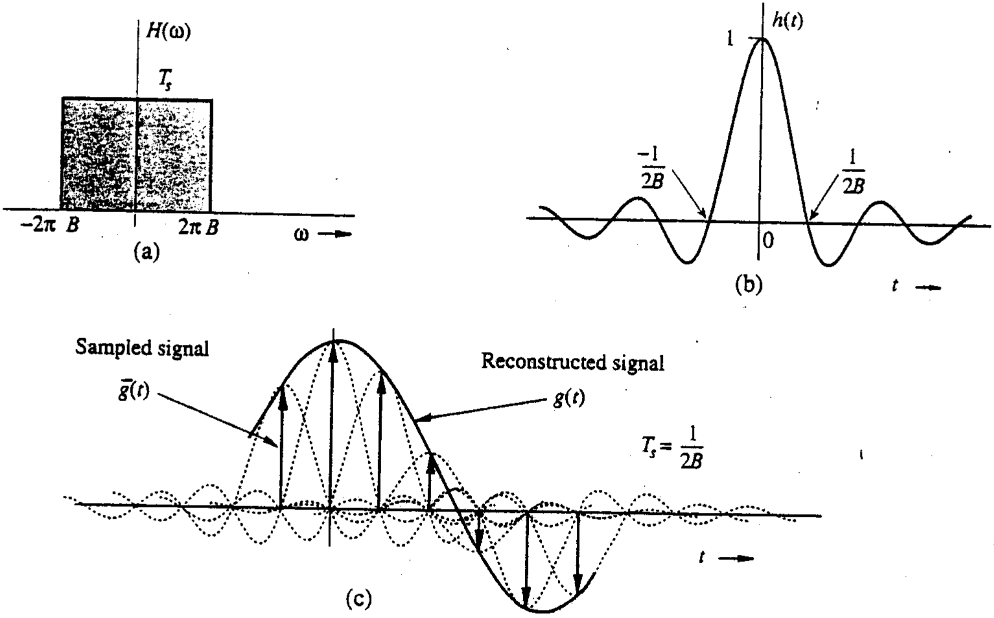


Figure 6.3 Ideal interpolation.

by passing the sampled signal through an ideal low-pass filter of bandwidth B Hz.

As seen earlier, the sampled signal contains a component $\left(\frac{1}{T_s}\right) g(t)$, and to recover $g(t)$, the sampled signal must be passed through an ideal low-pass filter of bandwidth B Hz and gain T_s.

Thus, the interpolating filter transfer function is:

$$H(\omega) = T_s \, \text{rect}\left(\frac{\omega}{4\pi B}\right).$$

Therefore, the impulse response of this filter:

given that $h(t) \Leftrightarrow H(\omega)$, hence

$$h(t) = 2BT_s \, \text{sinc}(2\pi Bt), \quad \text{since } 2BT_s = 1$$

$$\therefore \quad h(t) = \text{sinc}(2\pi Bt)$$

$$\bar{g}(t) \longrightarrow \boxed{h(t)} \longrightarrow g(t)$$

The k^{th} sample of the input $\bar{g}(t)$ is the impulse $g(kT_s)\delta(t-kT_s)$; the filter output of this impulse is $g(kT_s)\, h(t-kT_s)$.

Hence, the filter output can be expressed as a sum; that is,

$$\begin{aligned}
g(t) &= \sum_k g(kT_s)\, h(t-kT_s) \\
&= \sum_k g(kT_s)\, \text{sinc}\left[2\pi B(t-kT_s)\right] \\
&= \sum_k g(kT_s)\, \text{sinc}\left[2\pi Bt - 2\pi BkT_s\right] \\
&= \sum_k g(kT_s)\, \text{sinc}\left[2\pi Bt - k\pi\right]
\end{aligned}$$

The above result is the interpolation formula, which yields values of $g(t)$ between samples as a weighted sum of all sample values.

1.2 Practical Difficulties in signal reconstruction:

It is impossible in practice to recover a band-limited signal $g(t)$ exactly from its samples, even if the sampling rate is higher than the Nyquist rate. However, as the sampling rate increases, the recovered signal approaches the desired signal more closely.

* The Treachery of aliasing:

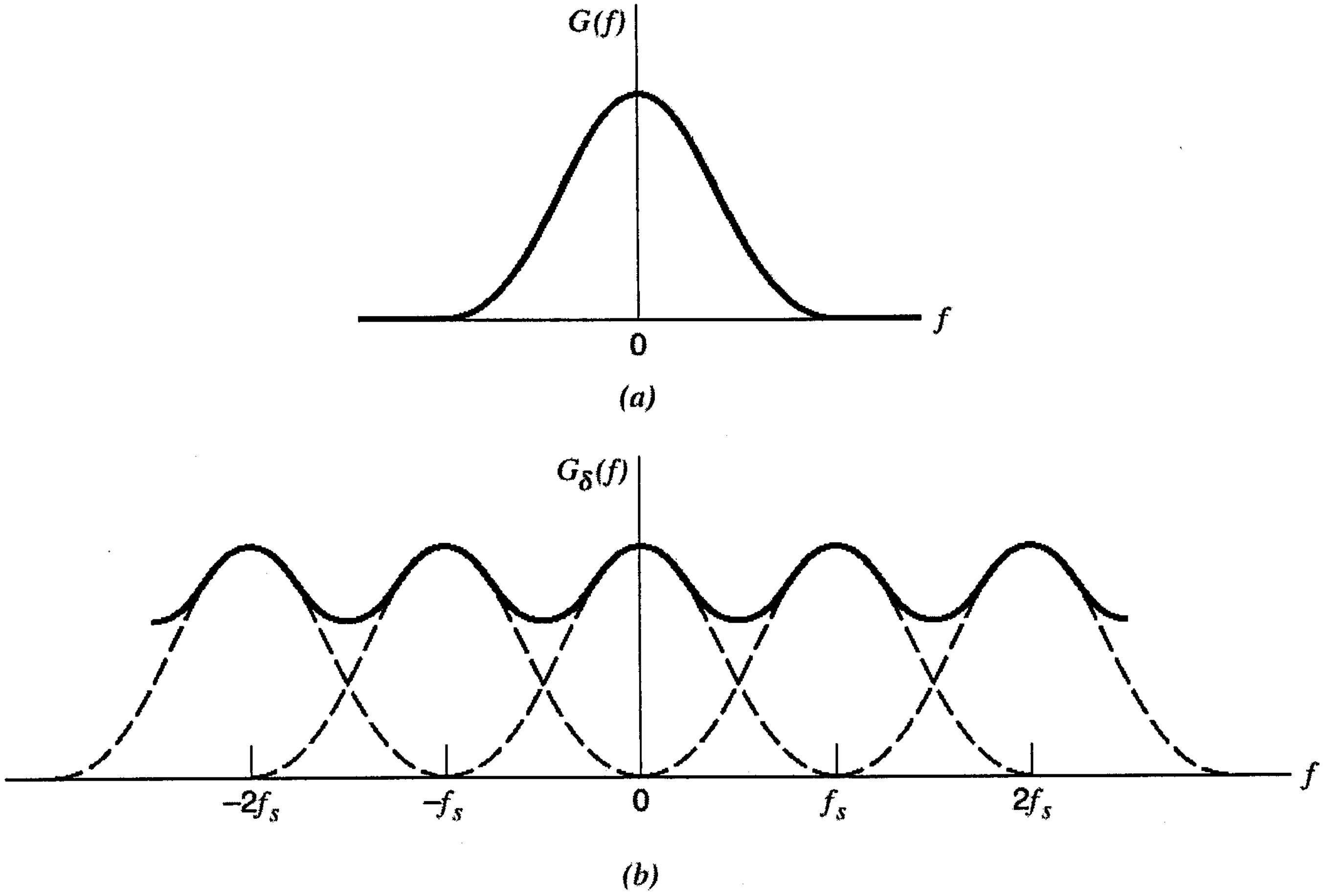
There is another fundamental practical difficulty in reconstructing a signal from its samples. The sampling theorem was proven on the assumption that the signal $g(t)$ is band-limited. In fact, all practical signals are time-limited. Since a signal cannot be time-limited and band-limited simultaneously. Therefore, time-limited signals are non-band limited; they have an infinite bandwidth, and the spectrum $\bar{G}(\omega)$ consists of overlapping cycles of $G(\omega)$ repeating every f_s Hz (Figure 6.6). Because of the overlapping tails, $\bar{G}(\omega)$ no longer has complete information about $G(\omega)$, and it no longer possible to recover $g(t)$ from the sampled $\bar{g}(t)$.

The solution: The antialiasing Filter:

Pass the signal $g(t)$ through an ideal low-pass filter with bandwidth $\frac{f_s}{2}$ Hz, then the signal is sampled at f_s Hz.

1.3 Maximum information rate:

A knowledge of the maximum rate of information that can be transmitted over a channel of bandwidth B Hz is important in digital communication. A channel of bandwidth B Hz can transmit a signal of bandwidth B Hz error free. Since a signal of bandwidth B Hz can be reconstructed from its Nyquist samples, (*a*) Spectrum of a signal. (*b*) Spectrum of an undersampled version of the signal exhibiting the aliasing phenomenon.



wich are at a rate of 2BHz. This means that a signal of bandwidth BHz can be completely specified by 2B independent pieces of information per second. Since the channel is capable of transmitting this signal error free, it follows that the channel should be able to transmit, error free, 2B independent pieces of information per second.

∴ We can transmit error free at most 2 pieces of information per second per hertz bandwidth.

1.4 Some applications of the sampling theorem:

Processing a continuous-time signal is equivalent to processing a discrete sequence of numbers. In communication, the transmission of a continuous-time (CT) message reduces to the transmission of a sequence of numbers wich opens doors to techniques of communicating CT signals by pulse trains. The continuous-time signal $g(t)$ is sampled, and sample values are used to modify certain parameters of a periodic pulse train such as amplitude, widths, or positions of the pulses in proportion to the sample values of the signal $g(t)$. Accordingly, we have:

- pulse-amplitude modulation (PAM),
- pulse-width modulation (PWM), or
- pulse-position modulation (PPM).

The most important form of pulse modulation today is:

- pulse-code modulation (PCM).

One advantage of using pulse modulation is that it permits the simultaneous transmission of several signals on a

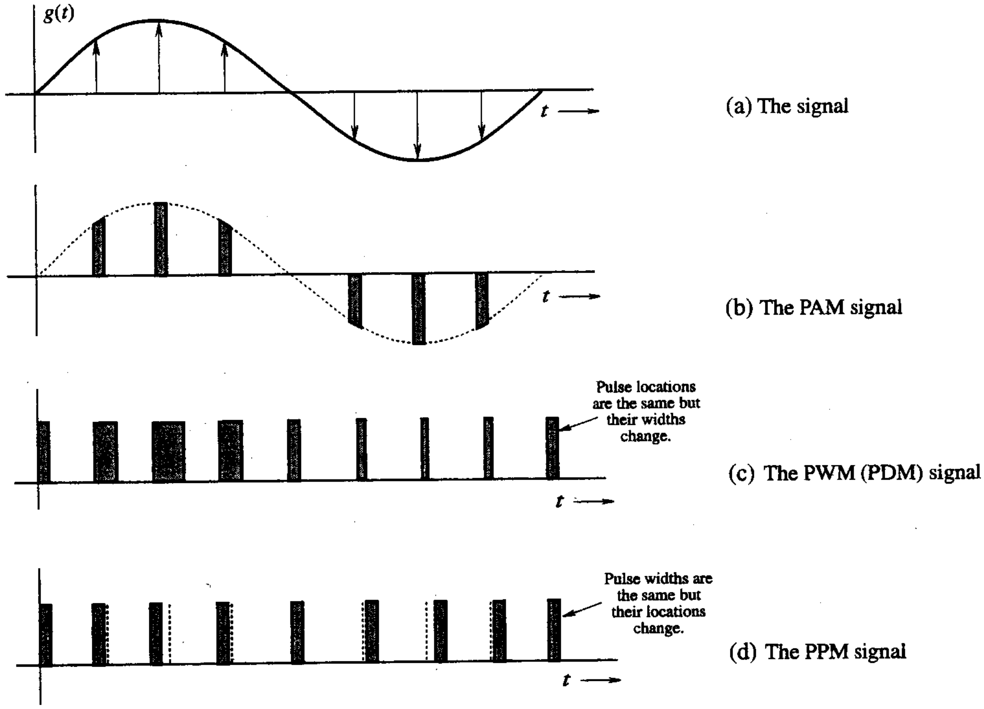


Figure 6.8 Pulse-modulated signals.

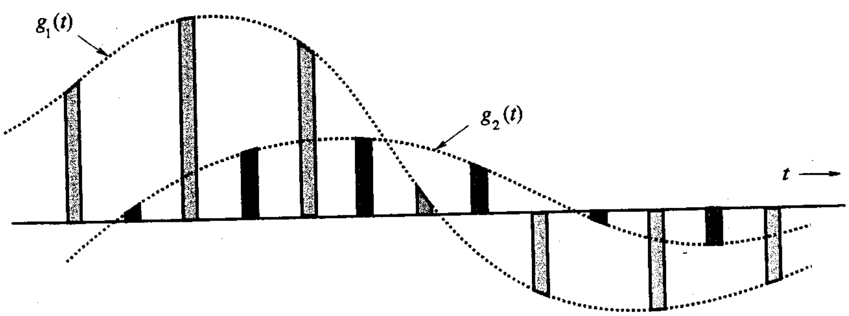


Figure 6.9 Time-division multiplexing of two signals.

time-sharing basis — time-division multiplexing (TDM).
Because a pulse-modulated signal occupies only a part of the channel time, we can transmit several pulse modulated signals on the same channel by interweaving them. We can multiplex several signals on the same channel by reducing pulse widths.

* Another technique is frequency-division multiplexing (FDM) where various signals are multiplexed by sharing the channel bandwidth.

A **baseband** is the original information signal in a communication system. The band of frequencies it occupies is the **baseband bandwidth** which usually starts from 0 Hz and is often quoted as a single figure, B Hz.

Classification of modulation types

1. **Frequency translation**: The baseband is moved to a higher-frequency range by arranging for it to alter some property of a higher-frequency carrier.
2. **Sampling**: The baseband waveform voltage is allowed through for short periods of time at regular intervals and these values only are sent, either coded or uncoded. The signal, although much changed in form, still remains essentially baseband in nature.

Advantages of using modulation

There are several general reasons for modulating a baseband signal. Some have been mentioned already but are included again in this summary.

1. Advantages produced by frequency translation

(i) Use of frequency division multiplexing (FDM): This allows many signals to be sent simultaneously down the same communication channel. It gives economic use of equipment and enables complex systems to be designed.

(ii) Use of correct transmission frequency to give best transmission conditions: This is especially important in radio links where aerial efficiency increases with frequency and the best frequencies may need to be selected for propagation through the troposphere or via the ionosphere.

2. Advantages produced by sampling

(iii) Use of time division multiplexing (TDM): This allows many signals to be sent simultaneously along the same communication link by interleaving them in time. Gives similar economic and design advantages as (i).

3. Advantages produced by coding

(iv) Reliability of transmission greatly increased. Noise corruption very much less likely. Received baseband reproduces original signal very accurately.

(v) Signal processing much easier using standard logic and computing techniques. Facilitates design and production of complex systems at their most economic and reliable as in modern telephone systems.

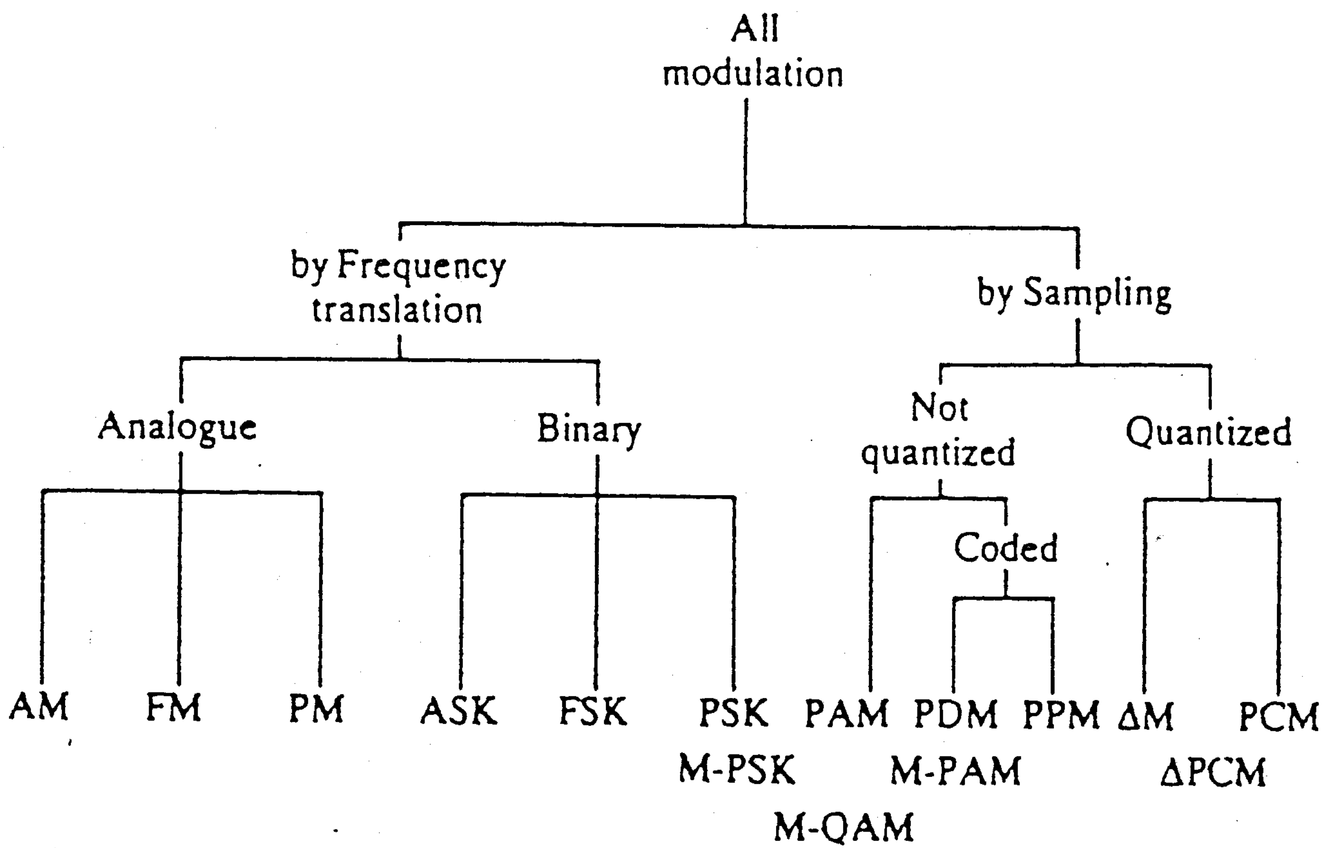


Figure Family tree of modulation methods

Pulse Code Modulation (PCM):

PCM is a technique of converting an analog signal into a digital signal. Hence, PCM process is equivalent to A/D conversion

PCM can be generated by:

1) sampling the analog signal
2) quantizing the samples by rounding off their values to the closest quantized levels.

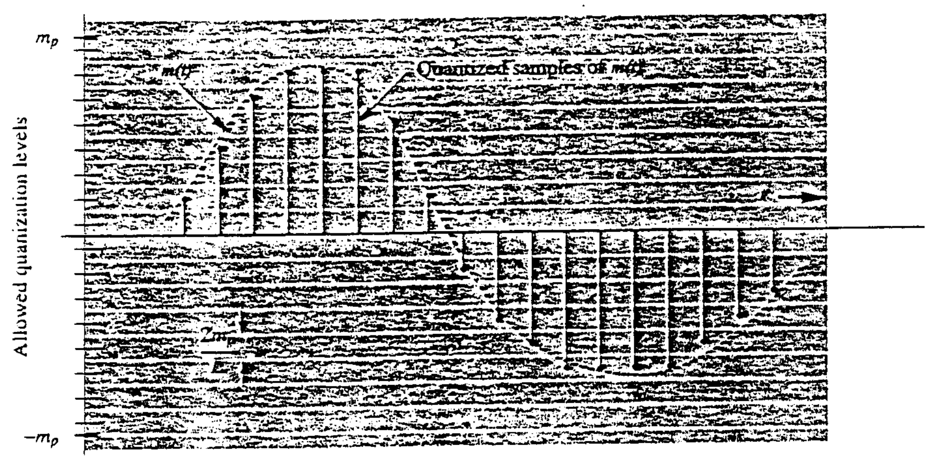
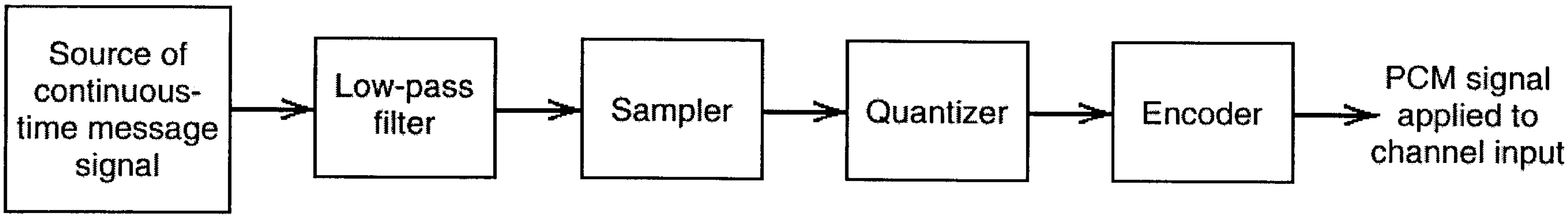


Figure 6.10 Quantization of a sampled analog signal.

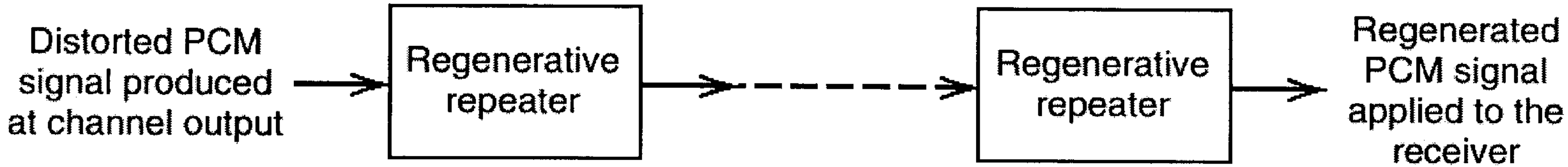
The range of the analog signal $(-m_p, m_p)$ is divided into L subintervals, each of magnitude $\Delta v = \frac{2m_p}{L}$. $L = 16$ above.

3) encoding each quantized sample by a binary code, where each level is assigned one binary code of four digits.

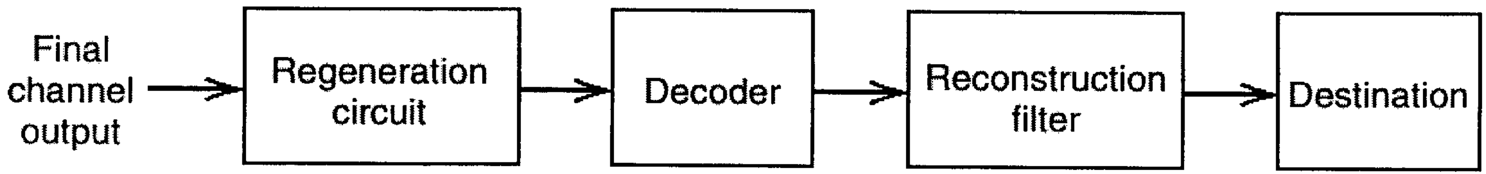
The basic elements of a PCM system.



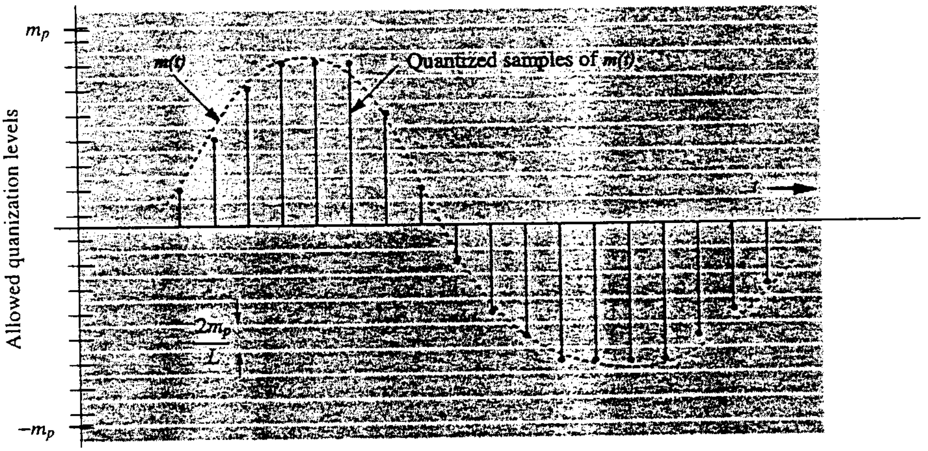
(*a*) Transmitter



(*b*) Transmission path



(*c*) Receiver



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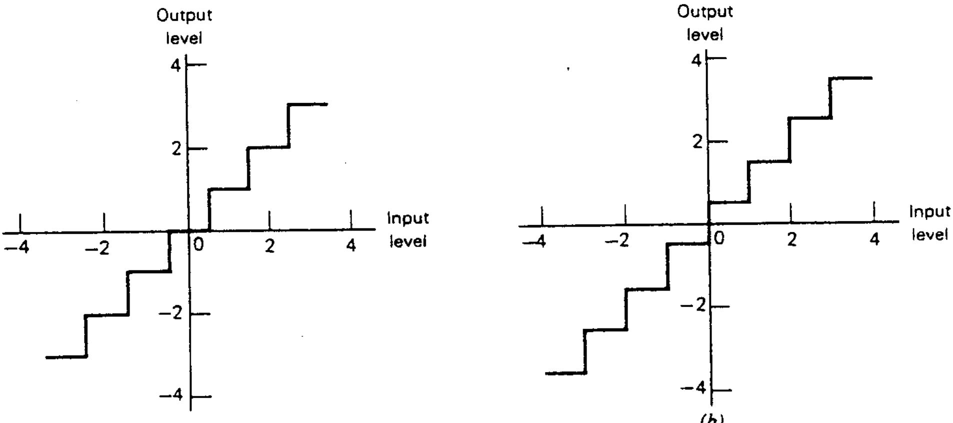


Figure 6.17 Two types of quantization: (*a*) midtread and (*b*) midrise.

2 - Pulse Code Modulation (PCM):

PCM is a method of converting an analog signal into a digital signal (A/D conversion).

PCM can be generated by means of:

1- Sampling the analog signal.

2- quantizing the samples, that is, rounding off their values to one of the closest quantized levels.

The range of the analog signal $(-m_p, m_p)$ is divided into L subintervals, each of magnitude $\Delta v = \frac{2m_p}{L}$.

3- Encoding each quantized sample by a binary code.

2-1 Quantizing:

For quantizing, we limit the amplitude of the message signal $m(t)$ to the range $(-m_p, m_p)$. Note that m_p is not necessarly the peak of $m(t)$. The amplitudes of $m(t)$ beyond $\pm m_p$ are chopped off. Thus, m_p is not a parameter of the signal $m(t)$, but is a constant of the quantizer. The amplitude range $(-m_p, m_p)$ is divided into L uniformly spaced intervals, each of magnitude $\Delta v = \frac{2m_p}{L}$. The quantized samples are coded and transmitted as binary pulses.

At the receiver some pulses will be detected incorrectly. Hence, there are 2 sources of error in this scheme:

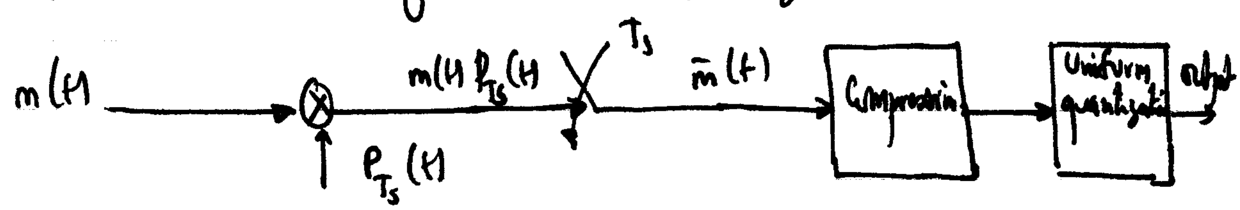
1- quantization error, and $\rightarrow$ ($\frac{\Delta v}{2}$ is the quantization error)

2- pulse detection error.

2-2 Nonuniform quantization:

Ideally, the SNR should be constant for all values of the message signal power, which varies from talker to talker, the quantizing steps (uniform values) $\Delta v = \frac{2m_p}{L}$ are made nonuniform, and therefore smaller steps for smaller amplitudes. This process is called nonuniform quantization.

An equivalent result can be obtained by first compressing the signal samples and then using a uniform quantizer.



This approach of equalizing the SNR appears as the following: the loud talkers and stronger signals are penalized with higher noise steps Δv in order to compensate for soft talkers and weaker signals.

There are two compression laws:

- The μ-law: $$y = \frac{1}{\ln(1+\mu)} \ln\left(1+\mu \frac{m}{m_p}\right), \quad 0 \le \frac{m}{m_p} \le 1$$

- and the A-law:

$$y \begin{cases} \dfrac{A}{1+\ln A}\left(\dfrac{m}{m_p}\right), & 0 \le \dfrac{m}{m_p} \le \dfrac{1}{A} \\[2ex] \dfrac{1}{1+\ln A}\left(1+\ln \dfrac{Am}{m_p}\right), & \dfrac{1}{A} \le \dfrac{m}{m_p} \le 1 \end{cases}$$

The compression parameters μ or A determine the degree of compression. $\mu = 255$ and $A = 87.6$ are almost standard values.

At the receiver, the compressed samples must be restored to their original values by using an expander. The expander has complementary characteristics to that of the compressor.

Remember:

① Sampling, which operates in the time domain: the sampling process is the link between an analog waveform and its discrete-time representation.

Quantization, which operates in the amplitude domain: the quantization process is the link between an analog waveform and its discrete-amplitude representation.

Quantization Error:

During quantization, the amplitude range $(-m_p, m_p)$ is divided into L uniformly spaced intervals, each of width $\Delta V = \frac{2m_p}{L}$.

If $g(nT_s)$ is the nth sample of the signal $g(t)$, and if $\hat{g}(nT_s)$ is the corresponding quantized sample, then according to the interpolation formula:

$$g(t) = \sum_n g(nT_s)\ \mathrm{sinc}(2W(t-nT_s))$$

$$\hat{g}(t) = \sum_n \hat{g}(nT_s)\ \mathrm{sinc}(2W(t-nT_s))$$

where $\hat{g}(t)$ is the reconstructed signal from quantized samples.

$$\begin{aligned}\text{Quantization noise} &= q(t) \\ &= \hat{g}(t) - g(t) \\ &= \sum_n [\hat{g}(nT_s) - g(nT_s)]\ \mathrm{sinc}(2W(t-nT_s)) \\ &= \sum q(nT_s) \cdot \mathrm{sinc}(2W(t-nT_s))\end{aligned}$$

$$\therefore\ \overline{q^2(t)} = \lim_{T\to\infty} \int_{-T/2}^{T/2} q^2(t)\, dt$$

$$= \frac{(\Delta V)^2}{12} = \frac{m_p^2}{3L^2} = N_q \text{ (quantization noise power)}$$

The desired signal at the output is $g(t)$. Hence, the power of the message signal is:

$$S_o = \overline{g^2(t)}$$

$$\Rightarrow \quad \frac{S_o}{N_o} = 3L^2 \, \frac{\overline{g^2(t)}}{m_p^2}$$

Nonuniform Quantization:

Ideally, S_o/N_o should be constant for all values the message signal power $\overline{g^2(t)}$. Also, note that $N_q = \frac{(\Delta v)^2}{12}$ is proportional to the square of the step size (Δv).

It will be better to use smaller steps for smaller signal amplitudes, and this is called nonuniform quantization. An equivalent result can be obtained by first compressing signal samples and then using a uniform quantization.

There are two compression laws: the μ-law and the A-law.

$$y = \frac{1}{\ln(1+\mu)} \cdot \ln\left(1 + \frac{\mu g}{m_p}\right), \quad 0 \le \frac{g}{m_p} \le 1$$

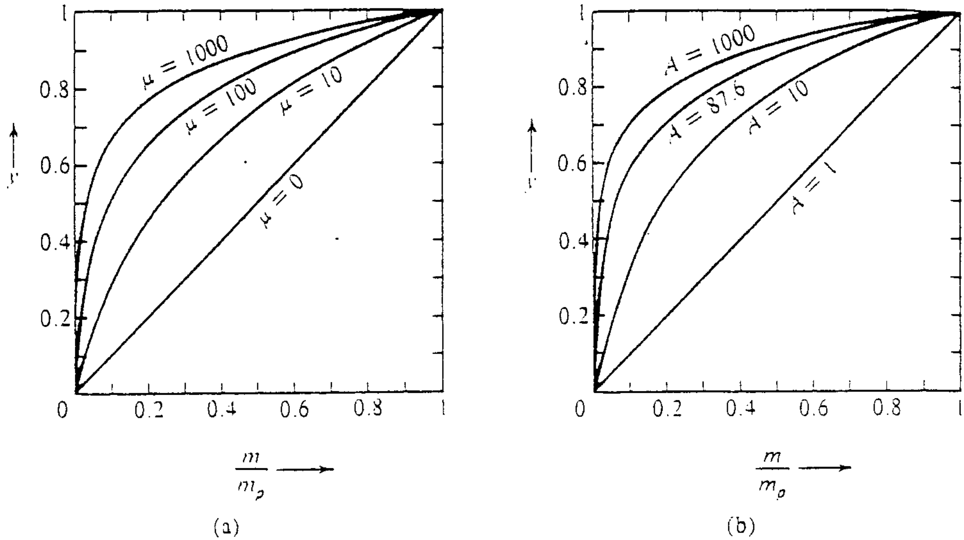


Figure 6.12 (a) μ-law characteristic. (b) A-law characteristic.

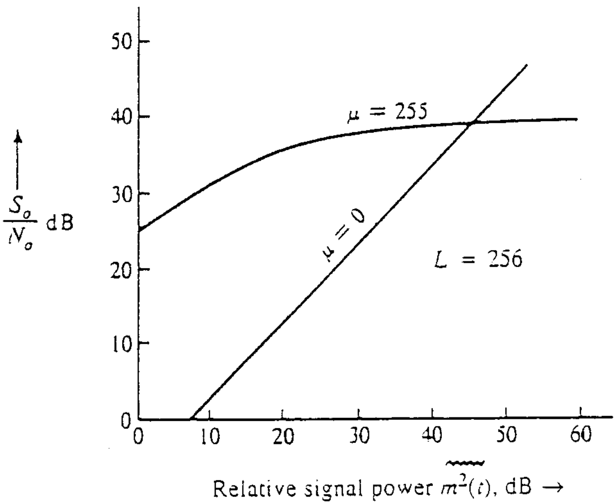
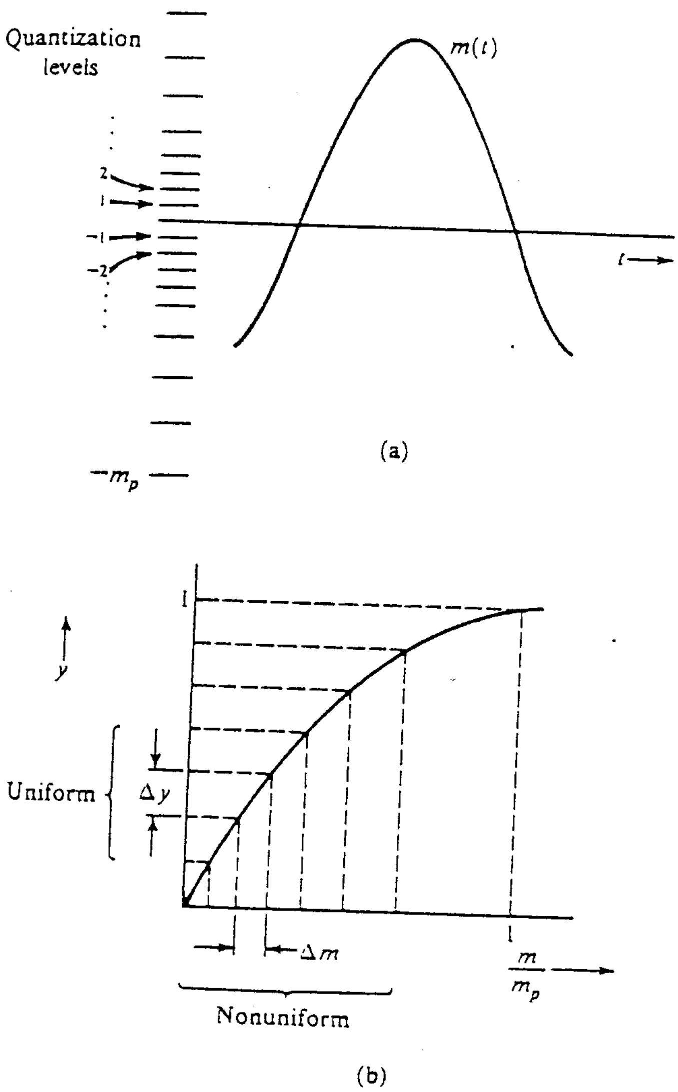


Figure 6.13 Signal-to-quantization-noise ratio in PCM with and without compression.

Figure 6.11 Nonuniform quantization.



$$y = \begin{cases} \frac{A}{1+\ln A} \cdot \left(\frac{g}{m_p}\right), & 0 \le \frac{g}{m_p} \le \frac{1}{A} \\ \frac{1}{1+\ln A} \cdot \left(1 + \ln \frac{A \cdot g}{m_p}\right), & \frac{1}{A} \le \frac{g}{m_p} \le 1 \end{cases}$$

The compression parameters μ or A determine the degree of compression.

$\mu = 255$ and $A = 87.6$ are almost standard values.

At the receiver, the compressed samples must be restored to their original values by using an expander. The expander has complementary characteristics to that of the compressor.

Transmission Bandwidth & Output SNR :

In binary PCM, we assign n binary digits (bits) to each of the L quantization levels.

$$L = 2^n$$

$$\Rightarrow \quad n = \log_2(L)$$

Each quantized sample is encoded into n bits. The analog signal is bandlimited to W Hz, and hence the minimum sampling freq. is 2W Hz. The resulting bit rate is <u>2nW</u> bit/s.

Because a unit bandwidth (Hz) can transmit a maximum of two pieces of information per second, we will require a minimum channel Bandwidth (B_T) given by :

$$B_T = nW \qquad (\text{Hz})$$

This is the theoretical minimum transmission bandwidth required to transmit the PCM signal.

Digit	Binary equivalent	Pulse code waveform
0	0000	
1	0001	
2	0010	
3	0011	
4	0100	
5	0101	
6	0110	
7	0111	
8	1000	
9	1001	
10	1010	
11	1011	
12	1100	
13	1101	
14	1110	
15	1111	

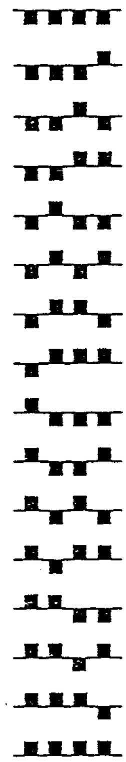


Figure 1.6 Example of a binary pulse code.

A negative pulse is assigned to binary 0, and a positive pulse is assigned to binary 1.

Example 1:

In telephony, the speech signal is bandlimited to 3.4 kHz. Then, a sampling rate of 8000 samples per second (8 kHz) is used to sample the speech. Each sample is quantized into $L = 256$ level, which requires 8 binary pulses to encode each sample ($2^8 = 256$).

$\Rightarrow$ The resulting bit rate $= 8 \times 8000 = 64$ kbit/s

Example 2 :

Music compact disc (CD) is a recent application of PCM. The audio bandwidth is 15 kHz, and hence the Nyquist sampling rate is 30 kHz. However, the <u>actual</u> sampling rate is 44.1 kHz. The number of quantization levels is $L = 65,536$.

So, each quantized sample is encoded by 16 binary pulses $2^{16} = 65,536$.

$\Rightarrow$ Resulting bit rate $= 44.1 \times 16 = 705.6$ kbit/s

It was found that, for uncompressed PCM, the output SNR is given by

$$\frac{S_o}{N_o} = 3L^2 \frac{\overline{g^2(t)}}{m_p^2} = 3\,\frac{\overline{g^2(t)}}{m_p^2}\cdot L^2 = 3\,\frac{\overline{g^2(t)}}{m_p^2}\cdot (2)^{2n}$$

Also, for compressed PCM using a μ-law compandor:

$$\frac{S_o}{N_o} \simeq \frac{3L^2}{[\ln(1+\mu)]^2} = \frac{3}{[\ln(1+\mu)]^2}\cdot (2)^{2n}$$

$$= \frac{3}{[\ln(1+\mu)]^2}\cdot (2)^{2B_T/W}$$

Note that the SNR increases exponentially with B_T.

Example:
A signal $m(t)$ is bandlimited to 3kHz, and is sampled at $33\frac{1}{3}\%$ higher than the Nyquist rate. The maximum acceptable quantization error is 0.5% of the peak amplitude m_p. The quantized samples are binary coded. Find the minimum bandwidth of a channel required to transmit the PCM signal. If 24 such signals are time-division-multiplexed, determine the minimum transmission bandwidth required to transmit the multiplexed signal.

Nyquist rate $= 2 \times 3000 = 6000$ Hz.

Actual rate $= 6000 \times \left(1\frac{1}{3}\right) = 8000$ Hz.

$$\Delta v = \frac{2m_p}{L}$$

$$\therefore \frac{\Delta v}{2} = \frac{m_p}{L} = \frac{0.5}{100} m_p \Rightarrow L = 200$$

But L must be a power of 2,

$\therefore L = 256 = 2^n$

$\therefore n = 8$ bits/sample.

Bit Rate $= 8 \times 8000 = 64$ kbit/s.

$B_T = 32$ kHz.

For the multiplexed signal:

$B_T = 24 \times 32 = 768$ kHz.

Example:

A signal $m(t)$ of 4 kHz bandwidth is transmitted using a binary companded PCM with $\mu = 100$. Compare the case of $L = 64$ with that of $L = 256$ from the point of view of transmission bandwidth and output SNR.

For $L = 64$, $n = 6$

$\therefore B_T = nW = 24$ kHz.

$$\frac{S_o}{} = \frac{3L^2}{} = 561.1 = 27.49 \text{ dB}$$

For $L = 256$; $n = 8$

$$B_T = nW = 8 \times 4000 = 32 \text{ kHz}$$

$$\frac{S_o}{N_o} = \frac{3L^2}{[\ln(1+\mu)]^2} = 8892 = 39.49 \text{ dB}$$

Note the increase in B_T and SNR for $L = 256$.

A T1 Carrier System:

In this system, 24 audio voice channels are sampled in sequence by using high speed electronic switching circuits.

The sampler output produces a time-division-multiplexed PAM signal. Then, this multiplexed PAM is applied at quantizer and encoder, where each quantized sample is encoded into eight binary pulses. The T1 multiplexed signal has a bit rate of 1.544 Mbit/s. [24 voice signals. Each sampled at 8 kHz and encoded as 8 bits/sample. 1 bit is added for synchronization per frame to give: Rate = 8000 (24×8 +1)

= 1.544 Mbit/s.

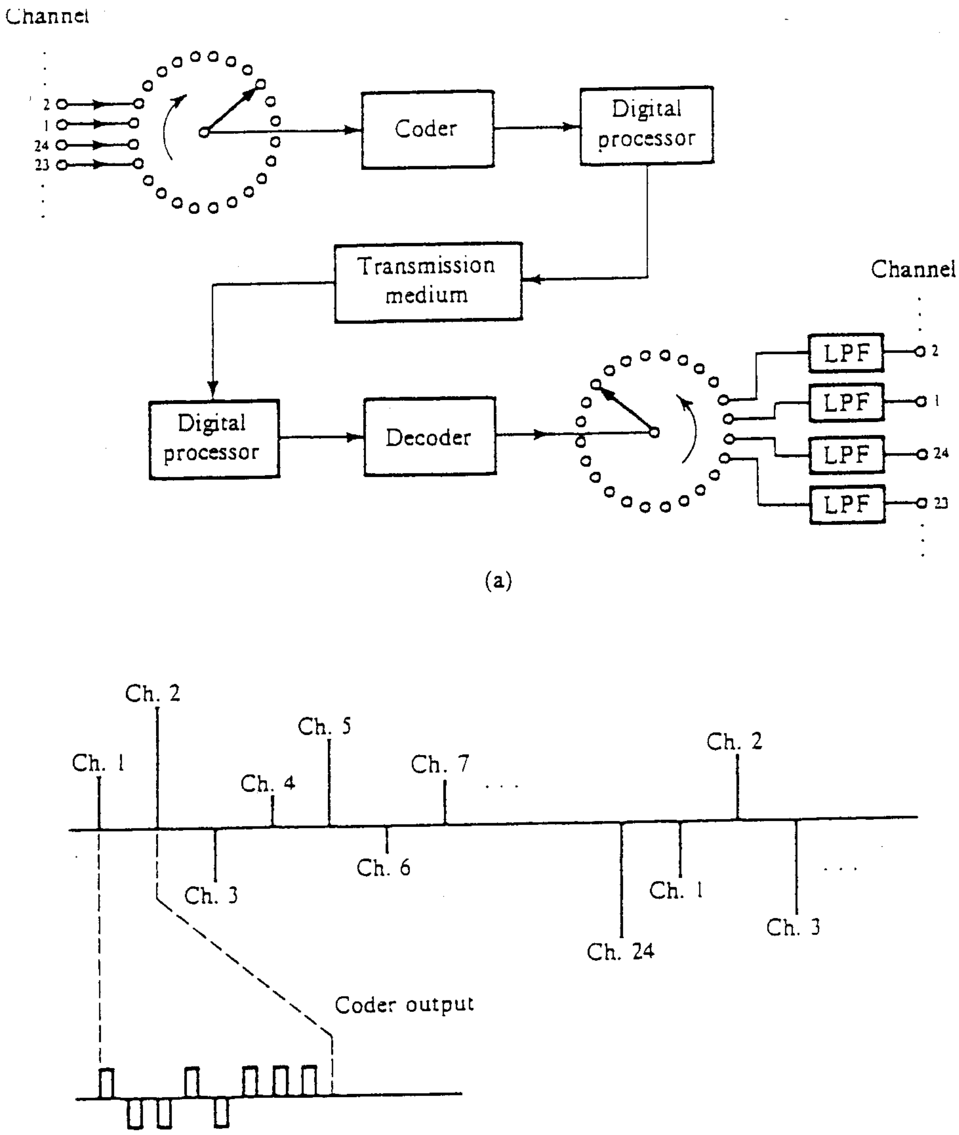


Figure 6.15 T1 carrier system.

In the transmission medium, regenerative repeaters, spaced 6000 feet apart, detect the pulses and transmit new pulses. At the receiver, the decoder converts the binary pulses into samples. The samples are demultiplexed and distributed to each each of the 24 channels. The desired audio signal is reconstructed by passing the samples through a LPF in each channel.

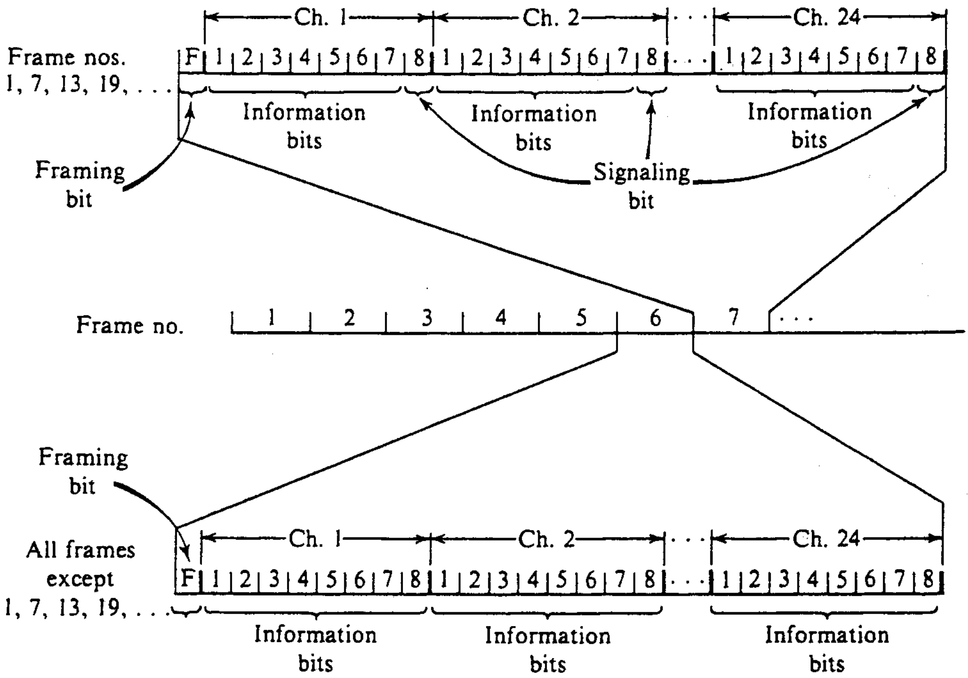


Figure 6.16 T1 system signaling format.

Differential Pulse Code Modulation (DPCM):

This technique is a variation of normal PCM, and it is particularly more efficient when the sampled signal has high sample – to – sample correlation. In the transmission of picture information, considerable portions of the signal describe background information containing very little changes. Hence, in such a situation if we use normal PCM, then we are essentially transmitting repeated sample values. We can improve the situation by sending **only** the digitally encoded **differences** between **successive samples**.

In conventional PCM, the bit rate is determined by the highest frequency and highest amplitude of the source signal. For radio and video signals, neither the maximum amplitude nor the highest frequency is continuously present in the signal. Hence, the majority of the adjacent PCM words will have only small differences.

It can be readily understood that a considerable amount of bit rate reduction is achieved when the differences between successive samples rather than the sample values themselves are encoded.

A picture that has been quantized to 256 levels (8 bits) may be transmitted with comparable fidelity using 4 bits DPCM. This reduces the transmission bandwidth by a **factor** of **2**.

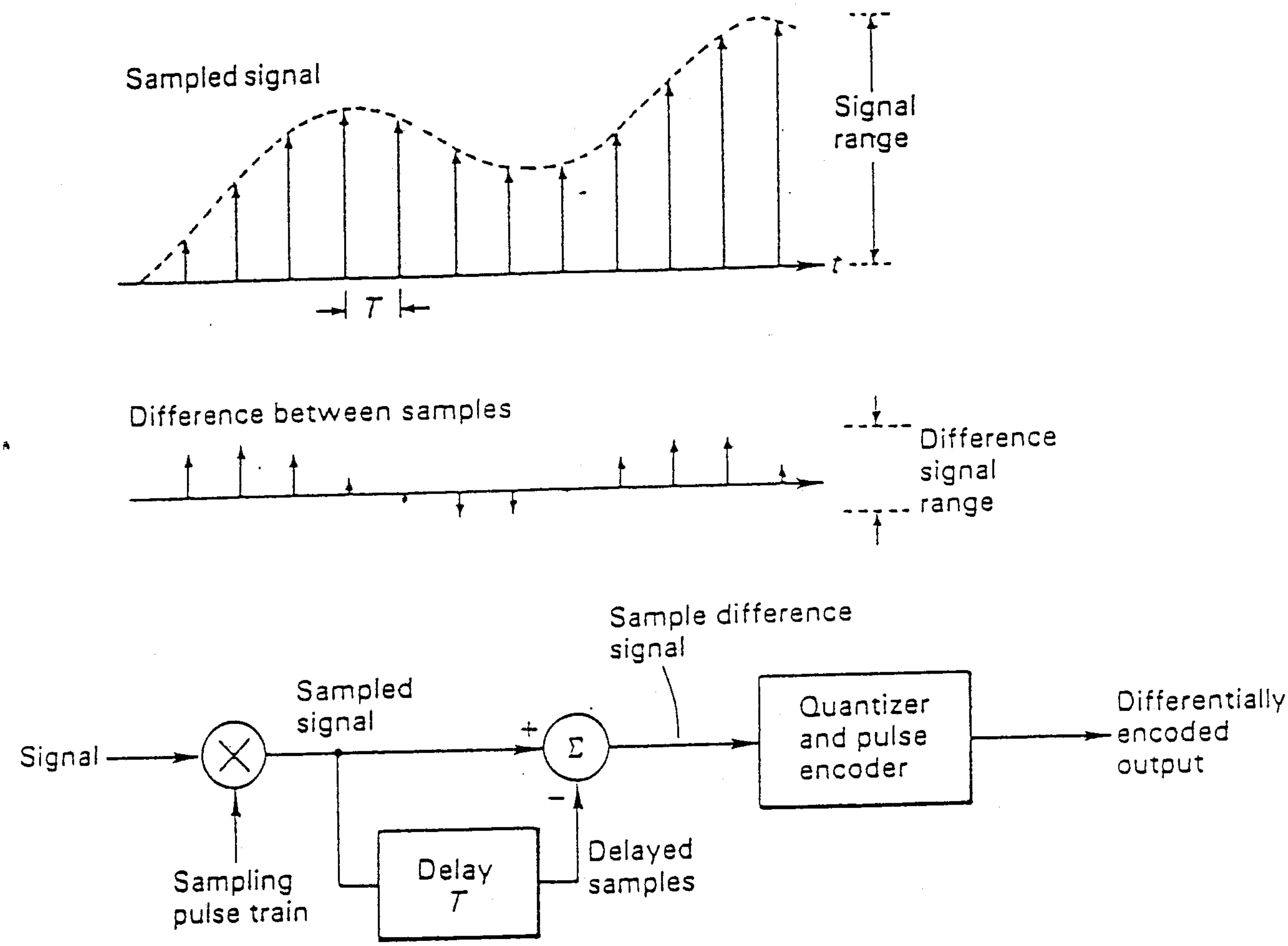


Fig. 42 Principles of DPCM

Delta Modulation (DM):

In this scheme, sample correlation used in DPCM is further increased by oversampling (about 4 times the Nyquist rate) the baseband signal. This increased correlation between adjacent samples minimizes the prediction error, which can be encoded using only one bit $(L = 2)$.

Thus, DM is a 1-bit DPCM that uses only two levels $(L=2)$ for quantization of $m(k)-\hat{m}_q(k)$

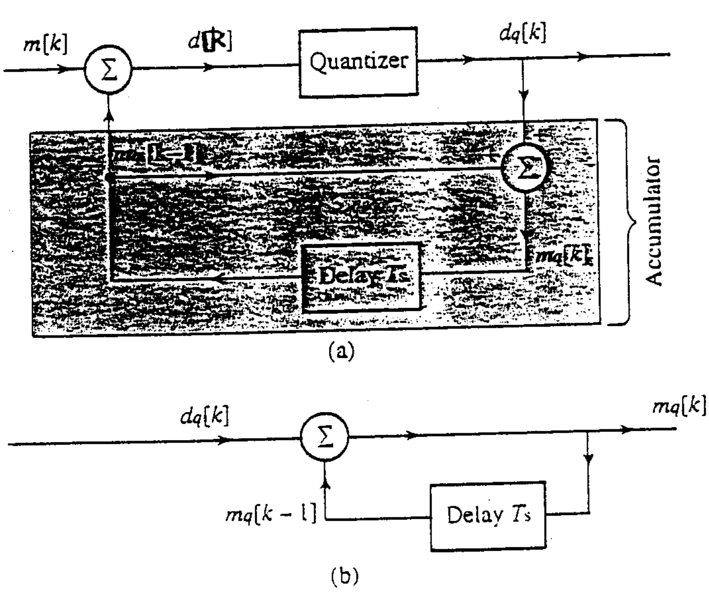


Figure 6.19 Delta modulation is a special case of DPCM.

DM is very easy and inexpensive way of A/D conversion.

In DM, we use a time delay of T_s.

$$m_q[k] = m_q[k-1] + d_q[k] \quad \text{------} (1)$$

$$\therefore \quad m_q[k-1] = m_q[k-2] + d_q[k-1] \quad \text{----} (2)$$

Substituting (2) in (1) gives:

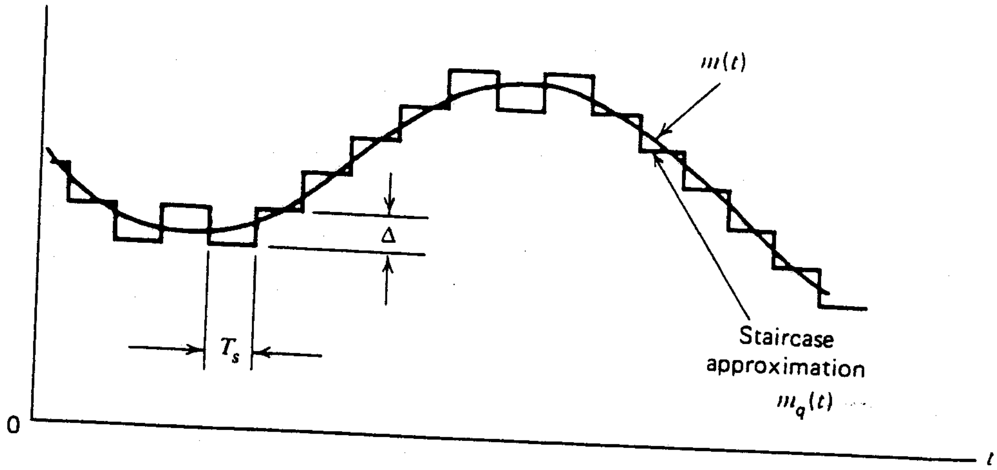
$$m_q[k] = m_q[k-2] + d_q[k] + d_q[k-1]$$

Assuming $m_q[0] = 0$,

$$\therefore \quad m_q[k] = \sum_{m=0}^{k} d_q[m]$$

This shows that the receiver is just an accumulator (adder). Hence, the feedback portion of the receiver can be replaced by an integrator. The same thing applies to the DM transmitter.

The pulse train $d_q[k]$ is the delta-modulated pulse train.

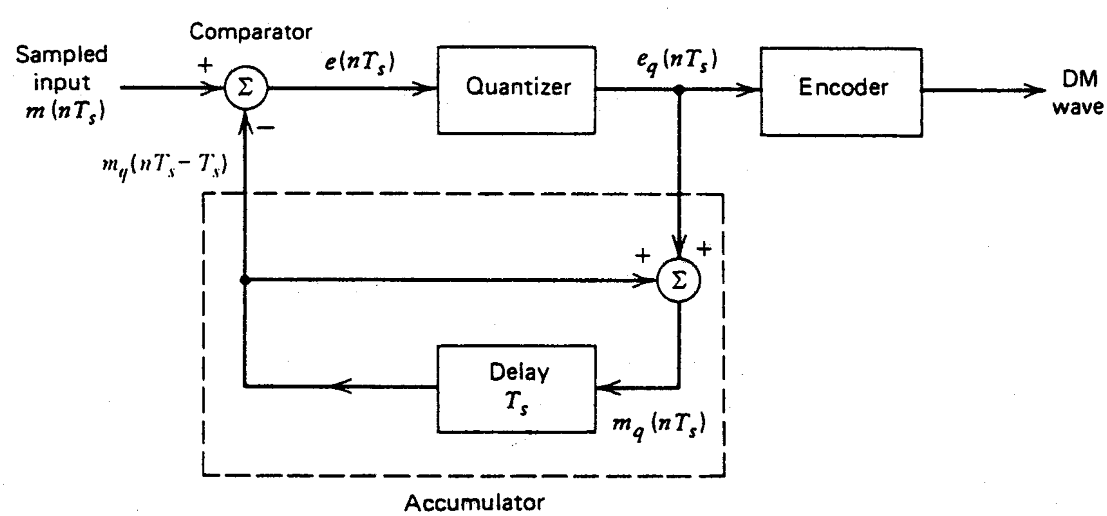


(a)

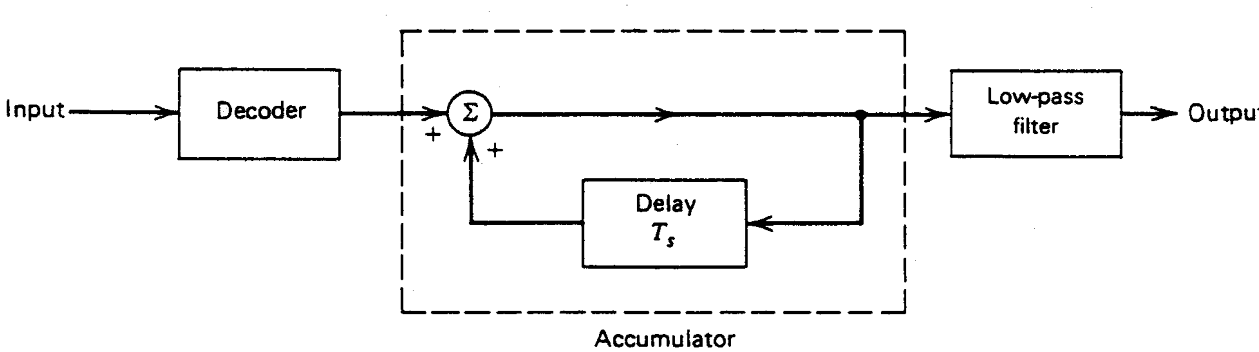
Binary sequence at modulator output   0 0 1 0 1 1 1 1 1 0 1 0 0 0 0 0 0

(b)

Figure 6.23 Illustration of delta modulation.



(a)



(b)

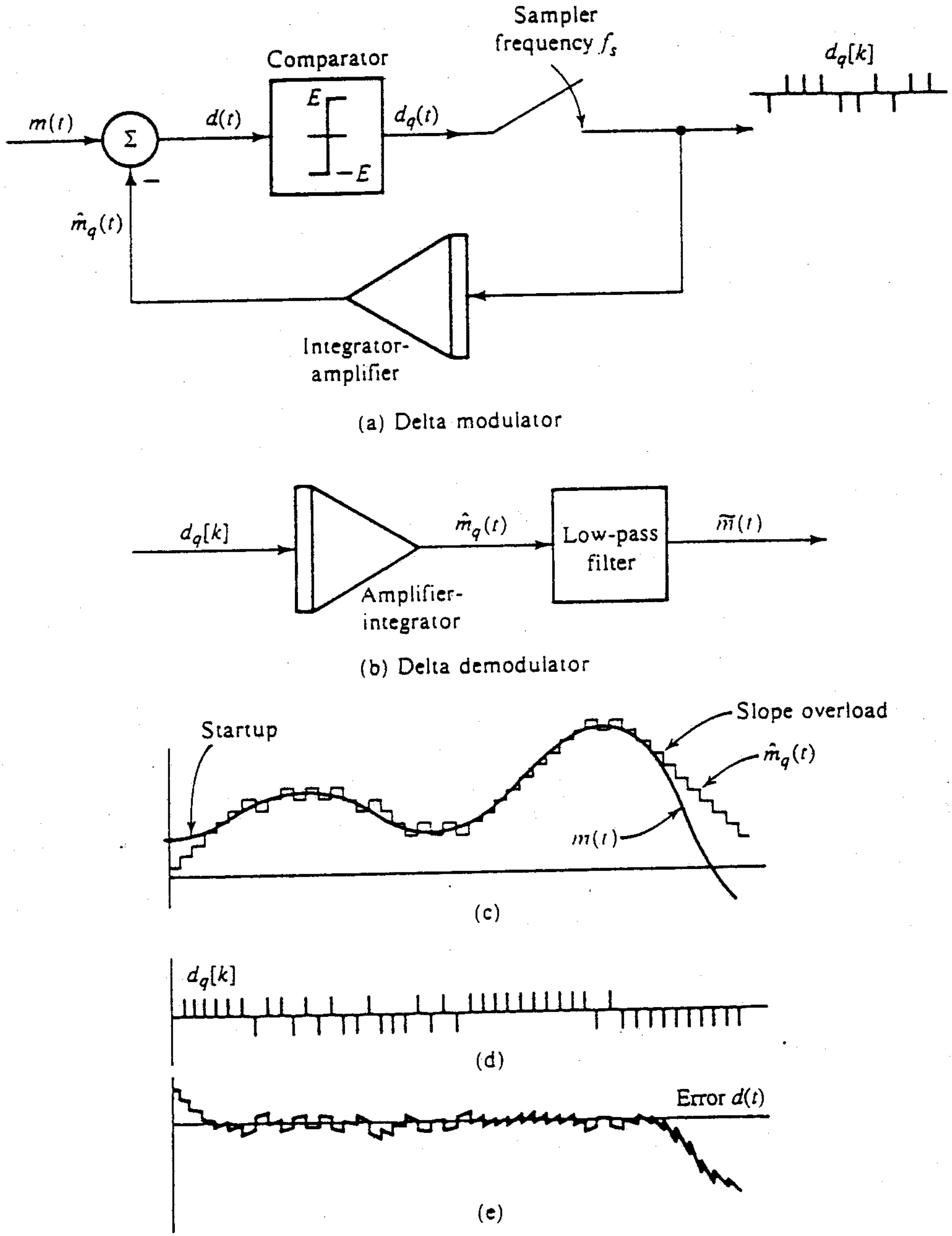


Figure 6.20 Delta modulation.