King Fahd University of Petroleum & Minerals
Department of Electrical Engineering

Communications Engineering I
EE 370

Course Notes
Chapter 5

Prepared By:
Dr. Azzedine Zerguine

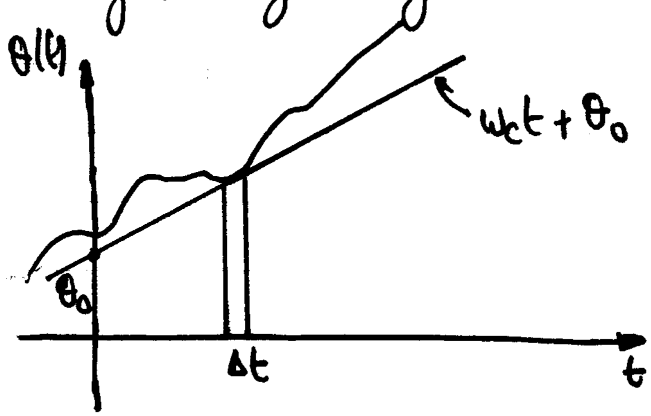
Chapter 5 — Angle (Exponential) Modulation

1. Concept of instantaneous frequency:

Consider a generalized sinusoidal signal $\varphi(t)$ given by:

$$\varphi(t) = A\cos\theta(t)$$

where $\theta(t)$ is the generalized angle and is a function of time.



The instantaneous frequency ω_i at any instant t is:

$$\omega_i(t) = \frac{d\theta(t)}{dt}$$

equivalently $\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\,d\alpha$

Now we can see the possibility of transmitting the information $m(t)$ by varying the angle θ of a carrier. Such techniques are known as angle modulation or exponential modulation.

Two simple possibilities are:

1. phase modulation,
2. frequency modulation.

In PM, the angle $\theta(t)$ is: $\theta(t) = \omega_c t + \theta_0 + k_p m(t)$

where k_p is a constant and ω_c is the carrier. Assuming $\theta_0 = 0$, then

$$\varphi_{PM}(t) = A\cos\theta(t)$$
$$= A\cos[\omega_c t + k_p m(t)]$$

$$\Rightarrow \quad \omega_i(t) = \frac{d\theta(t)}{dt}$$
$$= \omega_c + k_p \frac{dm(t)}{dt}$$

In the case of FM the instantaneous frequency is:

$$\omega_i(t) = \omega_c + k_f\, m(t)$$, where k_f is a constant.

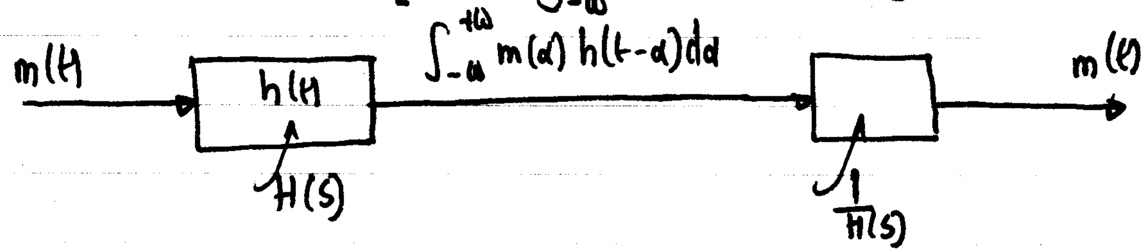
The angle $\theta(t)$ is now given by:

$$\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f\, m(\alpha)]\, d\alpha$$
$$= \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha$$, assuming constant term $= 0$.

$$\Rightarrow \varphi_{FM}(t) = A \cos \theta(t)$$
$$= A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

The generalized angle-modulated carrier $\varphi_{EM}(t)$ is given by:

$$\varphi_{EM}(t) = A \cos\left[\omega_c t + \int_{-\infty}^{t} m(\alpha)\, h(t-\alpha)\, d\alpha\right]$$



If $h(t) = k_p\, \delta(t) \quad \Rightarrow \varphi_{PM}(t)$

If $h(t) = k_f\, u(t) \quad \Rightarrow \varphi_{FM}(t)$

Example 5:1 and Example 5:2.

Example:

Sketch the FM and PM waves for the modulating signal shown.

$k_f = 2\pi \times 10^5$, $k_p = 10\pi$, $f_c = 100$ MHz.

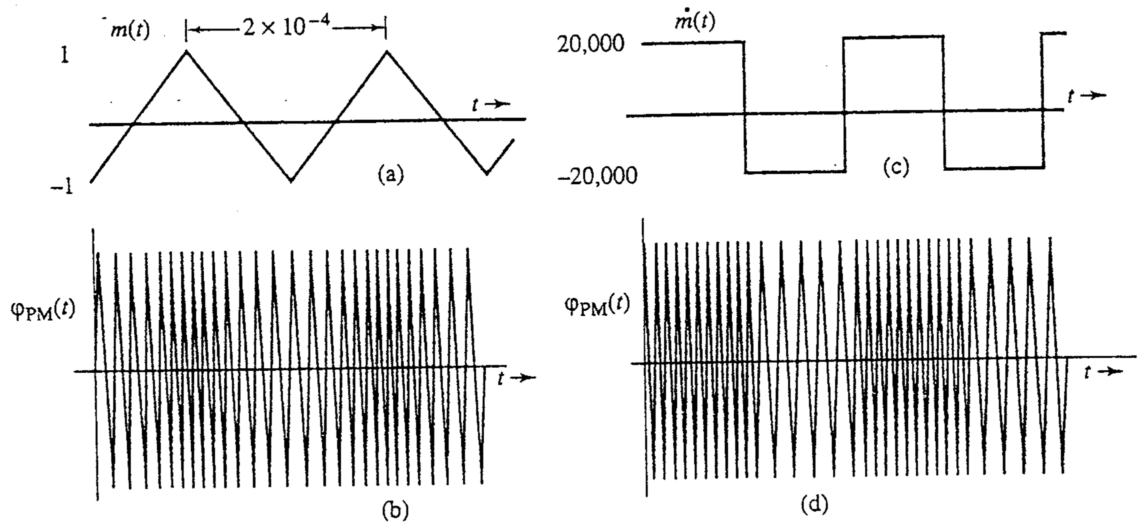


Figure 5.4 FM and PM waveforms.

For FM: $w_i = w_c + k_f\, m(t)$

$$f_i = f_c + \frac{k_f}{2\pi}\, m(t)$$

$$= 10^8 + 10^5\, m(t)$$

$$(f_i)_{min} = 10^8 + 10^5\, [m(t)]_{min}$$

$$= 99.9 \text{ MHz}$$

$$(f_i)_{max} = 10^8 + 10^5\, [m(t)]_{max}$$

$$= 100.1 \text{ MHz}$$

For PM: $f_i = f_c + \frac{k_p}{2\pi}\, \dot{m}(t)$

$$= 10^8 + 5\, \dot{m}(t)$$

$$(f_i)_{min} = 10^8 + 5\, [\dot{m}(t)]_{min} = 99.9 \text{ MHz}.$$

$$(f_i)_{max} = 10^8 + 5\, [\dot{m}(t)]_{max} = 100.1 \text{ MHz}.$$

Example:

Sketch FM and PM waves for the digital modulating signal m(t) shown below.
$k_f = 2\pi \times 10^5$, $k_p = \pi/2$, $f_c = 100$ MHz.

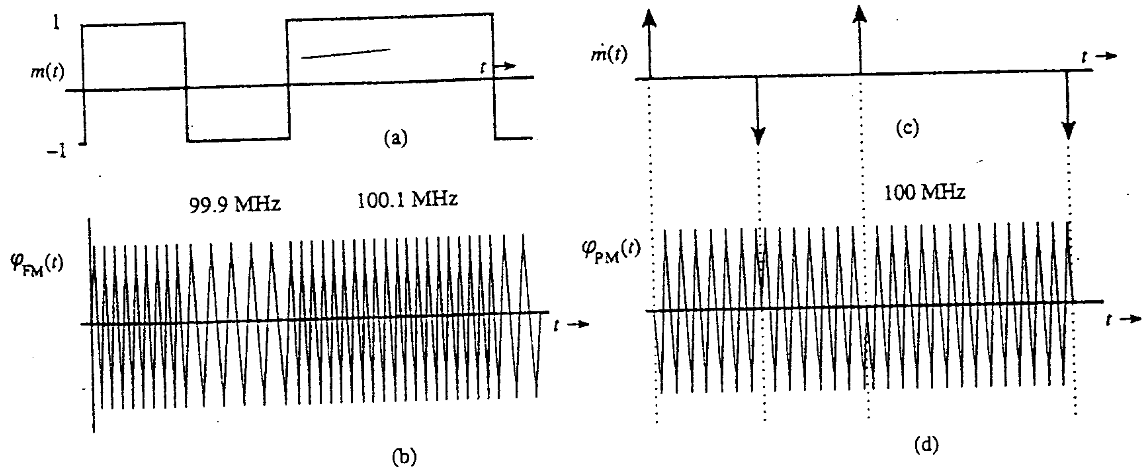


Figure 5.5 FM and PM waveforms.

For FM:

$$f_i = f_c + \frac{k_f}{2\pi} m(t)$$
$$= 10^8 + 10^5 m(t)$$
$$(f_i)_{min} = 10^8 - 10^5 = 99.9 \text{ MHz}$$
$$(f_i)_{max} = 10^8 + 10^5 = 100.1 \text{ MHz}$$

For PM:

$$f_i = f_c + \frac{k_p}{2\pi} \dot{m}(t)$$
$$= 10^8 + \frac{1}{4} \dot{m}(t)$$
$$\varphi_{PM}(t) = A \cdot \cos[w_c t + k_p m(t)]$$
$$= A \cos[w_c t + \frac{\pi}{2} m(t)]$$
$$= \begin{cases} A \sin w_c t, & \text{if } m(t) = -1 \\ -A \sin w_c t, & \text{if } m(t) = 1 \end{cases}$$

* Power of angle-modulated wave:

Although the instantaneous frequency and phase of an angle-modulated wave can vary with time, the amplitude A always remains constant. Hence, the power of an angle-modulated wave (PM or FM) is always $A^2/2$, regardless of the value of k_p or k_f.

2- Bandwidth of angle-modulated waves:

In order to determine the bandwidth of an FM wave, let us define:

$$a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha$$

and

$$\hat{\varphi}_{FM}(t) = A\, e^{j[\omega_c t + k_f a(t)]} = A e^{j\omega_c t} e^{jk_f a(t)}$$

Hence,

$$\varphi_{FM}(t) = \mathrm{Re}\left[\hat{\varphi}_{FM}(t)\right]$$

Expanding the exponential $e^{jk_f a(t)}$ in a power series yields:

$$\hat{\varphi}_{FM}(t) = A\left[1 + jk_f a(t) - \frac{k_a^2 a^2(t)}{2!} + \cdots + j^n \frac{k_f^n a^n(t)}{n!}\right] e^{j\omega_c t}$$

$$\therefore \varphi_{FM}(t) = A\left[\underset{\substack{\uparrow}}{\cos\omega_c t} - k_f a(t)\sin\omega_c t - \frac{k_f^2 a^2(t)}{2!}\cos\omega_c t + \frac{k_f^3 a^3(t)}{3!}\sin\omega_c t \cdots\right]$$

unmodulated carrier + various amplitude-modulated terms.

let $m(t) \Leftrightarrow M(\omega)$

$a(t) \Leftrightarrow A(\omega)$

The signal $a(t) = \int_{-\infty}^{t} m(\alpha)\, d\alpha$. If $M(\omega)$ is bandlimited to B Hz, $A(\omega)$ is also band limited to B Hz.

The spectrum of $a^2(t)$ is simply $\frac{A(\omega) * A(\omega)}{2\pi}$ is band-limited to $2B$.

Similarly, the spectrum of $a^n(t)$ is band-limited to nB.

Hence, the spectrum consists of unmodulated carrier plus spectra of $a(t), a^2(t), \ldots, a^n(t), \ldots$, centered at ω_c. Clearly, the modulated wave (FM wave) is not band-limited. It has an infinite bandwidth and it is not related to the modulating-signal spectrum in any simple way, as was the case in AM. Although the theoretical bandwidth of an FM wave is infinite, we shall see that most of the modulated-signal power resides in a finite bandwidth.

There are 2 distinct possibilities in terms of bandwidth:

- Narrow-band FM
- Wide-band FM

* **Narrow-band Angle modulation**: k_f is very small ($k_f |a(t)| \ll 1$)

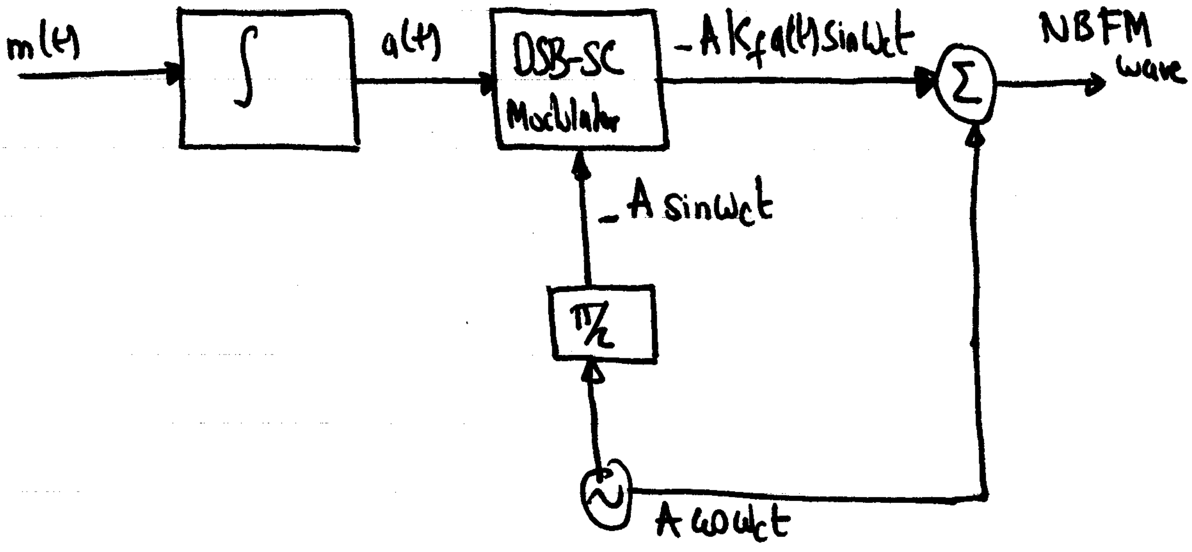
$$\therefore \varphi_{FM}(t) \simeq A\left[\cos\omega_c t - k_f a(t) \sin\omega_c t\right]$$

This is a linear modulation. The expression is similar to AM wave. Because the bandwidth of $a(t)$ is B, the bandwidth of $\varphi_{FM}(t)$ is $2B$.

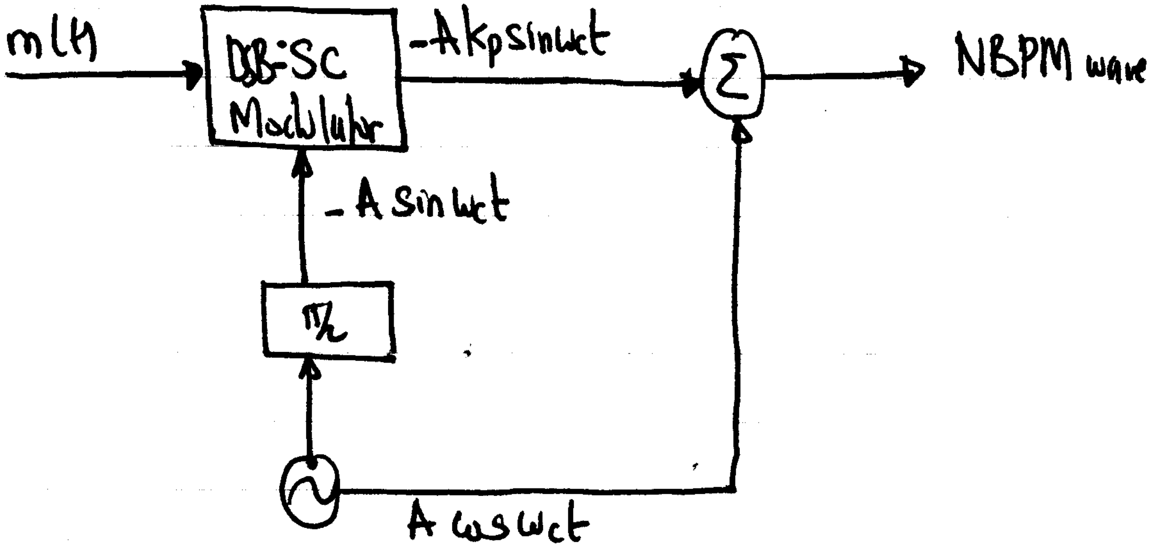
The narrow-band PM (NBPM) case is similarly given by:

$$\varphi_{PM}(t) = A\left[\cos\omega_c t - k_p m(t) \sin\omega_c t\right]$$

The NBFM and NBPM waves can be generated by using DBS-SC modulators.



Narrow-band FM generation.



Narrow-band PM generation.

* Wide-band FM (WBFM):

If the constant k_f is chosen large enough so that the condition $|k_f a(t)| \ll 1$ is not any more satisfied, we cannot ignore the higher horder terms in (5.8b).

$m(t)$ is bandlimited to B Hz. This signal is approximated by a staircase signal $\hat{m}(t)$, as shown in Figure 5.7a. The signal $m(t)$ is approximated by pulses of constant amplitudes. To ensure that $\hat{m}(t)$ has all the information of $m(t)$, the cell width in $\hat{m}(t)$ must be not greater than the Nyquist interval of $\frac{1}{2B}$ seconds. Thus, $m(t)$ is approximated by constant-amplitude cells of width $T = \frac{1}{2B}$ s. Consider a typical cell starting at $t = t_k$ with a constant amplitude $m(t_k)$. Hence, the FM signal corresponding to this cell is a sinusoid of instantaneous frequency $\omega_c + k_f m(t_k)$ and duration $T = \frac{1}{2B}$. The FM signal for $\hat{m}(t)$ consists of a sequence of sinusoidal pulses corresponding to various cells of $\hat{m}(t)$.

The FM spectrum for $\hat{m}(t)$ consists of the sum of the Fourier transforms of these sinusoidal pulses corresponding to all the cells. The Fourier transform of a sinusoidal pulse in Fig 5.7b is a sinc function shown in Fig 5.7c. Note that the spectrum of this pulse is spread out on either side of its frequency $\omega_c + k_f m(t_k)$ by $\frac{2\pi}{T} = 4\pi B$. The minimum and the maximum amplitudes of the cells are $-m_p$ and m_p, respectively. Therefore, the maximum and the minimum significant frequencies in the spectrum are $\omega_c + k_f m_p + 4\pi B$ and $\omega_c - k_f m_p - 4\pi B$, respectively.

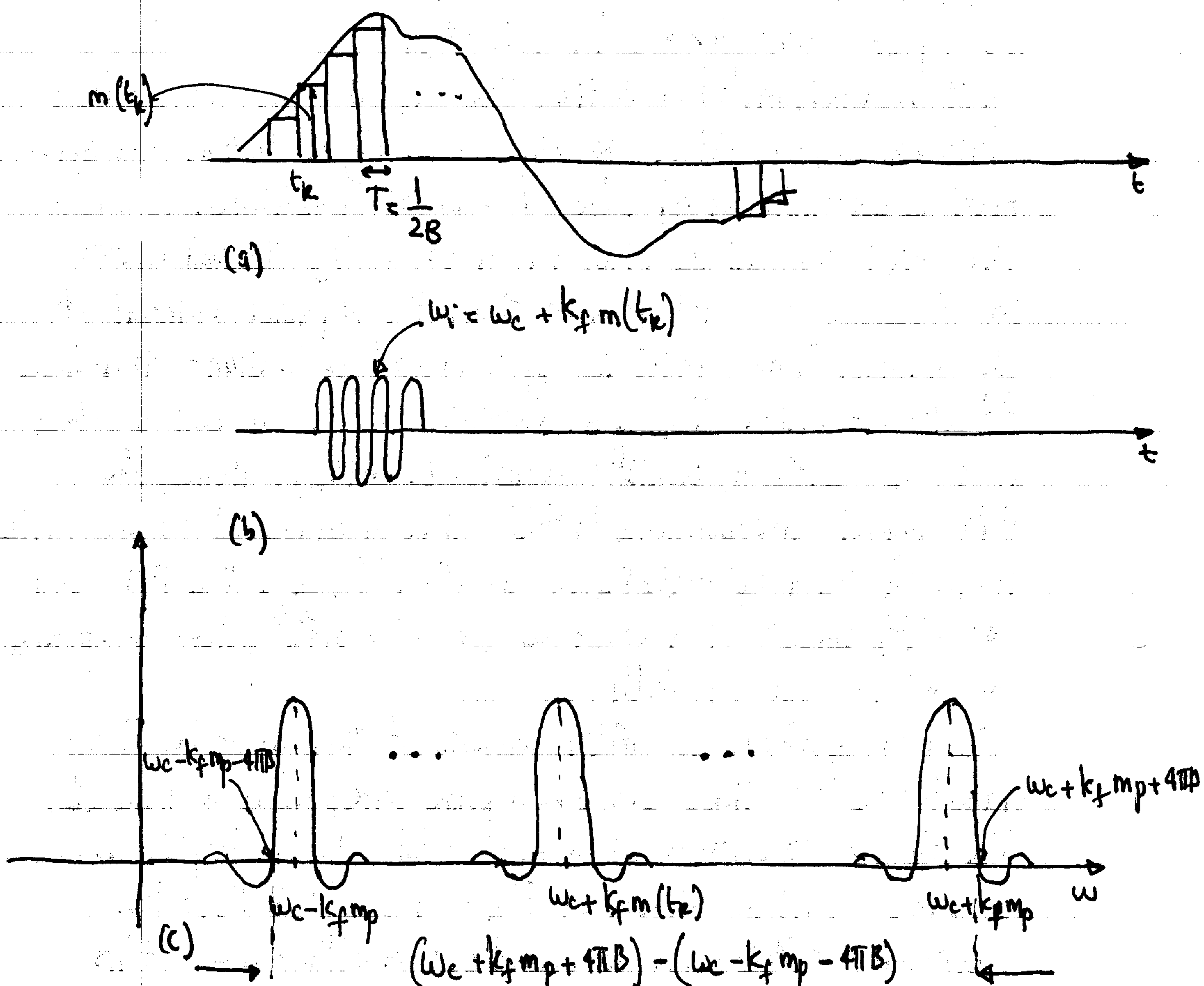
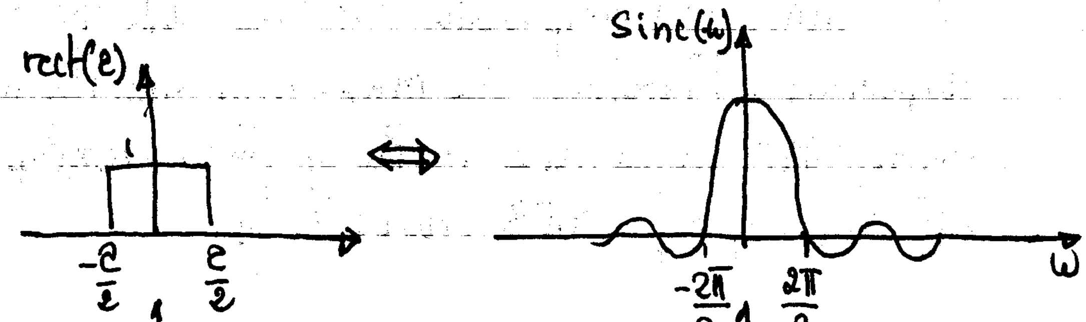
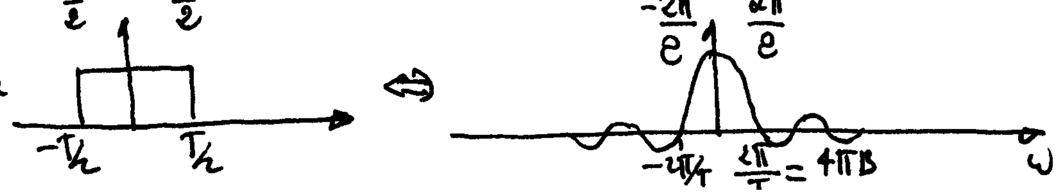


Figure 5.7: Estimation of FM wave Bandwidth.

Recall that:



in our case we have



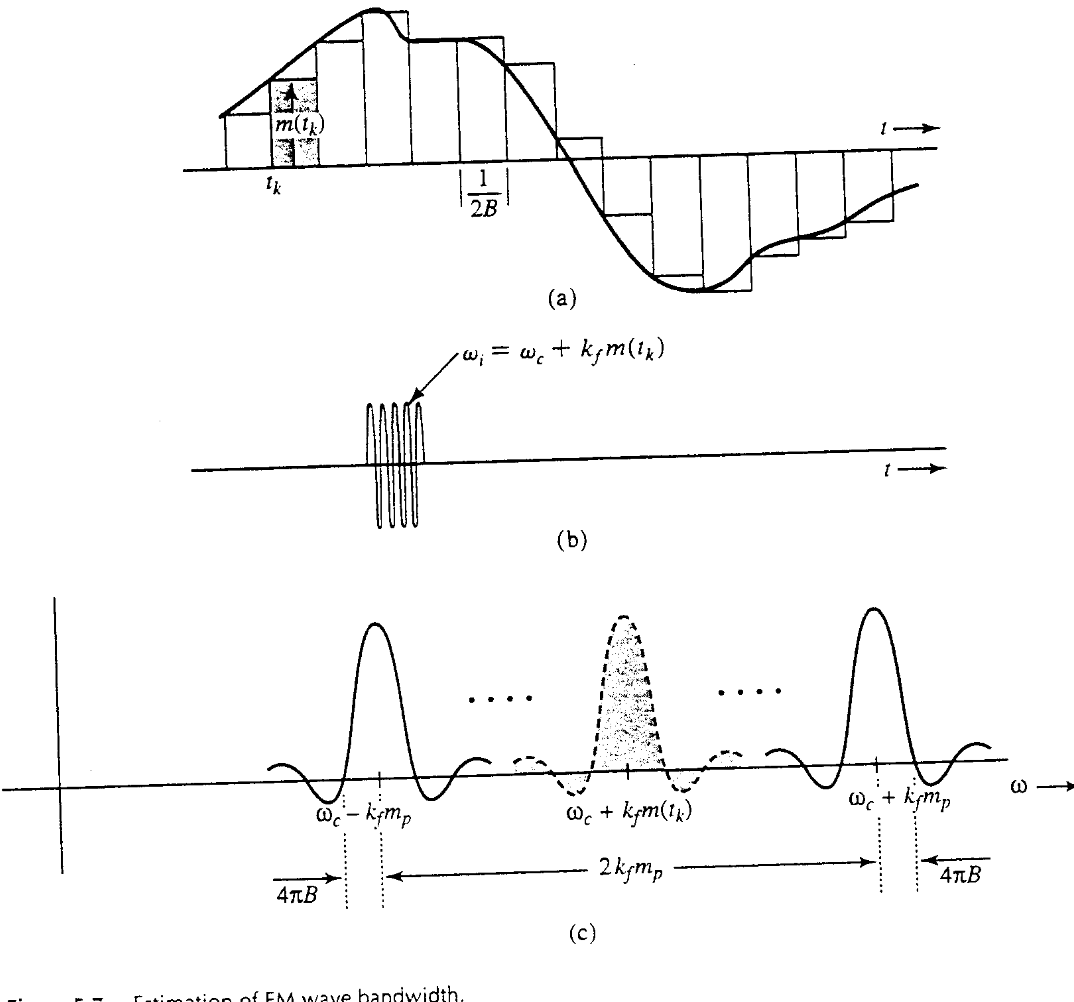


Figure 5.7 Estimation of FM wave bandwidth.

$\Rightarrow$ The spectrum Bandwidth is then:

$$(\omega_c + k_p m_p + 4\pi B) - (\omega_c - k_p m_p - 4\pi B) = 2k_f m_p + 8\pi B$$

The deviation of the carrier frequency is $\pm k_f m_p$. Hence, the carrier frequency deviation: $\Delta\omega = k_f m_p$ rad/s, and in Hertz it is:

$$\Delta f = \frac{k_f m_p}{2\pi}$$

The estimated FM Bandwidth can be expressed as:

$$B_{FM} = \frac{2k_f m_p + 8\pi B}{2\pi}$$

$$= 2\left[\Delta f + 2B\right]$$

This bandwidth corresponds to $\hat{m}(t)$ not to $m(t)$ and it is somewhat higher than the actual value. Hence, the actual bandwidth is somewhat smaller than this value. Therefore, we must readjust our bandwidth estimation. In order to make this mid course correction, we observe that for NBFM case, k_f is very small. Hence, Δf is very small (compared to B).

$$\Rightarrow B_{FM} \simeq 4B$$

which should have being 2B. This indicates that a better bandwidth estimate is given by Carson's Rule:

$$B_{FM} = 2(\Delta F + B).$$

Let $\beta = \frac{\Delta f}{B}$ be the deviation ratio, then

$$B_{FM} = 2B(1+\beta).$$

The deviation ratio controls the amount of modulation and consequently, plays a role similar to the modulation index in AM modulation.

* Wide-band Phase modulation (WBPM):

All the results derived for FM can be directly applied to PM with the following modifications in the instantaneous frequency:

$$\omega_i = \omega_c + k_p \dot{m}(t),$$

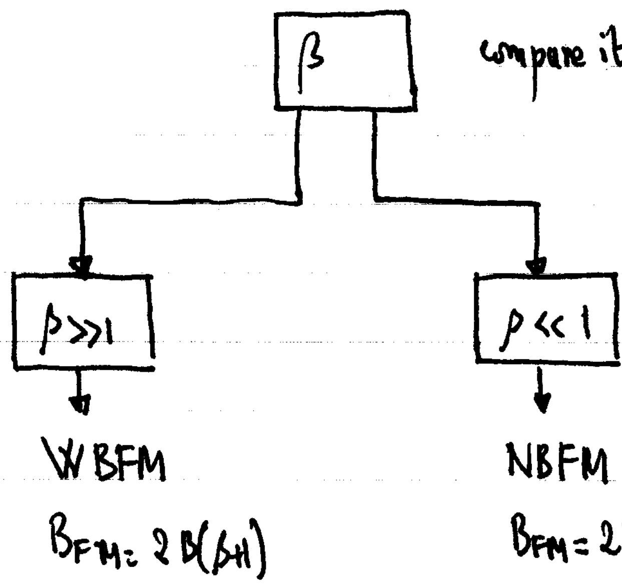
the frequency deviation becomes:

$$\Delta\omega = k_p \dot{m}_p \quad , \text{ where } \dot{m}_p = [\dot{m}(t)]_{max}$$

Therefore, the bandwidth for WBPM:

$$B_{PM} = 2(\Delta f + B)$$
$$= 2\left[\frac{k_p \dot{m}_p}{2\pi} + B\right]$$

Finally, to choose between narrowband FM or wideband FM (equally applicable to PM) one has to evaluate β and then decide, that is,



compare it to 1, this is from Carson's Rule:

$$2B\underbrace{(\beta+1)}_{\downarrow \text{ compare it to } 1}$$

3_ Generation of FM waves:

Basically, there are 2 ways of generating FM waves:

- Inderect generation, and
- Direct generation.

* Inderect method of Armstrong:

In this method, NBFM is generated by integrating $m(t)$ and using it to phase modulate a carrier, as shown in the Figure below. The NBFM is then converted to WBFM by using frequency multipliers.

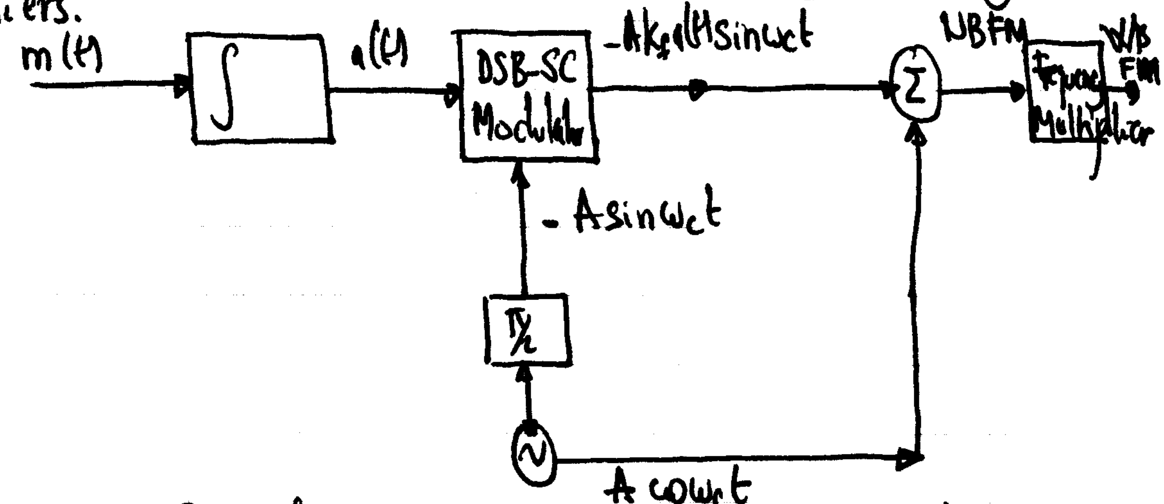


Figure: Simplified block diagram of Armstrong indirect FM wave generator.

The NBFM generated by Armstrong's method has some distortion because of the approximation of equation:

$$\varphi_{FM}(t) = A\left[\cos\omega_c t - k_f\, a(t) \sin\omega_c t - \frac{k_f^2\, a^2(t)}{2!} \cos\omega_c t + \cdots\right]$$

The output of the Armstrong NBFM modulator, as a result, also has some amplitude distortion. Amplitude limiting in the frequency multipliers removes most of this distortion.

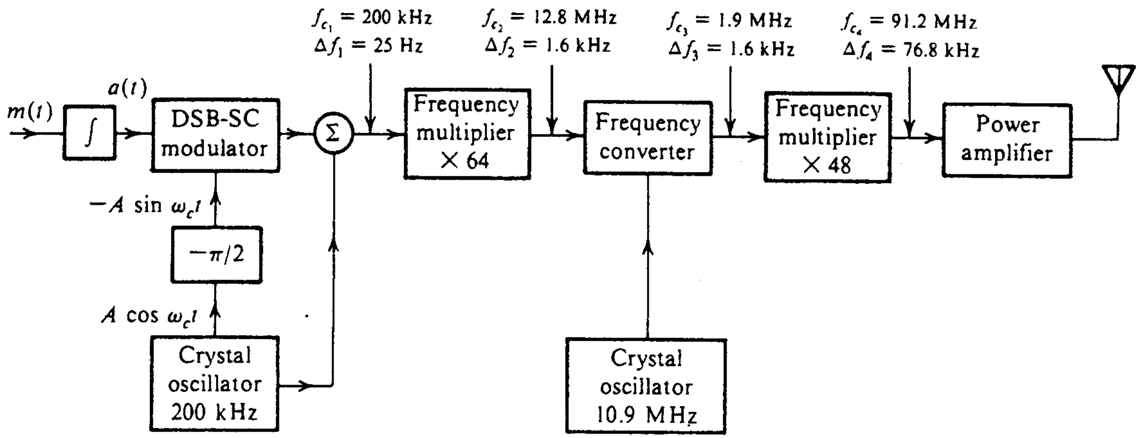


Figure 5.10 Armstrong indirect FM transmitter.

* Direct Generation:

In a voltage-controlled oscillator (VCO), the frequency is controlled by an external voltage. The oscillation frequency varies linearly with the control voltage. We can generate FM wave by using the modulating signal $m(t)$ a control signal. This gives:

$$\omega_i(t) = k_f\, m(t) + \omega_c .$$

This can be achieved by varying C in a Hartley or Colpitt oscillators, where the frequency of oscillation is given by:

$$\omega_0 = \frac{1}{\sqrt{LC}} .$$

The capacitance (known as varicaps) can be approximated as a linear function of the modulating signal $m(t)$ as:

$$C = C_0 - k\, m(t)$$

or

$$\omega_0 = \frac{1}{\sqrt{LC_0\left(1 - \frac{k\, m(t)}{C_0}\right)}}$$

$$= \frac{1}{\sqrt{LC_0}}\left(1 - \frac{k\, m(t)}{C_0}\right)^{-1/2}$$

$$\simeq \frac{1}{\sqrt{LC_0}}\left[1 + \frac{k\, m(t)}{2C_0}\right], \quad \text{for } \frac{k\, m(t)}{C_0} \ll 1$$

Thus $\omega_0 = \omega_c\left[1 + \dfrac{k\, m(t)}{2C_0}\right]$, with $\omega_0 = \dfrac{1}{\sqrt{LC_0}}$

$$\omega_0 = \omega_c + k_f\, m(t), \quad k_f = \frac{k\,\omega_c}{2C_0} \Rightarrow k = \frac{2C_0 k}{\omega_c}$$

Because $C = C_0 - k\, m(t)$, the maximum capacitance deviation is:

$$\Delta C = k\, m_p = \frac{2k_f C_0 m_p}{\omega_c}$$

Hence, $$\frac{\Delta C}{C_0} = \frac{2 k_f m_p}{\omega_c} = \frac{2 \Delta f}{f_c}$$

Direct FM generation generally produces sufficient frequency deviation and requires little frequency multiplication. But this method has poor frequency stability. The output frequency is compared with a constant frequency generated by a stable crystal oscillator. An error signal (error in frequency) is detected and fed back to the oscillator to correct the error.

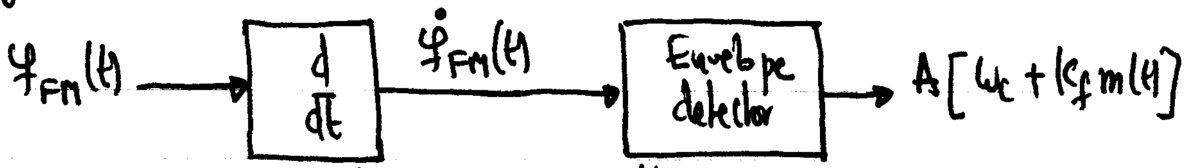
4- Demodulation of FM:

The information in an FM signal resides in the instantaneous frequency $\omega_i = \omega_c + k_f m(t)$. Hence, a frequency-selective network with a transfer function of the form $|H(\omega)| = a\omega + b$ over the FM band would yield an output proportional to the instantaneous frequency. The simplest among them is an ideal differentiator with the transfer function $j\omega$. If we apply $\varphi_{FM}(t)$ to an ideal differentiator, the output is:

$$\dot{\varphi}_{FM}(t) = \frac{d}{dt}\left\{A \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]\right\}$$

$$= A\left[\omega_c + k_f m(t)\right] \sin\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right]$$

$\dot{\varphi}_{FM}(t)$ is both amplitude and frequency modulated, the envelope being $A[\omega_c + k_f m(t)]$. Because $\Delta\omega = k_f m(t) \ll \omega_c$, $\omega_c + k_f m(t) > 0, \forall t$, therefore $m(t)$ can be obtained by envelope of $\dot{\varphi}_{FM}(t)$.



FM demodulation by direct differentiation.

Note that, any variation in A should be removed before applying the signal to the FM detector.

* Bandpass limiter: The amplitude variations of an angle-modulated carrier can be eliminated by what is known a bandpass limiter, which consists of a hard limiter followed by a bandpass filter. The figure below describes it very well.

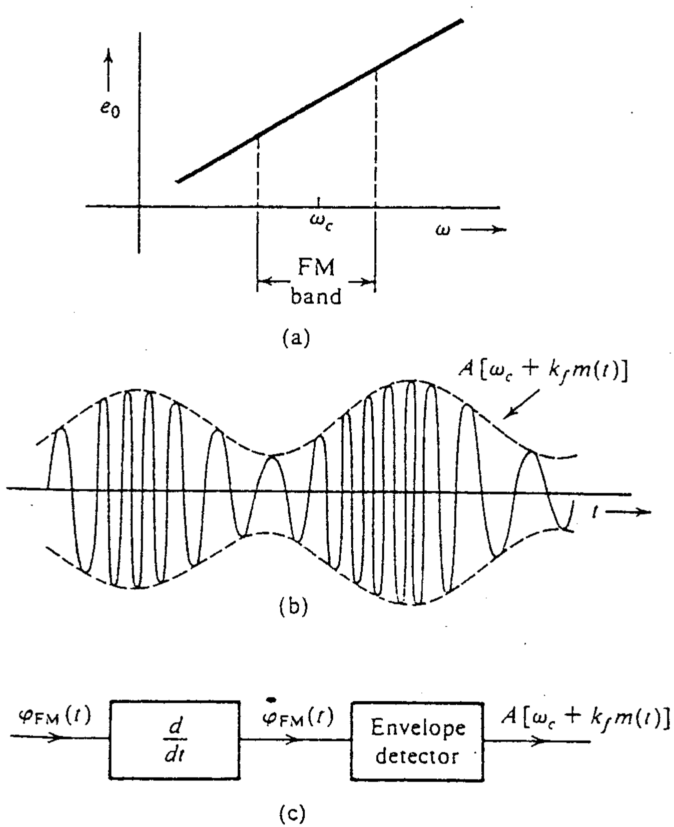
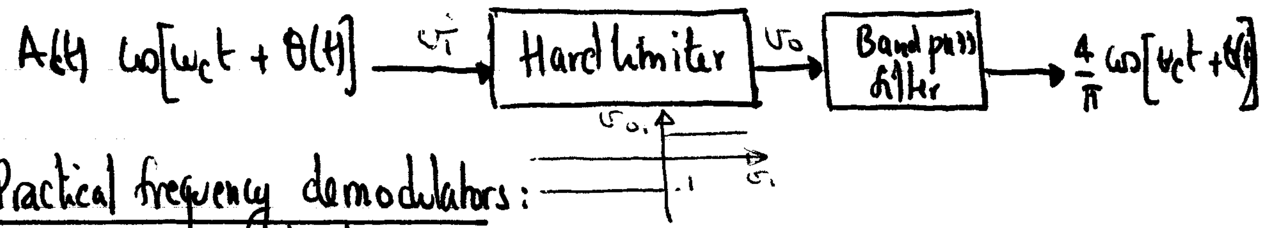


Figure 5.11 (a) FM demodulator frequency response. (b) Output of a differentiator to the input FM wave. (c) FM demodulation by direct differentiation.



* **Practical frequency demodulators:**
  - Slope detection.
  - Ratio detector.
  - Zero-Crossing detectors.
  - Phase-locked loop (PLL): today, this is the most widely used.

- **Phase-locked loop:**

The free-running frequency of VCO is set at the carrier frequency ω_c.
The instantaneous frequency of the VCO is given by:

$$\omega_{vco} = \omega_c + c\, e_o(t)$$

If the VCO output is $B\cos[\omega_c t + \theta_o(t)]$, then its instantaneous frequency is $\omega_c + \dot{\theta}_o(t)$, therefore, $\dot{\theta}_o(t) = c\, e_o(t)$ where c and B are constants of the PLL.

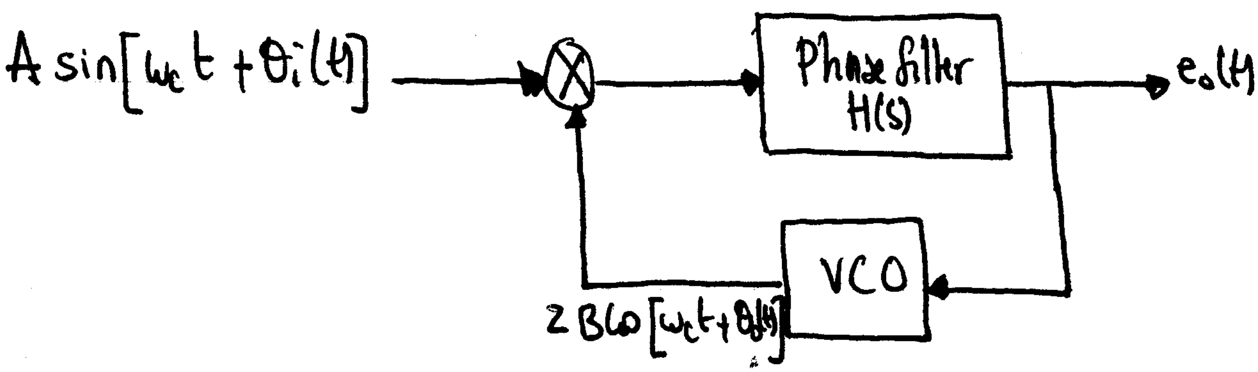


Figure: PLL circuit.

The multiplier output is:

$$AB\sin[\omega_c t + \theta_i(t)]\cos[\omega_c t + \theta_o(t)] = \frac{AB}{2}\left[\sin(\theta_i - \theta_o) + \sin(2\omega_c t + \theta_i + \theta_o)\right]$$

The term $\sin[2\omega_c t + \theta_i + \theta_o]$ is eliminated by the loop filter.

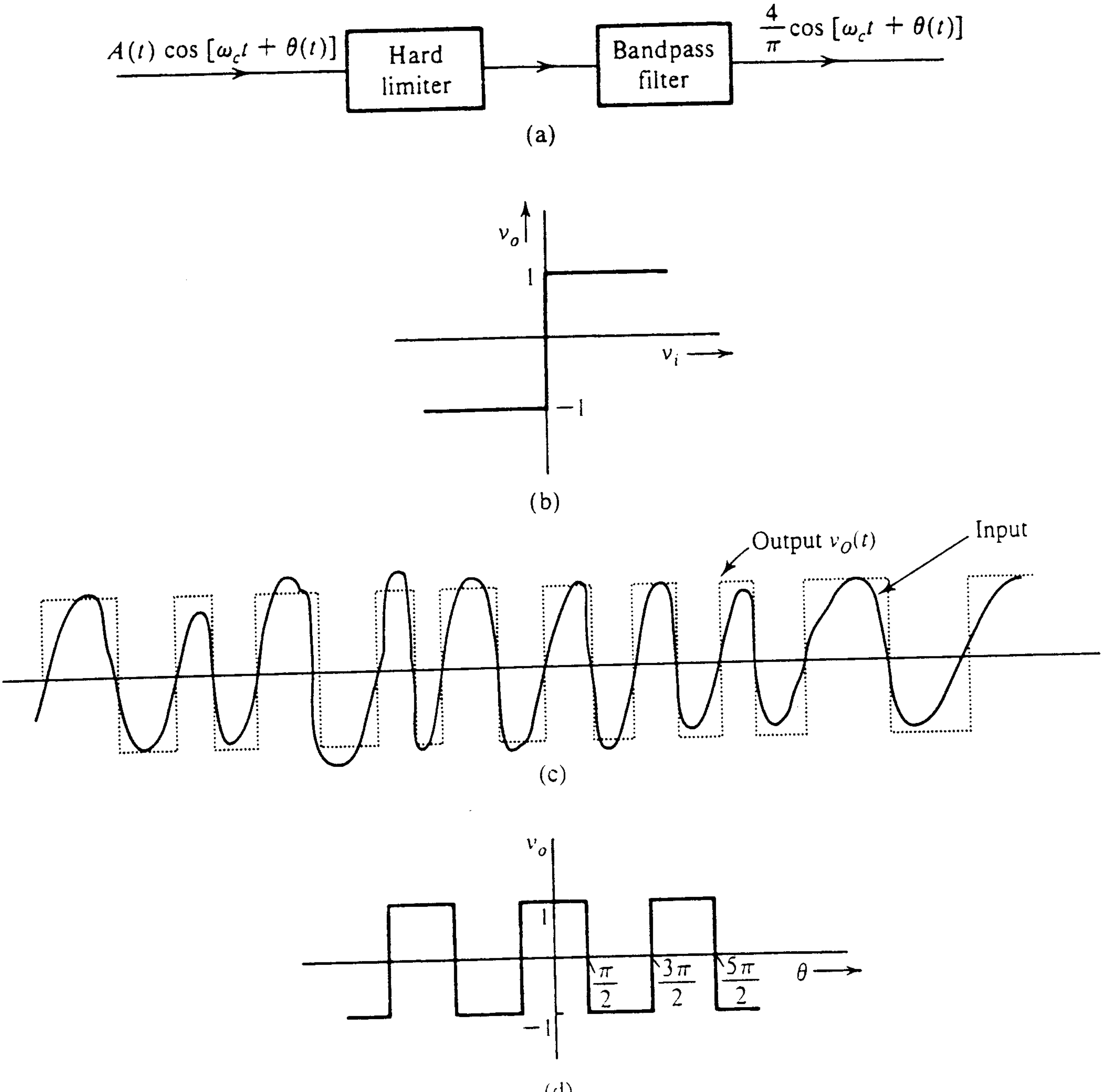


Figure 5.12 (a) Hard limiter and bandpass filter used to remove amplitude variations in FM wave. (b) Hard limiter input-output characteristic. (c) Hard limiter input and the corresponding output. (d) Hard limiter output as a function of θ.

Hence, the effective input to the loop filter is $\frac{1}{2} AB \sin[\theta_i - \theta_o]$.
If $h(t)$ is the unit impulse response of the loop filter,

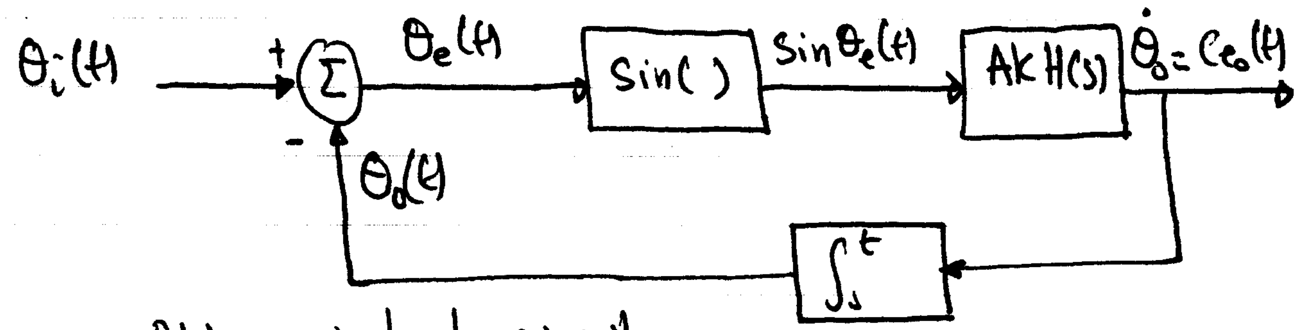
$$e_o(t) = h(t) * \frac{1}{2} AB \sin[\theta_i - \theta_o]$$

$$= \frac{1}{2} AB \int_0^t h(t-\tau) \sin[\theta_i(\tau) - \theta_o(\tau)]\, d\tau$$

Since $\dot{\theta}_o(t) = C e_o(t)$, we have then:

$$\dot{\theta}_o(t) = Ak \int_0^t h(t-\tau) \sin \theta_e(\tau)\, d\tau$$

where $k = \frac{1}{2} cB$ and $\theta_e(t) = \theta_i(t) - \theta_o(t)$.



PLL equivalent circuit.

When the incoming FM carrier is $A \sin[\omega_c t + \theta_i(t)]$,

where $\theta_i(t) = k_f \int_{-\infty}^t m(\alpha)\, d\alpha$, hence

$$\theta_o(t) = \theta_i(t) - \theta_e(t) = k_f \int_{-\infty}^t m(\alpha)\, d\alpha - \theta_e(t)$$

and, assuming a small error θ_e,

$$e_o(t) = \frac{p}{c} \theta_e(t) = \frac{k_f}{c} m(t)$$

Thus, the PLL acts as an FM demodulator. If the incoming signal is a PM wave, $\theta_o(t) = \theta_i(t) = k_p m(t)$ and $e_o(t) = \frac{k_p}{c} \dot{m}(t)$. In this case we need to integrate $e_o(t)$ to obtain the desired signal.

Preemphasis & Deemphasis in FM Broadcasting

It has been found that channel noise (interference) affects FM in a much different way than PM. For example, the interference (noise) at the FM receiver output increases linearly with frequency.

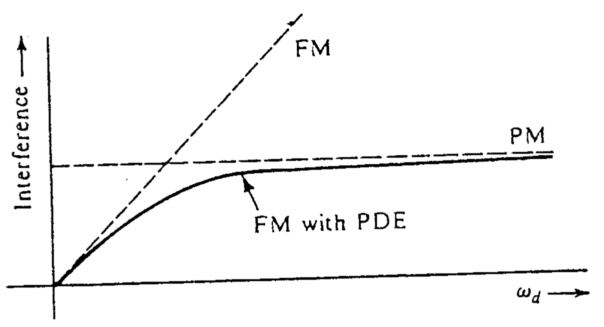
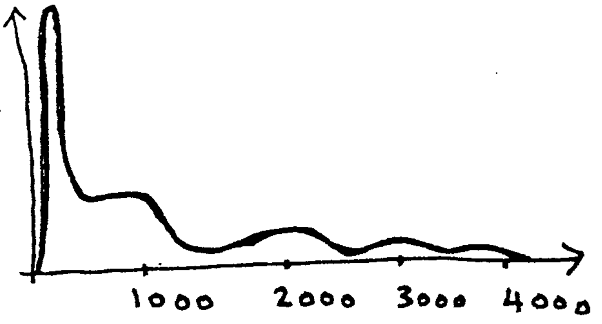


Figure 5.16 Effect of interference in PM, FM, and FM with preemphasis-deemphasis.

Hence, the noise power in the FM receiver output is concentrated at higher frequencies

Audio speech signals have most of their power spectral density (PSD) concentrated at lower frequencies (below 2.1 kHz).



Thus, the noise PSD is concentrated at higher frequencies where speech is weakest.

In order to mitigate this problem, at the transmitter, the weaker high frequency components (above 2.1 kHz) of speech are boosted before modulation by a preemphasis filter ($H_p(f)$). At the receiver, the demodulator output is passed through a deemphasis filter $\left(H_d(f) = \frac{1}{H_p(f)} \right)$. This $H_d(f)$ attenuates (deemphasizes) the higher frequency components (above 2.1 kHz), and hence restores the original signal.

This preemphasis - deemphasis process (PDE) protects the desired signal, and reduces the noise power significantly.

Preemphasis & Deemphasis Filters:

It is obvious that FM has smaller interference than PM at lower frequencies, while PM has smaller interference at higher frequencies. The PDE system used in commercial broadcasting uses preemphasis filter before modulation and deemphasis filter afer demodulation as shown below.

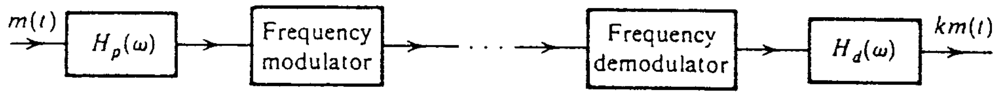


Figure 5.17 Preemphasis-deemphasis in an FM system.

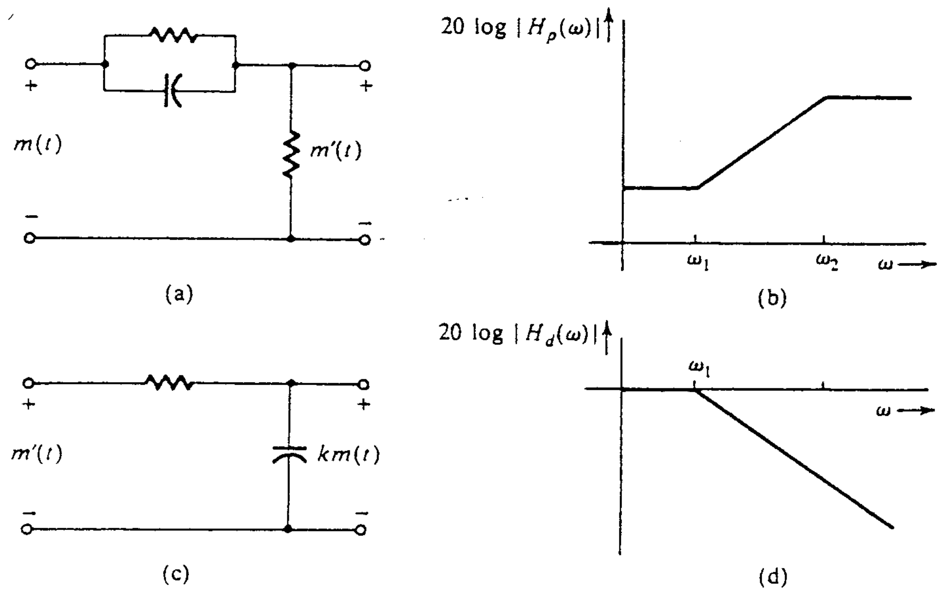


Figure 5.18 (a) Preemphasis filter. (b) Its frequency response. (c) Deemphasis filter. (d) Its frequency respons

The frequency $f_1 = 2.1$ kHz, and the other frequency f_2 is typically 30kHz or more.

$$H_p(w) = \left(\frac{w_2}{w_1}\right)\frac{jw + w_1}{jw + w_2}$$

For $w \ll w_1$

$H_p(w) \simeq 1$

For $w_1 \ll w \ll w_2$

$$H_p(w) \simeq \frac{jw}{w_1}$$

For $w \ll w_2$

$$H_p(w) \simeq \frac{jw + w_1}{w_1}$$

$$H_d(w) = \frac{w_1}{jw + w_1}$$

For $w \ll w_1$

$H_d(w) \simeq 1$

For $w_1 \ll w \ll w_2$

$$H_d(w) \simeq \frac{w_1}{jw}$$

For $w \ll w_2$

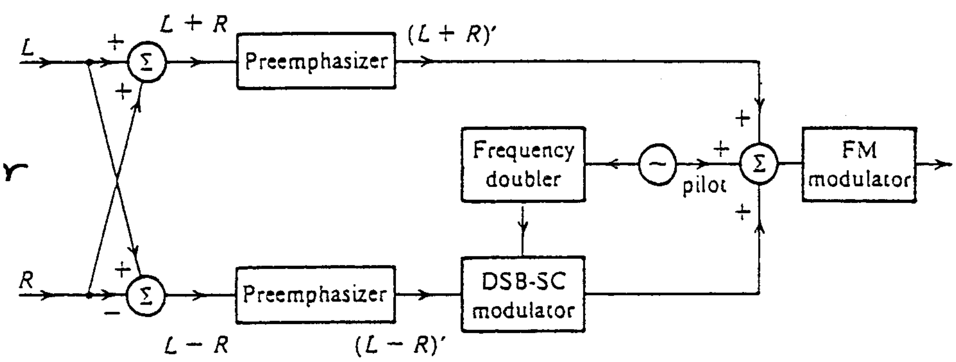
$$H_d(w) \simeq \frac{w_1}{jw + w_1}$$

$\Longrightarrow$ $H_p(w) \cdot H_d(w) \simeq 1$ over the baseband range of 0 to 15 kHz.

FM Receiver :

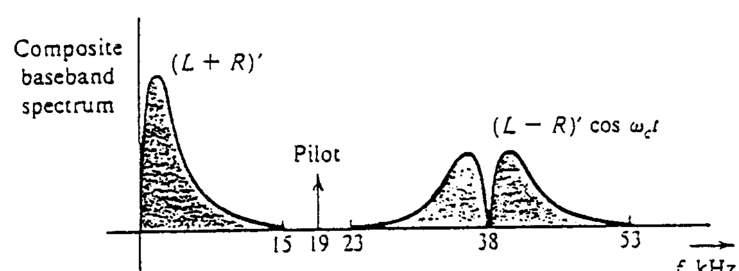
The frequency range 88 → 108 MHz is assigned for FM broadcasting, with a separation of 200 kHz between adjacent stations and a peak frequency deviation of $\Delta f = 75$ kHz.

FM stereo transmitter



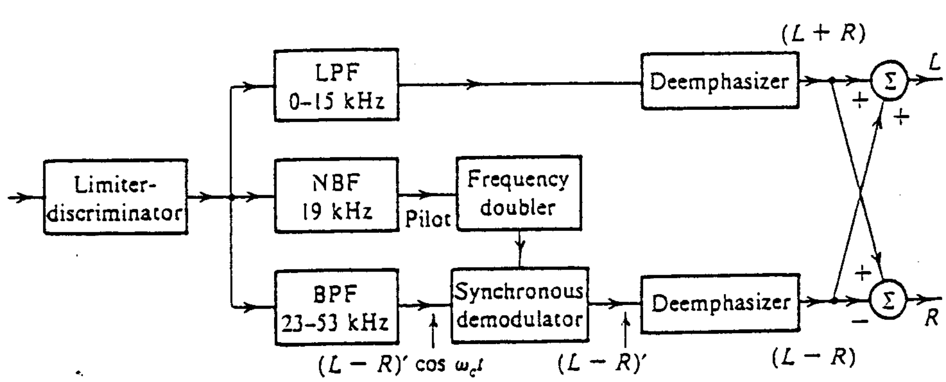
(a)

Spectrum of a baseband stereo signal



(b)

FM stereo receiver



(c)

The composite baseband signal is :

$$m(t) = (L+R)' + (L-R)' \cos(w_g t) + \alpha \cos\left(\frac{w_g t}{2}\right)$$

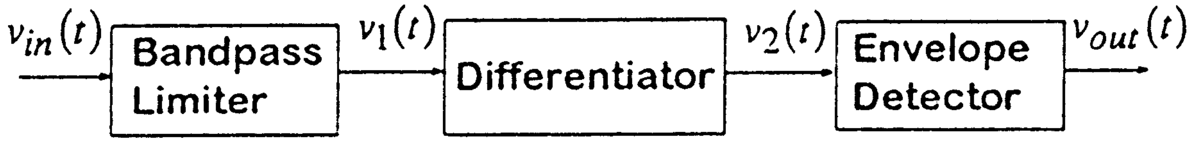
$$w_g = 2\pi f_g, \quad f_g = 38 \text{ kHz}.$$

The pilot carrier is at $\frac{f_g}{2} = 19$ kHz.

FM Broadcast Radio

- Frequency assignments:
  - 200 kHz increments from 88.1 MHz to 107.9 MHz
- Modulation parameters:
  - $B = 15$ kHz
  - $\Delta F = 75$ kHz
  - Modulation index: $\beta_f = \Delta F / B = 5$
  - Carson's Rule bandwidth: $BW = 2(\beta_f + 1)B = 180$ kHz
- Pre-emphasis/De-emphasis filtering is used
- Transmitter power: up to 100 kW

FM Receivers - Discriminator



- $v_{in}(t) = A(t)\cos[\omega_c t + \theta(t)]$
  - May contain amplitude variations due to fading
- $v_1(t) = V_L \cos[\omega_c t + \theta(t)]$
- $v_2(t) = -V_L[\omega_c + d\theta(t)/dt]\sin[\omega_c t + \theta(t)]$
- $v_{out}(t) = |-V_L[\omega_c + d\theta(t)/dt]| = V_L\omega_c + V_L D_f m(t)$
  - a *balanced discriminator* eliminates the carrier frequency term