King Fahd University of Petroleum & Minerals
Department of Electrical Engineering

Communications Engineering I
EE 370

Course Notes
Chapter 4

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Chapter 4 — Amplitude (Linear) modulation

Modulation is a process that causes a shift in the range of frequencies in a signal. Before discussing modulation, it is important to distinguish between communication that does not use modulation (baseband communication) and communication that uses modulation (carrier modulation).

1. Baseband and carrier communication:

The term baseband is used to designate the band of frequencies of the signal delivered by the source or the input transducer.

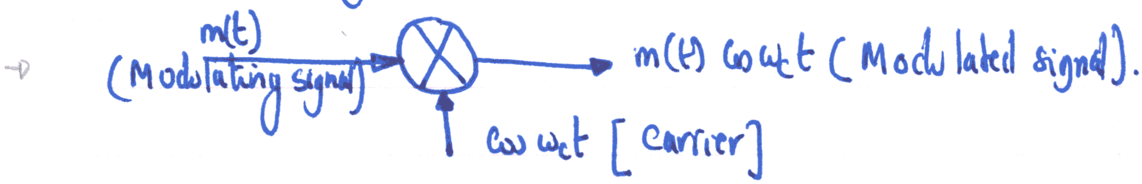
In baseband communication, baseband signals are transmitted without modulation, that is, without any shift in the range of frequencies of the signal.

Communication that uses modulation to shift the frequency spectrum of a signal is known as carrier communication. In this mode, one of the basic parameters (amplitude, frequency, or phase) of a sinusoidal carrier of high frequency w_c is varied in proportion to the baseband signal $m(t)$.

2- Amplitude modulation: Double sideband (DSB)

Amplitude modulation is characterized by the fact that the amplitude A of the carrier $A\cos(w_c t + \theta_c)$ is varied in proportion to the baseband (message) signal $m(t)$, the modulating signal. The frequency w_c and θ_c (phase) are constant. We can assume $\theta_c = 0$ without a loss of generality. If the carrier amplitude A

is made directly proportional to the modulating signal $m(t)$, the modulated signal is $m(t) \cos \omega_c t$.



[If $\quad m(t) \Longleftrightarrow M(\omega)$

then $\quad m(t) \cos \omega_c t \Longleftrightarrow \frac{1}{2}\left[M(\omega+\omega_c) + M(\omega-\omega_c)\right]$

Note that if the bandwidth of $m(t)$ is B Hz, then, as seen from the Figure (Fig. 4.1.c), the bandwidth of the modulated signal is $2B$ Hz.]

We also observe that the modulated signal spectrum centered as ω_c is composed of two parts: a portion that lies above ω_c, known as the upper sideband (USB), and a portion that lies below ω_c, known as the lower sideband (LSB). Similarly, the spectrum centered at $-\omega_c$ has upper and lower sidebands. Hence, this is a modulation scheme with double sidebands. We shall see a little later that the modulated signal in this scheme does not contain a discrete component of the carrier frequency ω_c. For this reason it is called "double-sideband suppressed carrier (DSB-SC) modulation."

see page (2) bring example Here

[The relationship of B to ω_c is of interest. It should be noted that $\omega_c \geq 2\pi B$ in order to avoid the overlap of the spectra centered at ω_c and $-\omega_c$.]

* Demodulation:

"To recover the original signal $m(t)$ from the modulated signal, it is necessary to retranslate the spectrum to its original position."

"Demodulation is almost identical to modulation," consists of multiplication

observe the following

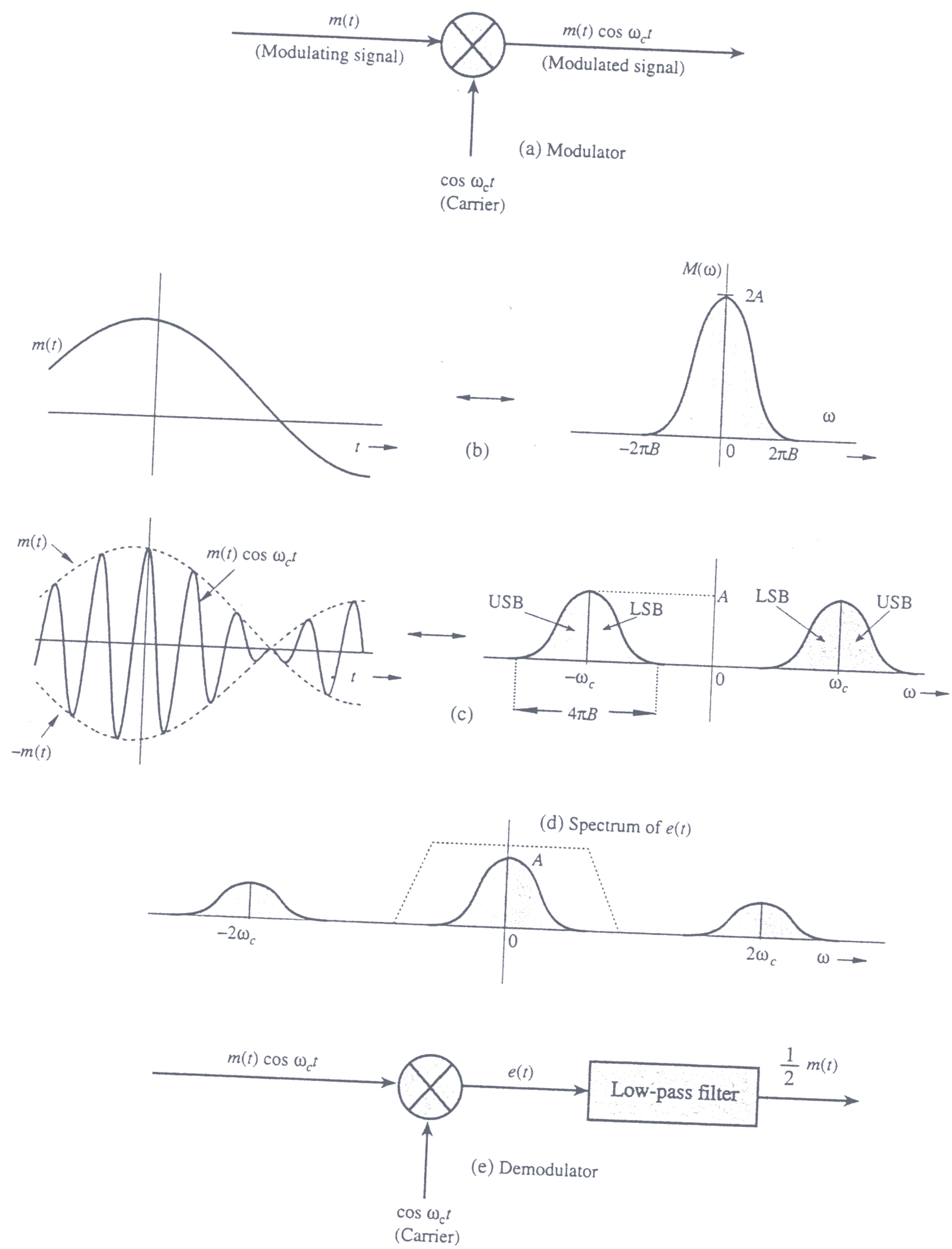


Figure 4.1 DSB-SC modulation and demodulation.

of the incoming modulated signal $m(t)\cos\omega_c t$ by a carrier $\cos\omega_c t$ followed by a low pass filter, as shown in Fig. 4.1e. [Observe that the signal

$$e(t) = m(t)\cos^2\omega_c t$$
$$= \frac{1}{2}\left[m(t) + m(t)\cos 2\omega_c t\right]$$

Therefore, the Fourier transform of the signal $e(t)$ is

$$E(\omega) = \frac{1}{2}M(\omega) + \frac{1}{4}\left[M(\omega+2\omega_c) + M(\omega-2\omega_c)\right]$$

The desired component $\frac{1}{2}M(\omega)$, being a low pass spectrum (centered at $\omega=0$), passes through the filter unharmed, resulting in the output $\frac{1}{2}m(t)$.

[This method of recovering the baseband signal is called synchronous detection, or coherent detection, where we use a carrier of exactly the same frequency (and phase) as the carrier used in modulation.]

Example: For $m(t) = \cos\omega_m t$, find the DSB-SC signal. This is referred to as tone modulation because the modulating signal is a pure sinusoid, or tone, $\cos\omega_m t$.

$$m(t) = \cos\omega_m t \iff M(\omega) = \pi\left[\delta(\omega-\omega_m) + \delta(\omega+\omega_m)\right]$$

Now, the DSB-SC signal is

$$\varphi_{DSB-SC}(t) = m(t)\cos\omega_c t$$
$$= \cos\omega_m t\cos\omega_c t$$
$$= \frac{1}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

Note the curious fact that there is no component of the carrier ω_c on the right-hand side of the preceding equation. As mentioned, this is why it is called double sideband-suppressed carrier (DSB-SC) modulation.

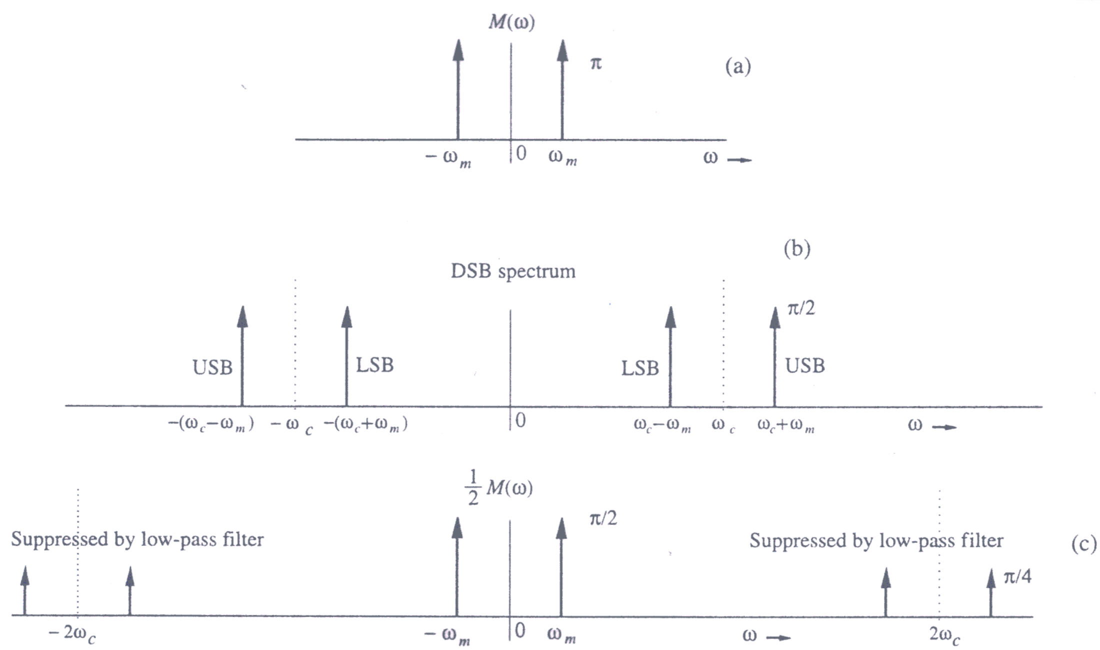


Figure 4.2 Example of DSB-SC modulation.

At the demodulator output, the signal $e(t)$ in Fig 4.1e is given by

$$e(t) = \cos\omega_m t \cos^2\omega_c t$$
$$= \frac{1}{2}\cos\omega_m t\,[1 + \cos 2\omega_c t]$$

The low-pass filter suppresses the spectrum centered at $\pm 2\omega_c$, yielding $\frac{1}{2}M(\omega)$.

• Modulators:

Modulation can be achieved in several ways. We shall discuss here some important categories of modulators:

- Multiplier modulators: Direct multiplication of $m(t)$ by $\cos\omega_c t$. Difficult to maintain linearity and expensive. Avoid them if possible.

- Nonlinear Modulators: Figure 4.3 shows one possible scheme.

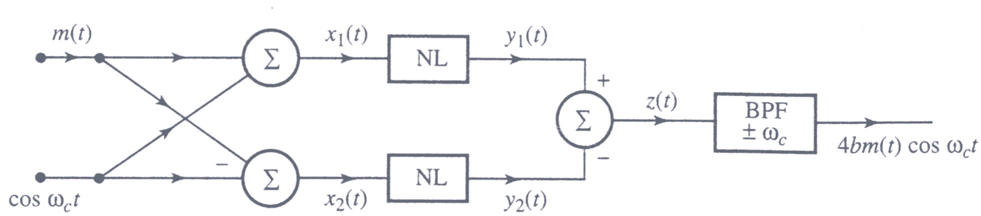


Figure 4.3 Nonlinear DSB-SC modulator.

$$y_1(t) = a x_1(t) + b x_1^2(t)$$
$$y_2(t) = a x_2(t) + b x_2^2(t)$$
$$\therefore \quad z(t) = y_1(t) - y_2(t)$$
with $x_1(t) = m(t) + \cos w_c t$ and $x_2(t) = \cos w_c t - m(t)$
$$\Rightarrow \quad z(t) = 2a\,m(t) + 4b\,m(t)\cos w_c t$$
The spectrum of $m(t)$ is centered at the origin, whereas the spectrum of $m(t)\cos w_c t$ is centered at $\pm w_c$. Consequently, when $z(t)$ is passed through a bandpass filter tuned at w_c, the signal $a\,m(t)$ is suppressed and the desired modulated signal is $4b\,m(t)\cos w_c t$ passes through.

$z(t)$ does not contain 1 of the inputs, i.e., $\cos w_c t$. The circuit acts as a balanced bridge for one of the inputs (the carrier). This modulator is known as a balanced modulator. And since it is balanced for one input it is called single balanced modulator. A circuit balanced with respect to both inputs is called a double balanced modulator.

- Switching modulators: the multiplication operation required for modulation can be replaced by a simpler switching operation if we realize that a modulated signal can be obtained by multipling $m(t)$ not only by a pure sinusoid but by any periodic signal $\phi(t)$ of the fundamental radian frequency w_c.
$$\phi(t) = \sum_{n=0}^{\infty} C_n \cos[w_c n t + \theta_n]$$
Hence,
$$m(t)\,\phi(t) = \sum_{n=0}^{\infty} C_n\, m(t) \cos[n w_c t + \theta_n]$$
This shows that the spectrum of the product $m(t)\,\phi(t)$ is the

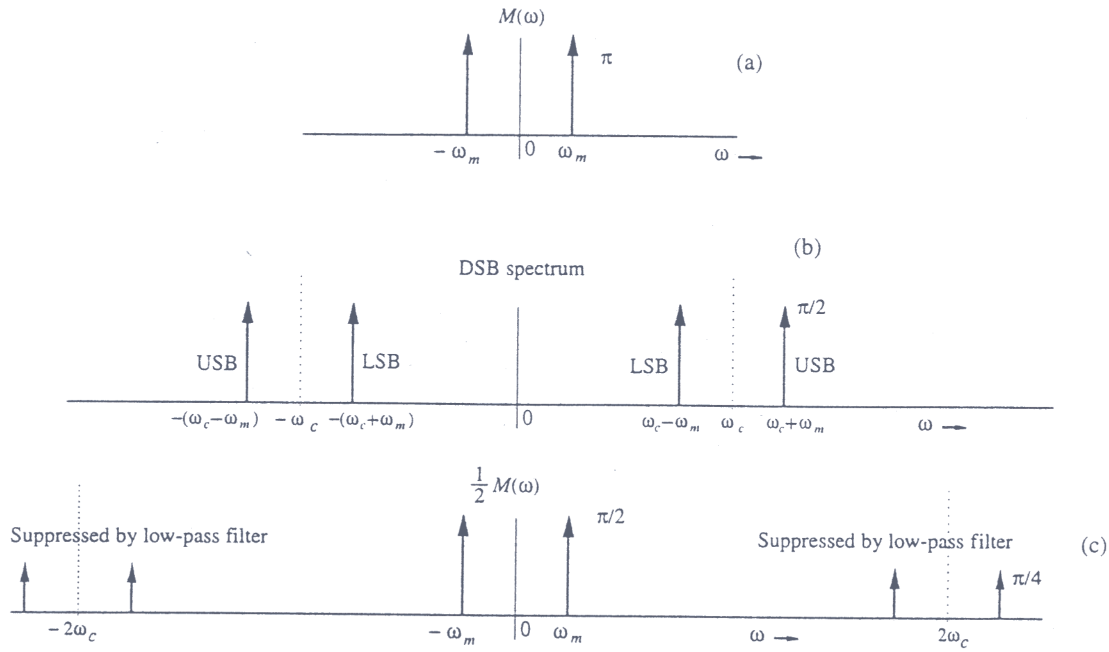


Figure 4.2 Example of DSB-SC modulation.

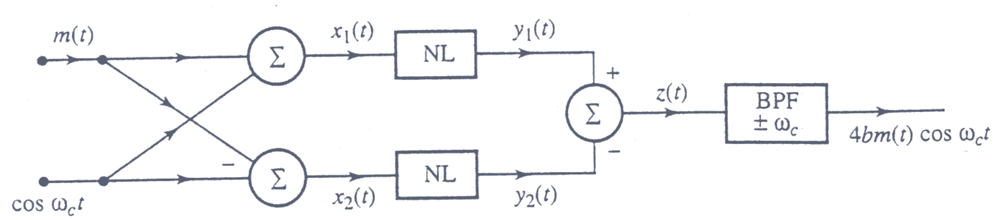


Figure 4.3 Nonlinear DSB-SC modulator.

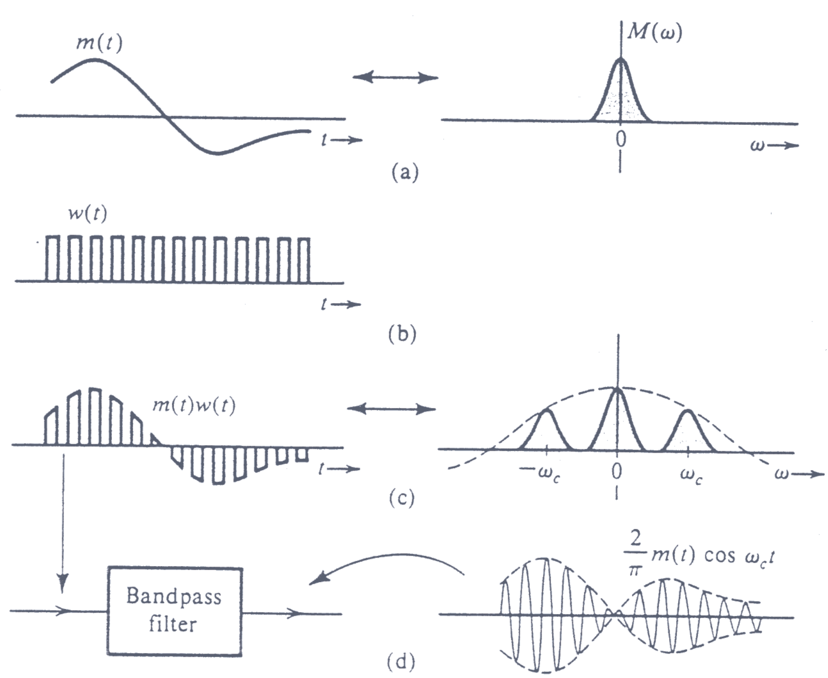


Figure 4.4 Switching modulator for DSB-SC.

spectrum $M(\omega)$ shifted to $\pm\omega_c, \pm 2\omega_c, \pm 3\omega_c, \dots, \pm n\omega_c, \dots$ If this signal is passed through a bandpass filter of bandwidth 2B Hz and tuned to ω_c, then we get the desired modulated signal $c_1 m(t) \cos[\omega_c t + \theta_1]$ - {the phase θ_1 is not important} -

$m(t) \xrightarrow{} \otimes \xrightarrow{} m(t)\,w(t) \rightarrow$ B.P.F. $\pm\omega_c$

Start here

$m(t) \cdot w(t)$

The square pulse train $w(t)$ in Fig. 4.4b is a periodic signal whose Fourier series representation is given by:

$$w(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t + \dots\right]$$

The signal $m(t)\,w(t)$ is then:

$$m(t)\,w(t) = \frac{m(t)}{2} + \frac{2}{\pi}\left[m(t)\cos\omega_c t - \frac{m(t)}{3}\cos 3\omega_c t + \frac{m(t)}{5}\cos 5\omega_c t + \dots\right]$$

We are interested in the modulated component $m(t)\cos\omega_c t$ only. A bandpass filter of bandwidth 2B Hz, centered at the frequency $\pm\omega_c$, will extract the signal $\frac{2}{\pi} m(t)\cos\omega_c t$.

Switch Mod

diode — ring modulator

Multiplication of a signal by a square pulse train is in reality a switching operation. Figure 4.5a shows one such electronic switch, the diode-bridge modulator, driven by a sinusoid $A\cos\omega_c t$ to produce the switching action. These modulators are known as the series-bridge diode modulator and the shunt-bridge diode modulator, respectively shown in Fig 4.5b and Fig 4.5c.

Another switching modulator, known as the ring modulator, is shown in Fig. 4.6a. Here the square pulse train $w_0(t)$ is given (Fourier series) by:

$$w_0(t) = \frac{4}{\pi}\left[\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \dots\right]$$

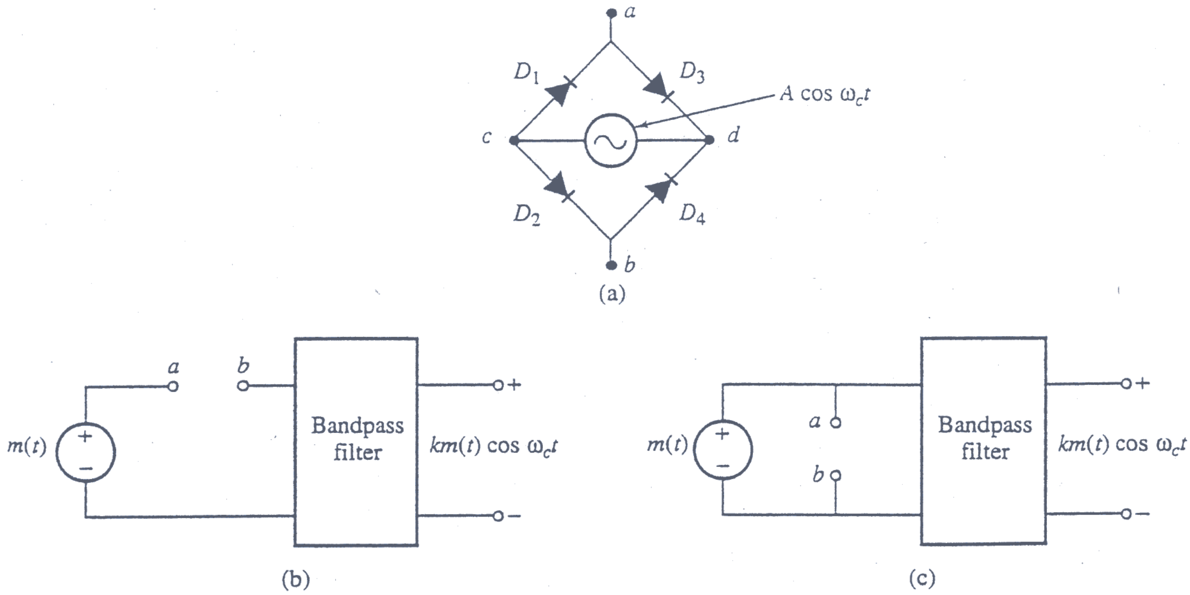


Figure 4.5 (a) Diode-bridge electronic switch. (b) Series-bridge diode modulator. (c) Shunt-bridge diode modulator.

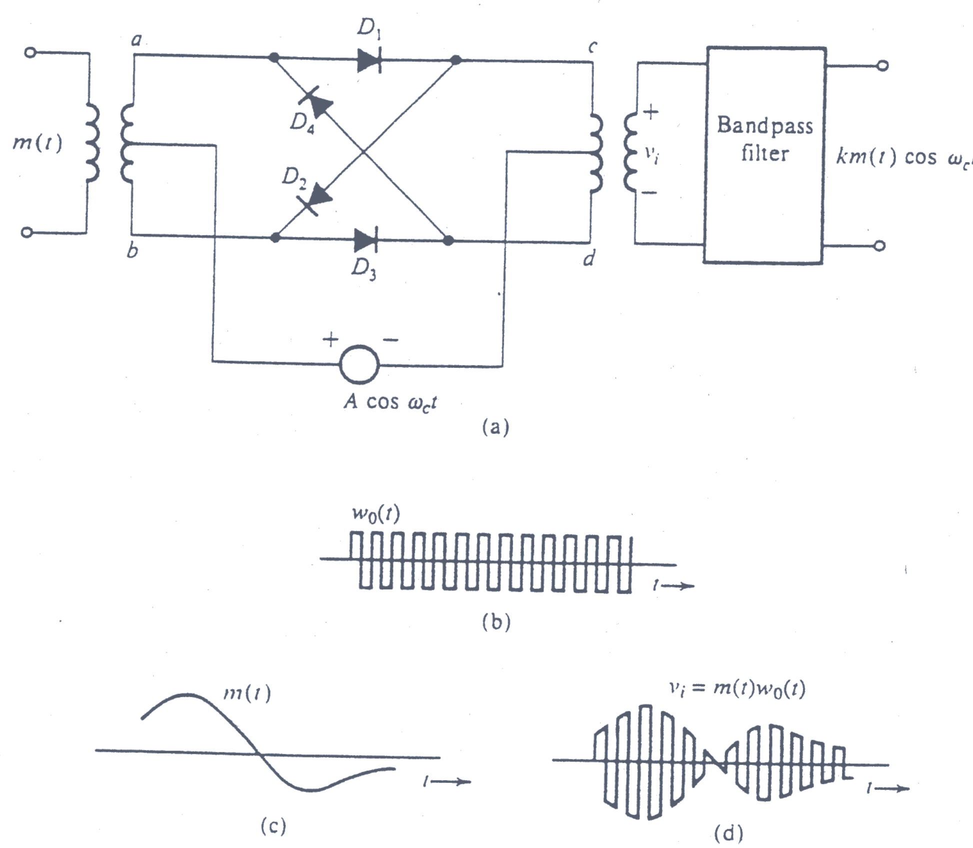


Figure 4.6 Ring modulator.

and $v_i(t) = m(t)\, w_o(t) = \frac{4}{\pi}\left[m(t)\cos\omega_c t - \frac{m(t)}{3}\cos 3\omega_c t + \frac{m(t)}{5}\cos 5\omega_c t - \dots \right]$

therefore, when this waveform is passed through a bandpass filter tuned to ω_c, the filter output will be the desired signal $\frac{4}{\pi} m(t)\cos\omega_c t$.

In this circuit there are 2 inputs: $m(t)$ and $\cos\omega_c t$. The input to the final bandpass filter does not contain either of these inputs. Consequently, this circuit is an example of a double balanced modulator.

In conclusion: The only difference between the modulator and the demodulator is the output filter. In the modulator a bandpass filter, tuned to ω_c, is used. However, in the demodulator a lowpass filter is used. The receiver must generate a carrier in phase and frequency synchronism with the incoming carrier. These demodulators are called synchronous or coherent (also homodyne) demodulators.

EXAMPLE 4.2 **Frequency Mixer or Converter**

We shall analyze a frequency mixer, or frequency converter, used to change the carrier frequency of a modulated signal $m(t)\cos\omega_c t$ from ω_c to some other frequency ω_I.

This can be done by multiplying $m(t)\cos\omega_c t$ by $2\cos\omega_{mix}t$, where $\omega_{mix} = \omega_c + \omega_I$ or $\omega_c - \omega_I$, and then bandpass-filtering the product, as shown in Fig. 4.7a.

The product $x(t)$ is

$$\begin{aligned} x(t) &= 2m(t)\cos\omega_c t\cos\omega_{mix}t \\ &= m(t)[\cos(\omega_c - \omega_{mix})t + \cos(\omega_c + \omega_{mix})t] \end{aligned}$$

If we select $\omega_{mix} = \omega_c - \omega_I$,

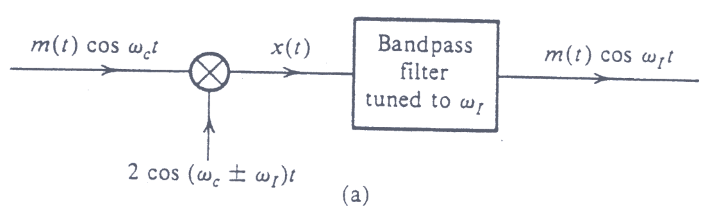
$$x(t) = m(t)[\cos\omega_I t + \cos(2\omega_c - \omega_I)t]$$

If we select $\omega_{mix} = \omega_c + \omega_I$,

$$x(t) = m(t)[\cos\omega_I t + \cos(2\omega_c + \omega_I)t]$$

In either case, a bandpass filter at the output, tuned to ω_I, will pass the term $m(t)\cos\omega_I t$ and suppress the other term, yielding the output $m(t)\cos\omega_I t$.* Thus, the carrier frequency has been translated to ω_I from ω_c.

The operation of frequency mixing, or frequency conversion (also known as heterodyning), is identical to the operation of modulation with a modulating carrier frequency (the mixer oscillator frequency ω_{mix}) that differs from the incoming carrier frequency by ω_I. Any one of the modulators discussed earlier can be used for frequency mixing. When we select the local carrier frequency $\omega_{mix} = \omega_c + \omega_I$, the operation is called **up-conversion**, and when we select $\omega_{mix} = \omega_c - \omega_I$, the operation is **down-conversion**.



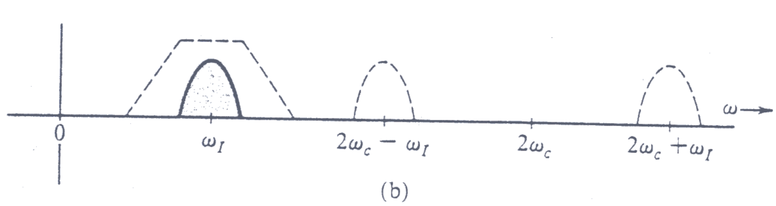


Figure 4.7 Frequency mixer or converter.

3- Amplitude modulation (AM):

$$\varphi_{AM}(t) = \underbrace{A \cos\omega_c t}_{\text{carrier}} + \underbrace{m(t) \cos\omega_c t}_{\text{Sidebands}}$$

$$= [A + m(t)] \cos\omega_c t \qquad , \text{A carrier Amplitude}$$

If $\quad m(t) \Leftrightarrow M(\omega)$

Then, $\varphi_{AM}(t) \Leftrightarrow \frac{1}{2}[M(\omega-\omega_c)+M(\omega+\omega_c)] + \pi A[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]$

* Two cases for $[A+m(t)]$:
  - A is large, $A+m(t) \geq 0$ (nonnegative) $\Rightarrow$ Detect $m(t)$ by detecting envelope.
  - A is not large enough, $A+m(t) \ngeq 0$ (negative) $\Rightarrow$ Impossible to detect $m(t)$

$\Rightarrow$ The condition for envelope detection of an AM signal is

$$A + m(t) \geq 0 \quad \forall t.$$

Let's assume $m(t)$ does take on negative values over some range of t. And let m_p be the peak amplitude (+ or −) of $m(t)$, that is $\quad m(t) \geq -m_p$.

Hence $\quad A + m(t) \geq 0 \quad \forall t$ becomes $A - m_p \geq 0$

$\Rightarrow \quad A \geq m_p$ which represents the minimum carrier amplitude required for the viability of envelope detection.

Define the modulation index $\mu = \frac{m_p}{A}$, it follows that the required condition for the viability of demodulation of AM by an envelope detector is: $\quad 0 \leq \mu \leq 1$.

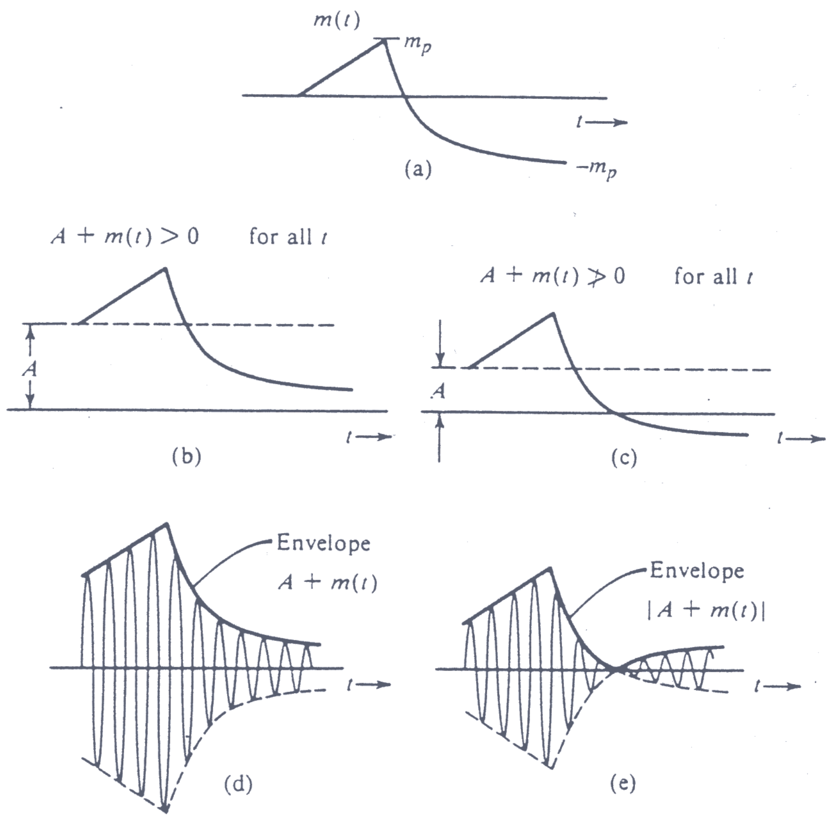


Figure 4.8 AM signal and its envelope.

* Sideband and carrier power :

$$\varphi_{AM}(t) = \underbrace{A\cos\omega_c t}_{\text{carrier}} + \underbrace{m(t)\cos\omega_c t}_{\text{sidebands}}$$

The carrier power : $P_c = E[A^2\cos^2\omega_c t]$
$= \frac{A^2}{2}$

The sideband power : $P_s = E[m^2(t)\cos^2\omega_c t]$
$= \frac{1}{2}\overline{m^2(t)}$

The power efficiency is : $\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_s+P_c} = \frac{\overline{m^2(t)}}{A^2+\overline{m^2(t)}}$

For the special case of tone modulation:

$$m(t) = \mu A\cos\omega_m t \Rightarrow \overline{m^2(t)} = \frac{(\mu A)^2}{2}$$

Hence $\eta = \frac{\mu^2}{\mu^2+2}$, $0 \le \mu \le 1$

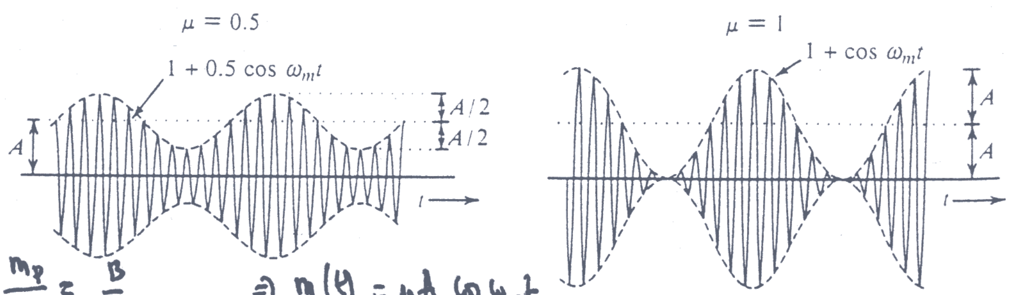
$$\eta_{max} = 33\% \text{ for } \mu = 1.$$

Thus, for tone modulation, under best conditions ($\mu=1$), only one-third of the transmitted power is used for carrying message. For practical signals, the efficiency is even worse – on the order of 25% or lower – compared to that of the DSB-SC case. The best condition implies $\mu=1$. Smaller values of μ degrade efficiency further. For this reason volume compression and peak limiting are commonly used in AM to ensure that full modulation ($\mu=1$) is maintained most of the time.

* Generation of AM signals :

AM signals can be generated by an DSB-SC modulators if

EXAMPLE 4.4 Sketch $\varphi_{AM}(t)$ for modulation indices of $\mu = 0.5$ and $\mu = 1$, when $m(t) = B \cos \omega_m t$. This case is referred to as **tone modulation** because the modulating signal is a pure sinusoid (or tone).



$\mu = \frac{m_p}{A} = \frac{B}{A}$ (a) $\Rightarrow m(t) = \mu A \cos \omega_m t$

(b) $\varphi_{AM}(t) = [A + m(t)] \cos \omega_c t = A[1 + \mu \cos \omega_m t] \cos \omega_c t$

Figure 4.9 Tone-modulated AM. (a) $\mu = 0.5$. (b) $\mu = 1$.

* In case the negative and the positive peak amplitudes are not identical, m_p in condition (4.9b) is the absolute negative peak amplitude.

$\mu = 1$. Smaller values of μ degrade efficiency further. For this reason volume compression and peak limiting are commonly used in AM to ensure that full modulation ($\mu = 1$) is maintained most of the time.

EXAMPLE 4.5 Determine η and the percentage of the total power carried by the sidebands of the AM wave for tone modulation when **(a)** $\mu = 0.5$ and **(b)** $\mu = 0.3$.

For $\mu = 0.5$,

$$\eta = \frac{\mu^2}{2 + \mu^2} 100\% = \frac{(0.5)^2}{2 + (0.5)^2} 100\% = 11.11\%$$

Hence, only about 11% of the total power is in the sidebands. For $\mu = 0.3$,

$$\eta = \frac{(0.3)^2}{2 + (0.3)^2} 100\% = 4.3\%$$

Hence, only 4.3% of the total power is the useful power (power in sidebands).

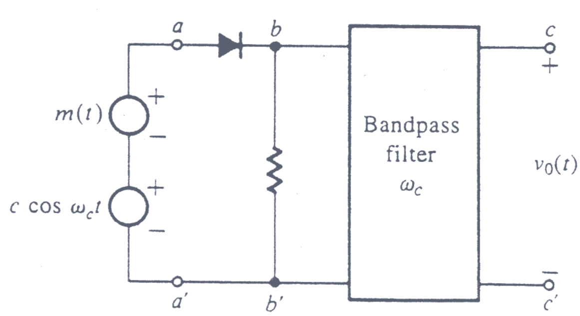


Figure 4.10 AM generator.

the modulating signal is $A + m(t)$ instead of just $m(t)$. But because there is no need to suppress the carrier in the output, the modulating circuits do not have to be balanced. This results in considerably simpler modulators for AM.

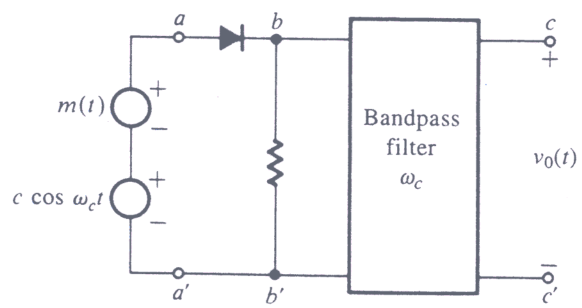


Figure 4.10 AM generator.

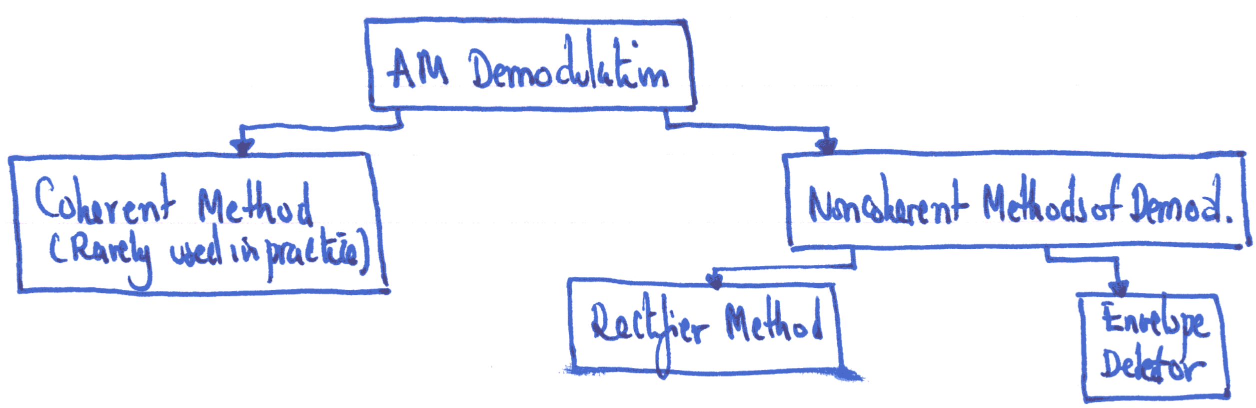
The input $= c\cos\omega_c t + m(t)$, with $c \gg m(t)$ so that the switching action of the diode is controlled by $c\cos\omega_c t$.

$$\Rightarrow \quad v_{bb'}(t) = \left[c\cos\omega_c t + m(t)\right] w(t)$$

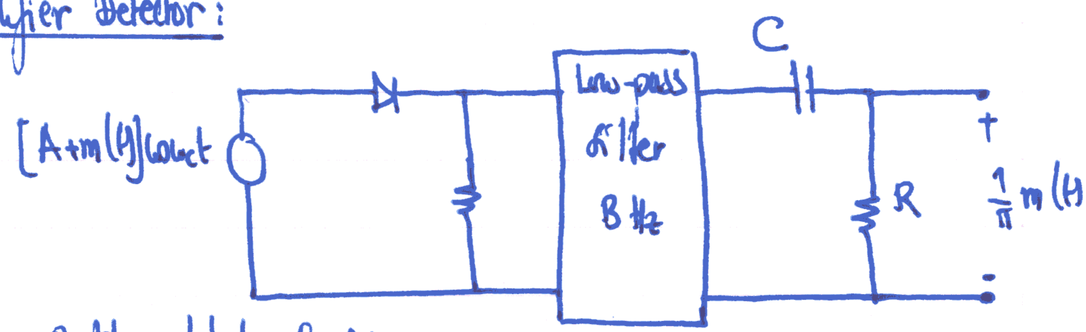
$$= \left[c\cos\omega_c t + m(t)\right]\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \dots\right)\right]$$

$$= \underbrace{\frac{c}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t}_{\text{AM signal}} + \underbrace{\text{other terms}}_{\text{suppressed by bandpass filter}}$$

* <u>Demodulation of AM signals:</u>



* **Rectifier Detector:**



Rectifier detector for AM.

The rectifier output is:

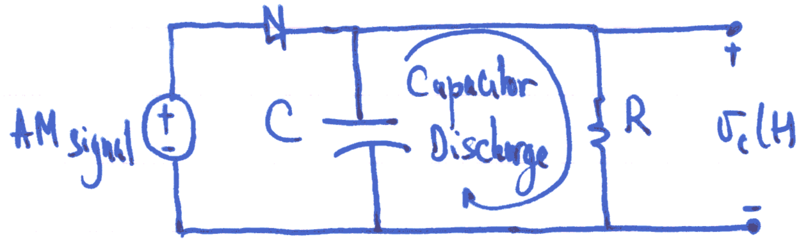
$$v_R(t) = \{[A+m(t)]\cos\omega_c t\}\, W(t)$$

$$= [A+m(t)]\cos\omega_c t\left[\frac{1}{2}+\frac{2}{\pi}\left(\cos\omega_c t-\frac{1}{3}\cos 3\omega_c t+\frac{1}{5}\cos 5\omega_c t - \dots\right)\right]$$

$$= \frac{1}{\pi}[A+m(t)] + \text{other terms of higher frequencies.}$$

The output of the low-pass is $\left[\frac{A}{\pi}+\frac{m(t)}{\pi}\right]$ where the DC part $\left(\frac{A}{\pi}\right)$ is blocked by the capacitor and the desired output $\frac{m(t)}{\pi}$.

* **Envelope detector:** In an envelope detector, the output of the detector follows the envelope of the modulated signal.



Envelope detector for AM.

Here, RC should be in the following range:

$$RC \gg \frac{1}{\omega_c}$$

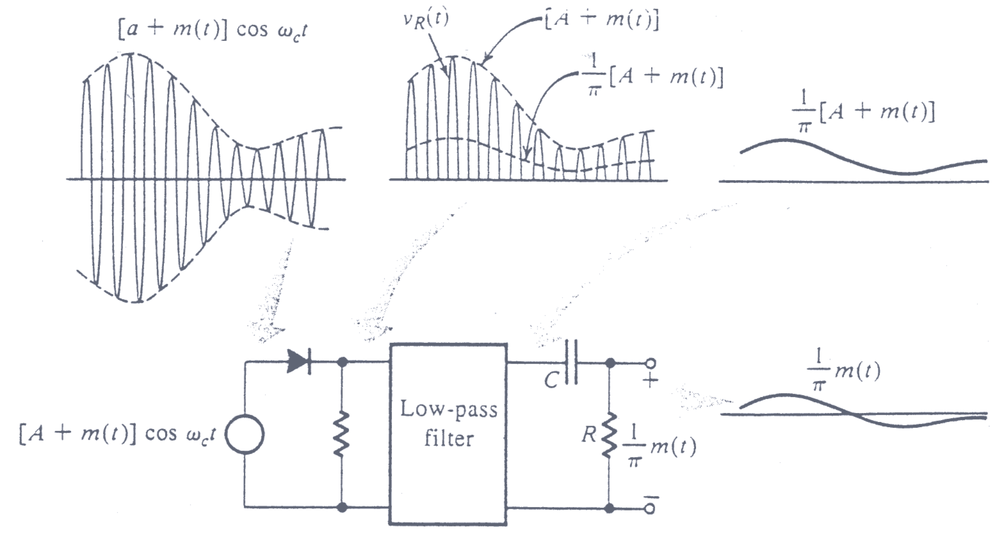


Figure 4.11 Rectifier detector for AM.

* By AM, we mean the case $\mu \leq 1$.

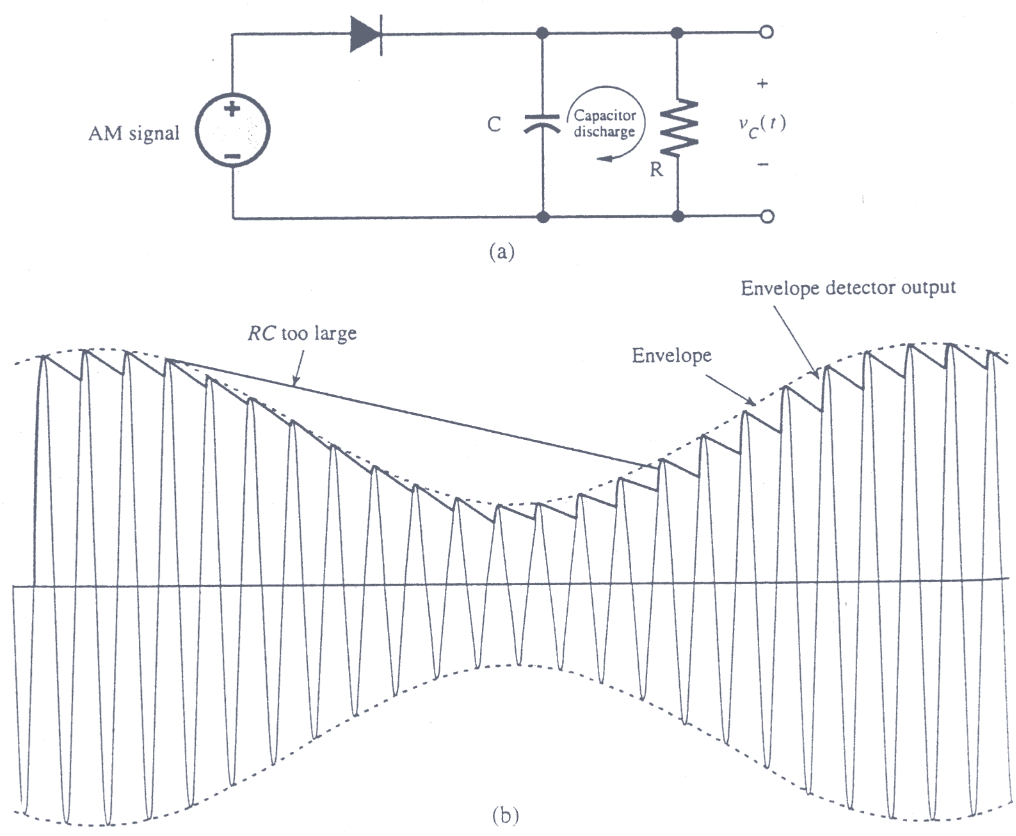


Figure 4.12 Envelope detector for AM.

$$RC \ll \frac{1}{2\pi B}$$, B is the highest frequency in $m(t)$.

$$\Rightarrow \quad \frac{1}{\omega_c} \ll RC \ll \frac{1}{2\pi B}$$

$v_c(t) = \underset{\downarrow}{A} + m(t)$ with a ripple of frequency of ω_c.

Blocked out by a capacitor or a simple RC High-pass filter. The ripple may be reduced further by another (Low-pass) filter.

EXAMPLE 4.6 For tone modulation (Example 4.4), determine the upper limit of RC to ensure that the capacitor voltage follows the envelope.

Figure 4.13 shows the envelope and the voltage across the capacitor. The capacitor discharges from the peak value E starting at some arbitrary instant $t = 0$. The voltage v_C across the capacitor is given by

$$v_C = Ee^{-t/RC}$$

Because the time constant is much larger than the interval between the two successive cycles of the carrier ($RC \gg 1/\omega_c$), the capacitor voltage v_C discharges exponentially for a short time compared to its time constant. Hence, the exponential can be approximated by a straight line obtained from the first two terms in Taylor's series for $Ee^{-t/RC}$,

$$v_C \simeq E\left(1 - \frac{t}{RC}\right)$$

The slope of the discharge is $-E/RC$. In order for the capacitor to follow the envelope $E(t)$, the magnitude of the slope of the RC discharge must be greater than the magnitude of the slope of the envelope $E(t)$. Hence,

$$\left|\frac{dv_C}{dt}\right| = \frac{E}{RC} \geq \left|\frac{dE}{dt}\right| \tag{4.12}$$

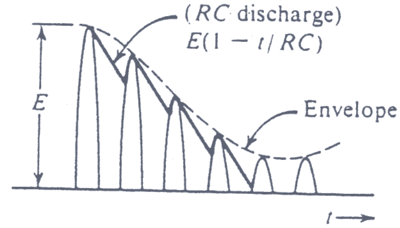


Figure 4.13 Capacitor discharge in an envelope detector.

But the envelope $E(t)$ of a tone-modulated carrier is [Eq. (4.11)]

$$E(t) = A[1 + \mu \cos \omega_m t]$$

$$\frac{dE}{dt} = -\mu A \omega_m \sin \omega_m t$$

Hence, Eq. (4.12) becomes

$$\frac{A(1 + \mu \cos \omega_m t)}{RC} \geq \mu A \omega_m \sin \omega_m t \qquad \text{for all } t$$

or

$$RC \leq \frac{1 + \mu \cos \omega_m t}{\mu \omega_m \sin \omega_m t} \qquad \text{for all } t$$

The worst possible case occurs when the right-hand side is the minimum. This is found (as usual, by taking the derivative and setting it to zero) to be when $\cos \omega_m t = -\mu$. For this case, the right-hand side is $\sqrt{(1-\mu^2)}/\mu\omega_m$. Hence,

$$RC \leq \frac{1}{\omega_m}\left(\frac{\sqrt{1-\mu^2}}{\mu}\right)$$

4- Quadrature amplitude Modulation (QAM):

The DSB signals occupy twice the bandwidth required for the baseband.

Solution: Transmit 2 DSB signals using carriers of the same frequency but in phase quadrature. If the two baseband signals to be transmitted are $m_1(t)$ and $m_2(t)$, the corresponding QAM signal is:

$$\varphi_{QAM}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t$$

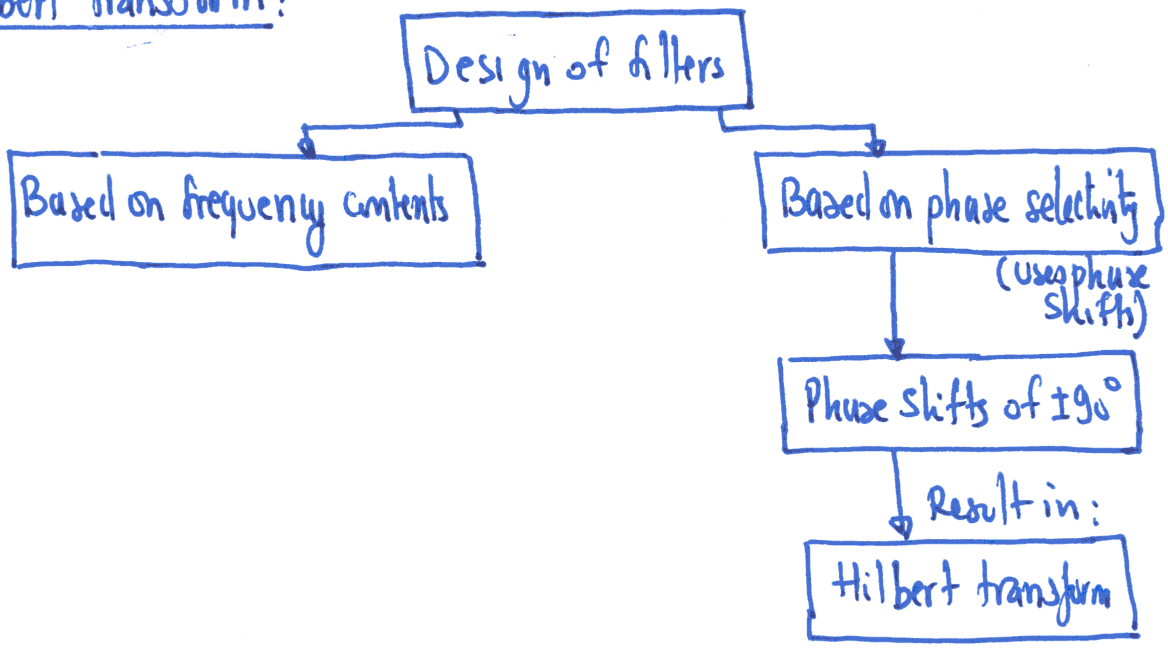
Both modulated signals occupy the same band. Yet two baseband signals can be seperated at the receiver by synchronous detection using two local carriers in phase quadrature, as shown in Figure 4.14.

Thus, two baseband signals, each of bandwidth B Hz, can be transmitted simultaneously over a bandwidth 2B using DSB transmission and quadrature multiplexing.

Quadrature multiplexing is used color television to multiplex the so-called chrominance signals.

5- Amplitude Modulation: Single sideband (SSB)

* Hilbert transform:



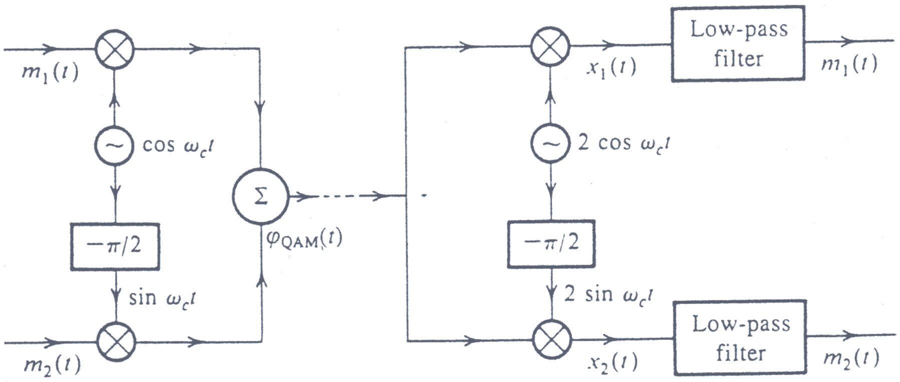


Figure 4.14 Quadrature amplitude multiplexing.

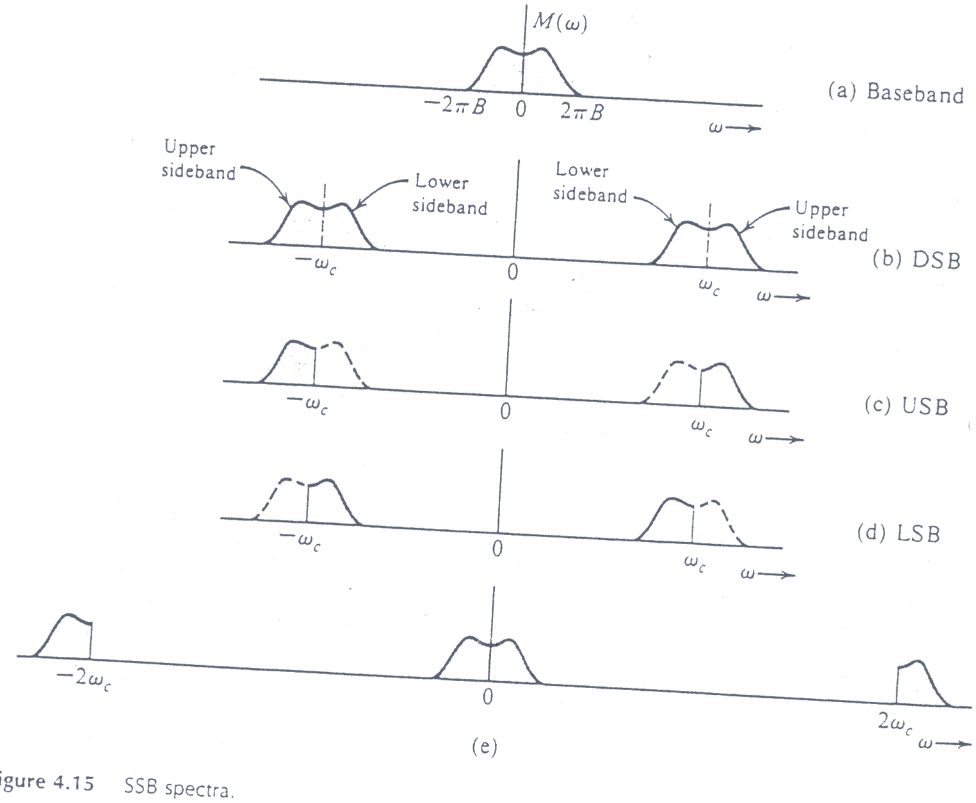


Figure 4.15 SSB spectra.

Let $g(t)$ with FT $G(\omega)$, then the Hilbert transform of $g(t)$, denoted as $\hat{g}(t)$ is defined as:

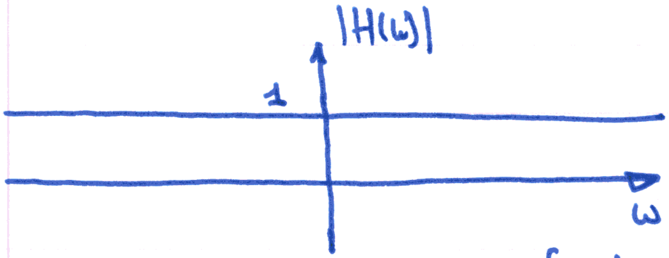
$$\hat{g}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\, d\tau$$

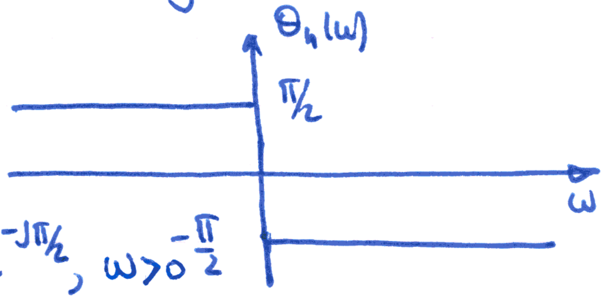
which is in fact:

$$g(t) \longrightarrow \boxed{\text{Hilbert transform filter}} \longrightarrow \hat{g}(t) = g(t) * h(t)$$

where $h(t) = \frac{1}{\pi t}$; then $H(\omega) = -j\, \mathrm{sgn}(\omega)$

$\therefore\ \hat{g}(t) \Leftrightarrow -j\, G(\omega)\, \mathrm{sgn}(\omega)$





Since $H(\omega) = \begin{cases} -j = 1 e^{-j\pi/2}, & \omega > 0 \\ j = 1 e^{j\pi/2}, & \omega < 0 \end{cases}$

The DSB spectrum has 2 sidebands: USB and LSB, both containing the complete information of the baseband. A scheme in which only one sideband is transmitted is known as single-sideband (SSB) transmission, which requires only $\frac{1}{2}$ the bandwidth of the DSB signal.

* <u>Time-Domain Representation of SSB Signals:</u>

From Figure 4.16, we have:

the USB $M_+(\omega) = M(\omega)\, u(\omega)$ and the LSB $M_-(\omega) = M(\omega)\, u(-\omega)$

let $m_+(t) \Leftrightarrow M_+(\omega)$ and $m_-(t) \Leftrightarrow M_-(\omega)$

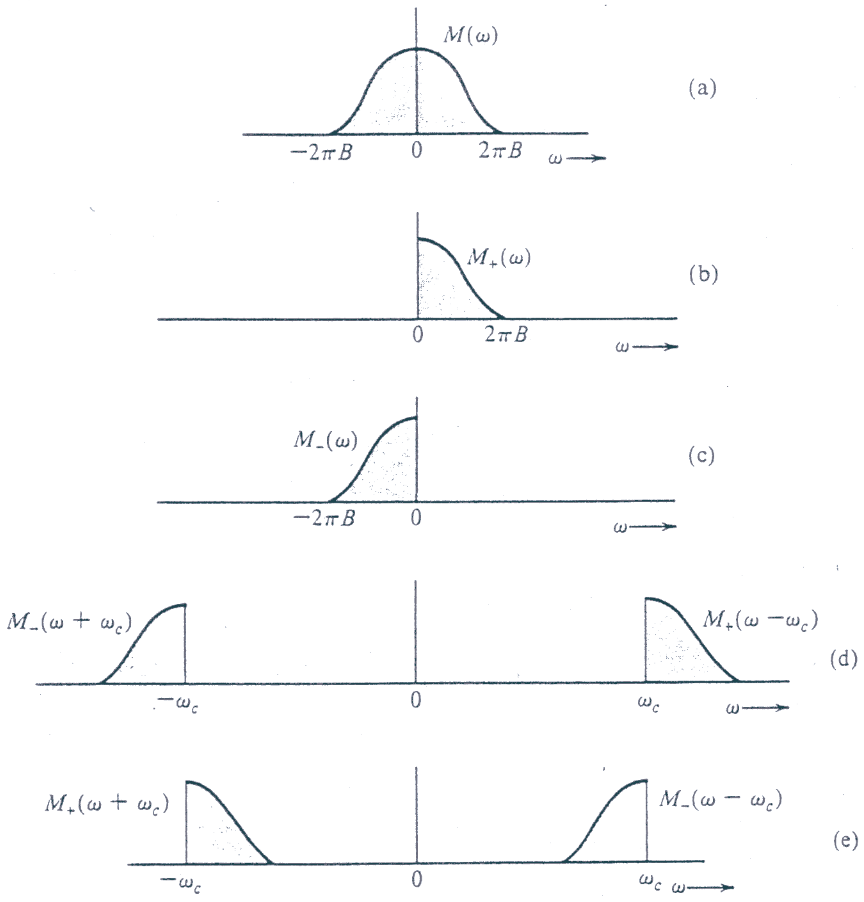


Figure 4.16 Expressing SSB spectra in terms of $M_+(\omega)$ and $M_-(\omega)$.

Since $|M_+(\omega)|$ and $|M_-(\omega)|$ are not even functions of ω, the signals $m_+(t)$ and $m_-(t)$ are complex. Moreover, $M_+(\omega)$ and $M_-(\omega)$ are two halves of $M(\omega)$. Hence, from Eqs (3.16), it follows that $M_+(-\omega)$ and $M_-(\omega)$ are conjugates. Consequently, $m_+(t)$ and $m_-(t)$ are conjugates. Because $m(t) = m_+(t) + m_-(t)$, we can express:

$$m_+(t) = \frac{1}{2}[m(t) + j\, m_h(t)]$$

and

$$m_-(t) = \frac{1}{2}[m(t) - j\, m_h(t)]$$

where $m_h(t)$ is unknown. Let's determine $m_h(t)$, we note that:

$$M_+(\omega) = M(\omega)\, u(\omega)$$
$$= \frac{1}{2} M(\omega)[1 + \operatorname{sgn}(\omega)]$$
$$= \frac{1}{2} M(\omega) + \frac{1}{2} M(\omega) \operatorname{sgn}(\omega)$$

Therefore, from $m_+(t) = \frac{1}{2} m(t) + \frac{1}{2} j m_h(t)$
we can express $j m_h(t) \Longleftrightarrow M(\omega)\,\mathrm{sgn}(\omega)$

Hence $m_h(t) \Longleftrightarrow M_h(\omega) = -j M(\omega)\,\mathrm{sgn}(\omega)$

$\therefore$ $m_h(t)$ and $m(t)$ are Hilber transform pairs.
Thus, if delay the phase of every component of $m(t)$ by $\frac{\pi}{2}$ (without changing its amplitude), the resulting signal is $m_h(t)$. Therefore, a Hilbert transformer is an ideal filter (phase shifter) that shifts the phase of every spectral component by $-\frac{\pi}{2}$.

We can now express the SSB signal in terms of $m(t)$ and $m_h(t)$.

$$\Phi_{USB}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c)$$

The inverse transform of this equation yields

$$\varphi_{USB}(t) = m_+(t)\, e^{j\omega_c t} + m_-(t)\, e^{-j\omega_c t}$$

which yields:

$$\varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

Using a similar argument, we can show that

$$\varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

Hence, a general SSB signal $\varphi_{SSB}(t)$ can be expressed as:

$$\varphi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t$$ $m_h(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{m(\alpha)}{t-\alpha}$

where the $-$ sign applies to USB and the $+$ sign applies to LSB.

Example 4.7.

* <u>Generation of SSB signals:</u>

Two methods are commonly used to generate SSB signals. The first method uses sharp cutoff filters to eliminate the undesired

Problem # 4.5.3

$$m(t) = B\,\text{sinc}(2\pi Bt)$$

let's find the Hilbert transform of $m(t)$. To do this, we have to resort of the Fourier transform of $m(t)$. That is:

$$M(\omega) = \frac{1}{2}\,\text{rect}\left[\frac{\omega}{4\pi B}\right]$$

Therefore,

$$M_h(\omega) = -j\, M(\omega)\,\text{sgn}\,\omega$$

$$= -j\,\frac{1}{2}\,\text{rect}\left[\frac{\omega}{4\pi B}\right]\text{sgn}\;\omega$$

$$= \begin{cases} -j\,\frac{1}{2}\,\text{rect}\left[\frac{\omega}{4\pi B}\right] & , \omega > 0 \\ j\,\frac{1}{2}\,\text{rect}\left[\frac{\omega}{4\pi B}\right] & , \omega < 0 \end{cases}$$

$$\therefore \quad m_h(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} M_h(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\times\frac{1}{2}\left[\int_{-2\pi B}^{0} j\, e^{j\omega t}\, d\omega + \int_{0}^{2\pi B} -j\, e^{j\omega t}\, d\omega\right]$$

$$= \frac{2}{2\pi t}\sin^2(\pi Bt)$$

$$= \frac{1}{\pi t}\sin^2(\pi Bt).$$

$$\Rightarrow \varphi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$$

$$= B\,\frac{\sin(2\pi Bt)}{(2\pi Bt)}\cos\omega_c t - \frac{\sin^2(\pi Bt)}{\pi t}\sin\omega_c t$$

$$= \frac{2\sin(\pi Bt)\cos(\pi Bt)}{2\pi t}\cos\omega_c t \quad \frac{\sin^2(\pi Bt)}{\pi t}\sin\omega_c t$$

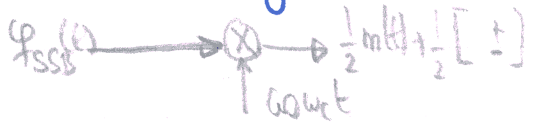
$$= B\times\frac{\sin(\pi Bt)}{\pi Bt}\left[\cos(\pi Bt)\cos\omega_c t - \sin(\pi Bt)\sin\omega_c t\right]$$

$$= B\,\text{sinc}(\pi Bt)\cos(\omega_c t + \pi Bt).$$

sideband, and the second method uses phase-shifting networks to achieve the same goal.

- Selective-filtering Method: this is the most commonly used method of generating SSB signals. In this method, a DSB-SC signal is passed through a sharp cutoff filter to eliminate the undesired sideband.

- Phase-shift Method: Figure 4.20 details the generation of SSB by this method.



- Demodulation of SSB-SC Signals:

6- Amplitude modulation: Vestigial Sideband (VSB)

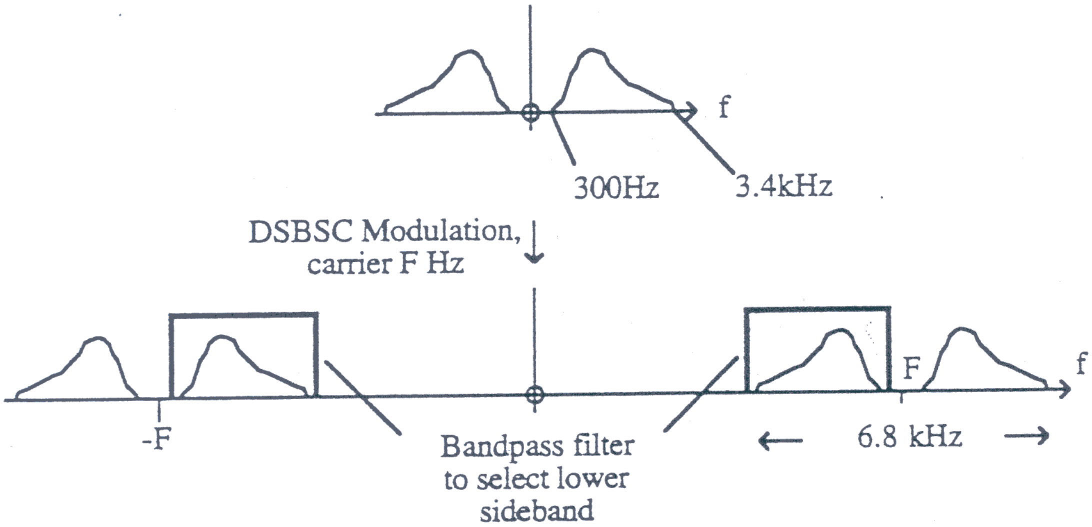
As seen earlier, the generation of SSB signals is rather difficult. The selective-filtering method requires DC null in the modulation signal spectrum. A phase shifter required in the phase-shift method is unrealizable, or realizable only approximately. The generation of DSB signals is much simpler, but requires twice the signal bandwidth. A vestigial-sideband (VSB), also called asymetric sideband system is a compromise between DSB and SSB. VSB signals are relatively easy to generate, and, at the same time, their bandwidth is only (typically 25%) greater than that of SSB signals. See Figure 4.21.

If the vestigial shaping filter that produces VSB from DSB is $H_i(\omega)$, then:

$$\Phi_{VSB}(\omega) = \left[M(\omega+\omega_c) + M(\omega-\omega_c) \right] H_i(\omega)$$

Single Sideband (SSB)

Both DSBSC and conventional AM are inefficient in their use of bandwidth. In a situation where it is necessary to carry as many communication channels as possible in a limited spectral space, this inefficiency cannot be tolerated. For intellegible speech over telephone circuits a bandwidth of 300 to 3.4kHz is adequate.



SSB for telephone-quality speech

The bandwidth occupied by the DSBSC modulated signal is twice that at baseband, or 6.8kHz. This is an obvious waste, as the SAME information on which frequency components are present is available in each sideband: removing one or other of them would halve the frequency space required. The sideband selected can be isolated by a sharp-cut bandpass filter.

In practice, the design of the isolating filter is not straightforward, because of the need for a very narrow transition region from stopband to passband. This is assisted by the 600Hz gap between the sidebands, and means that SSB is only really suitable for signals without significant low-frequency energy.

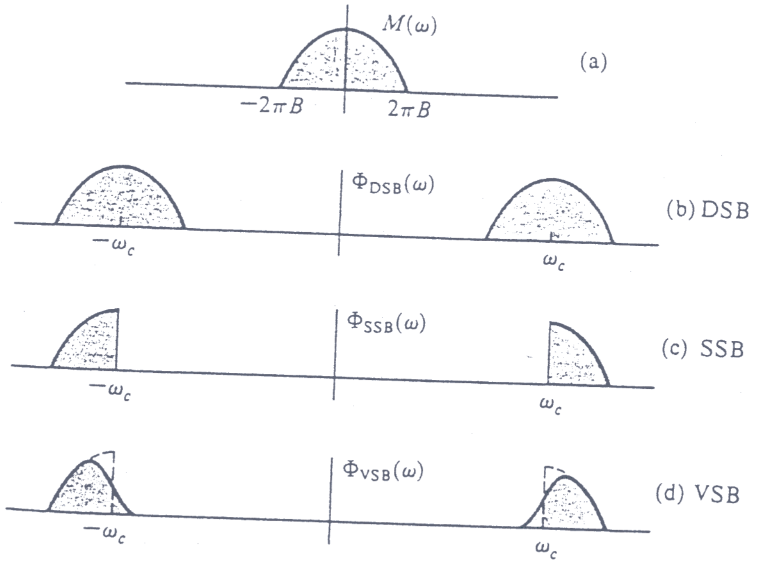


Figure 4.21 Spectra of the modulating signal and corresponding DSB, SSB, and VSB signals.

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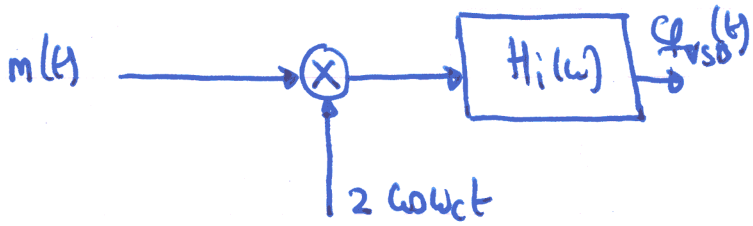
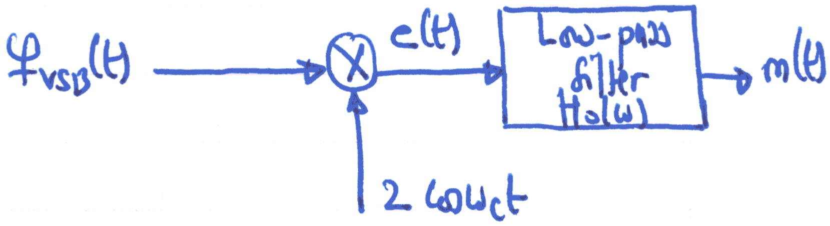
* This is true for the North American hierarchy. In the CCITT hierarchy, a basic mastergroup is formed by multiplexing five supergroups (300 voice channels).



Transmitter.

This VSB shaping filter $H_i(\omega)$ allows the transmission of one sideband, but suppresses the other sideband, not completely, but gradually. This makes it easy to realize such a filter, but the transmission bandwidth is now somewhat higher than that of the SSB. The bandwidth of the VSB signal is typically 25 to 33% higher than that of the SSB signals.

We require that $m(t)$ be recoverable from $\varphi_{VSB}(t)$ using synchronous demodulation at the receiver.



Reciver.

$$e(t) = 2\varphi_{VSB}(t)\cos\omega_c t \Leftrightarrow E(\omega) = \left[\Phi_{VSB}(\omega-\omega_c) + \Phi_{VSB}(\omega+\omega_c)\right]$$

The output of the equalizer filter (low-pass filter) is required to be $m(t)$. Hence, the output signal spectrum is:

$$m(t) = e(t) * h_o(t) \Leftrightarrow M(\omega) = E(\omega)H_o(\omega)$$

$$M(\omega) = M(\omega)\left[H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\right]H_o(\omega) +$$
$$\left[M(\omega+2\omega_c)H_i(\omega+\omega_c) + M(\omega-2\omega_c)H_i(\omega-\omega_c)\right]H_o(\omega)$$

After the action of the low-pass filter:

$$M(\omega) = M(\omega)\left[H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\right] H_o(\omega)$$

For the output to be $m(t)$ then:

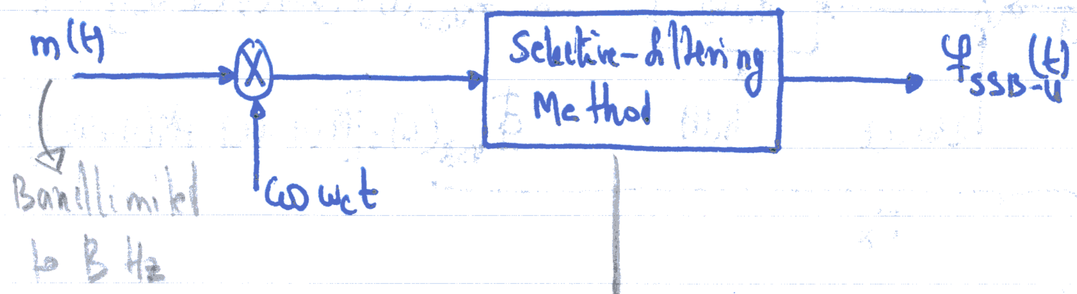
$$\left[H_i(\omega+\omega_c) + H_i(\omega-\omega_c)\right] H_o(\omega) = 1$$

$$\therefore \quad H_o(\omega) = \frac{1}{H_i(\omega+\omega_c) + H_i(\omega-\omega_c)}, \quad -2\pi B \leq \omega \leq 2\pi B$$

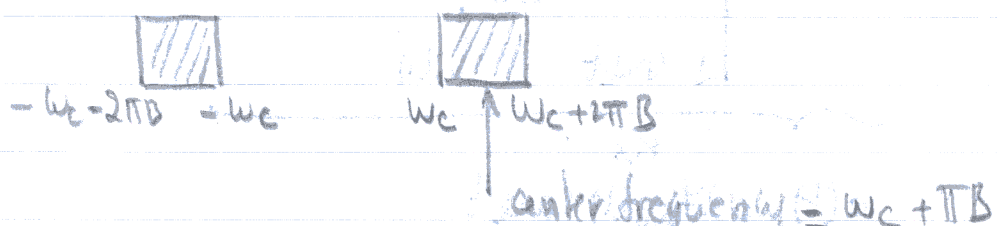
7. Carrier Acquisition

In the suppressed-carrier amplitude-modulated system (DSB-SC, SSB-SC, and VSB-SC), one must generate a local carrier at the receiver for the purpose of synchronous demodulation. Ideally, the local carrier must be in frequency and phase synchronism with incoming carrier. Any discrepancy in the frequency or phase of the local carrier gives rise to distortion in the detector output. To ensure identical carrier frequencies at the transmitter and the receiver, we can use crystal oscillators, which generally are very stable. At very high carrier frequencies, where the crystal dimensions become too small to match exactly, quartz-crystal performance may not be adequate. In such a case, a carrier, or pilot, is transmitted at a reduced level (usually about -20dB) along with the sidebands. The pilot is seperated at the receiver by a very narrow-band filter tuned to the pilot frequency. It is amplified and used to

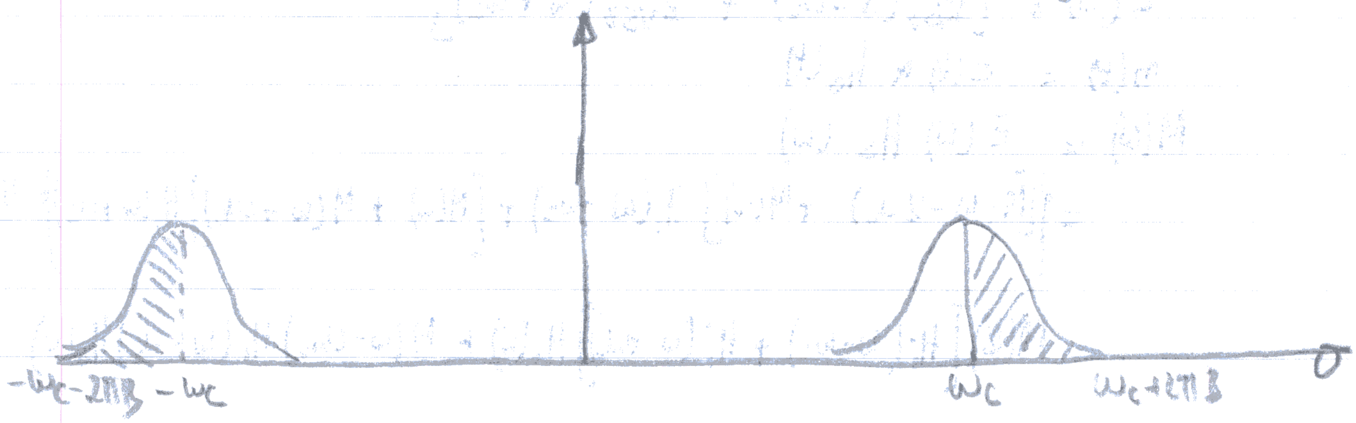
SSB :



m(t)
Bandlimited to B Hz
cos $\omega_c t$
Selective-filtering Method
$\varphi_{SSB-U}(t)$
BPF

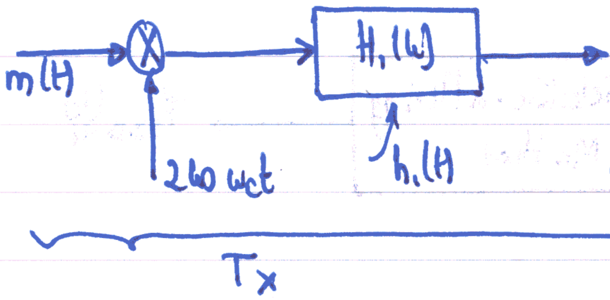


$-\omega_c - 2\pi B$
$-\omega_c$
ω_c
$\omega_c + 2\pi B$
center frequency = $\omega_c + \pi B$



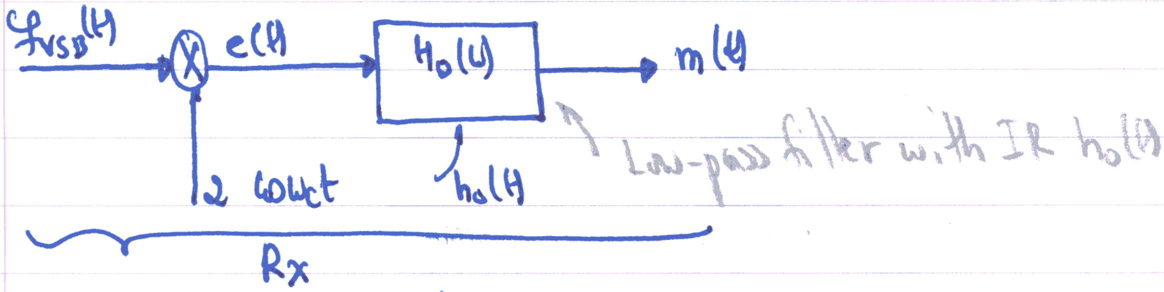
$-\omega_c - 2\pi B$
$-\omega_c$
ω_c
$\omega_c + 2\pi B$
ω

VSB:



$$\varphi_{VSB}(t) = [2m(t)\cos\omega_c t] * h_1(t)$$

$$\Phi_{VSB}(\omega) = [M(\omega-\omega_c) + M(\omega+\omega_c)]\, H_1(\omega)$$



$$e(t) = 2\varphi_{VSB}(t)\cos\omega_c t$$

$$E(\omega) = [\Phi_{VSB}(\omega-\omega_c) + \Phi_{VSB}(\omega+\omega_c)]$$

$$m(t) = e(t) * h_0(t)$$

$$M(\omega) = E(\omega)\, H_0(\omega)$$

$$= \left\{[M(\omega-2\omega_c) + M(\omega)]\, H_1(\omega-\omega_c) + [M(\omega) + M(\omega+2\omega_c)]\, H_1(\omega+\omega_c)\right\} H_0(\omega)$$

$$= M(\omega)\left[H_1(\omega-\omega_c) + H_1(\omega+\omega_c)\right] H_0(\omega) + \cancel{M(\omega-2\omega_c)}\, H_1(\omega-\omega_c)\, H_0(\omega)$$

$$+ \cancel{M(\omega+2\omega_c)}\, H_1(\omega+\omega_c)\, H_0(\omega)$$

$$\therefore\ H_0(\omega) = \frac{1}{H_1(\omega-\omega_c) + H_1(\omega+\omega_c)}, \qquad |\omega| \le 2\pi B$$

EXAMPLE 4.8 The carrier frequency of a certain VSB signal is $\omega_c = 20$ kHz, and the baseband signal bandwidth is 6 kHz. The VSB shaping filter $H_i(\omega)$ at the input, which cuts off the lower sideband gradually over 2 kHz, is shown in Fig. 4.23a. Find the output filter $H_o(\omega)$ required for distortionless reception.

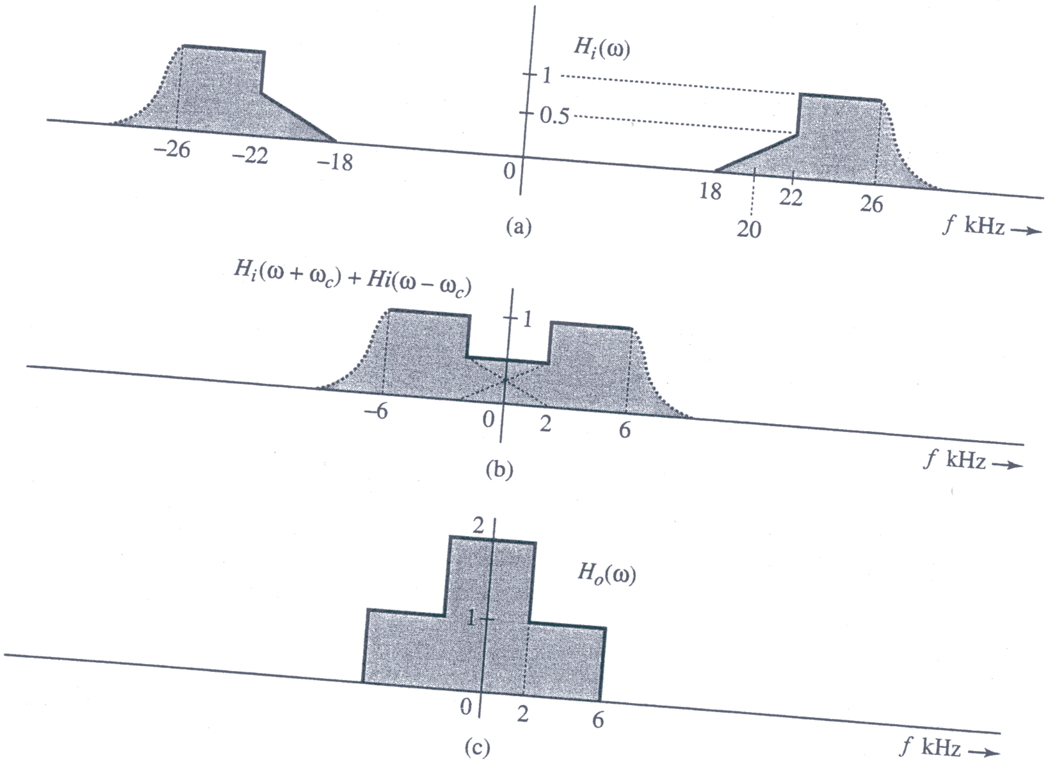


Figure 4.23 VSB out filter.

Figure 4.23b shows the low-pass segments of $H_i(\omega+\omega_c)+H_i(\omega-\omega_c)$. We are interested in this spectrum only over the baseband (the remaining undesired portion is suppressed by the output filter). This spectrum is 0.5 over the band of 0 to 2 kHz, and is 1 over 2 to 6 kHz, as shown in Fig. 4.23b. Figure 4.23c shows the desired output filter $H_o(\omega)$, which is the reciprocal of the spectrum in Fig. 4.23b [see Eq. (4.20)].

synchronize the local oscillator.

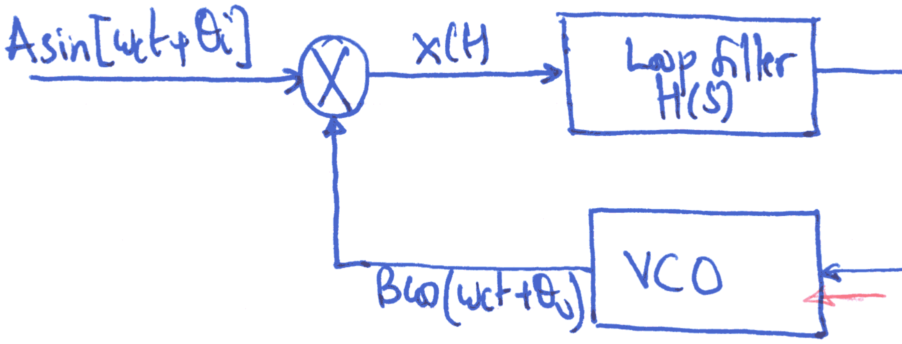
The phase-locked loop (PLL), which plays an important role in carrier acquisition, will now be discussed. It can be used to track the phase and the frequency of the carrier component of an incoming signal. It is, therefore, a useful device for synchronous demodulation of AM signals with suppressed carrier or with a little carrier (the pilot).

A PLL has 3 basic components:

1. A voltage-controlled oscillator (VCO). The VCO adjusts its own frequency until it is equal to that of the input sinusoid.

2. A multiplier, serving as a phase detector (PD) or phase comparator.

3. A loop filter $H(S)$.



The oscillation frequency varies according to: $\omega(t) = \omega_c + c\,e_0(t)$

Phase-locked loop configuration.

$$x(t) = \frac{AB}{2}\left[\sin(\theta_i - \theta_0) + \underbrace{\sin[2\omega_c t + \theta_i + \theta_0]}_{\text{suppressed by the loop filter.}}\right]$$

Carrier Acquisition:

For demodulation of DSBSC and SSB, it is essential to generate a local carrier at the receiver for the purpose of synchronous detection.

This local carrier must have the <u>same</u> frequency and phase as those of the incoming carrier.

<u>e.g.</u> DSBSC signal

$$e(t) = 2\, m(t) \cdot \cos(w_c t) \cdot \cos[(w_c + \Delta w)t + \phi]$$

$$= m(t) \cdot \cos(\Delta w t + \phi) + m(t) \cdot \cos[(2w_c + \Delta w)t + \phi]$$

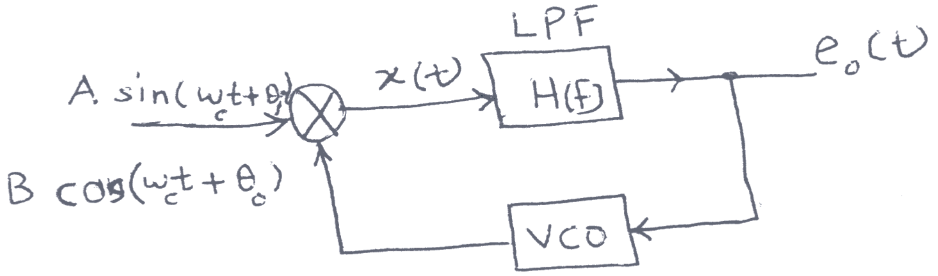
$$\therefore e_o(t) = m(t) \cdot \cos(\Delta w t + \phi)$$

Ideally, $\Delta w = 0$ and $\phi = 0$.

Phase Locked Loop (PLL):

This is composed of

1) voltage-controlled oscillator (VCO)
2) multiplier, serving as phase comparator
3) A loop filter



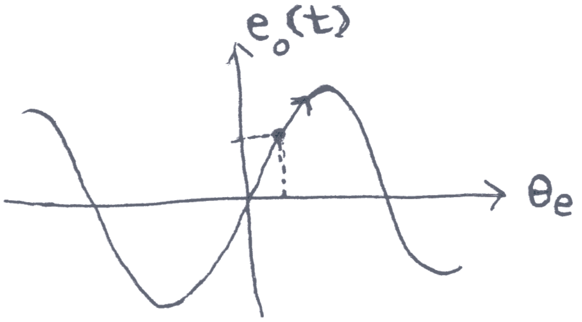
$$x(t) = AB \sin(\omega_c t + \theta_i)\cos(\omega_c t + \theta_o)$$

$$= \frac{AB}{2}\left[\sin(\theta_i - \theta_o) + \sin(2\omega_c t + \theta_i + \theta_o)\right]$$

The loop filter will remove the second term. Hence,

$$e_o(t) = \frac{AB}{2}\sin(\theta_i - \theta_o)$$

$$= \frac{AB}{2}\sin\theta_e, \quad \theta_e = \theta_i - \theta_o = \text{phase error}$$



Lock Range: It is a finite frequency range over which the PLL can track the incoming frequency.

Carrier Acquisition in DSBSC:

- Signal Squaring Method

$$m(t)\cdot\cos 2\pi f_c t \longrightarrow \boxed{(\ )^2} \xrightarrow{x(t)} \boxed{\text{BPF}}_{\substack{\text{at}\\ 2f_c}} \longrightarrow \boxed{\text{PLL}} \xrightarrow{C\cdot\cos 4\pi f_c t} \boxed{\text{2:1 frequency divider}} \longrightarrow k\cos 2\pi f_c t$$

The received DSBSC signal is $m(t)\cdot\cos 2\pi f_c t$.

$$x(t) = m^2(t)\cos^2 2\pi f t$$
$$= \frac{1}{2}m^2(t) + \frac{1}{2}m^2(t)\cos 4\pi f_c t$$

Note that $m^2(t)$ is a non-negative signal and hence has a nonzero average value.

$\Rightarrow \frac{1}{2}m^2(t) = k + \phi(t)$, where k is constant and $\phi(t)$ is a zero mean baseband signal.

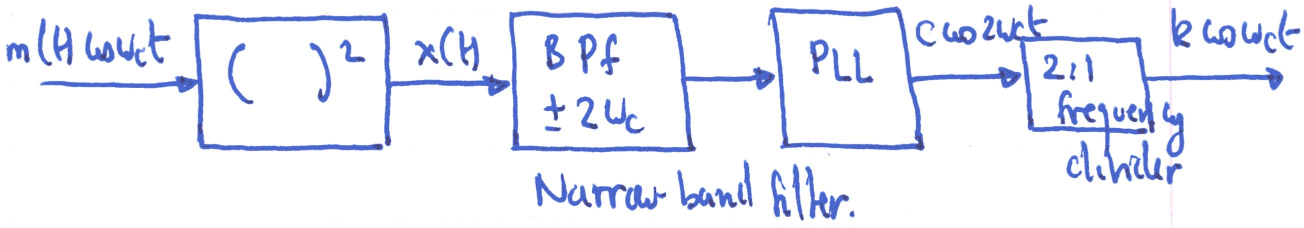
$$\therefore\ x(t) = \frac{1}{2}m^2(t) + k\cos 4\pi f_c t + \phi(t)\cos 4\pi f_c t$$

The BPF will pass $k\cos 4\pi f_c t$ whose spectrum is centered at $2f_c$.
However, the filter output will contain $k\cos 4\pi f_c t$, in addition to some undesired components from $\phi(t)\cos 4\pi f_c t$.
This undesired output can be minimised by using a PLL, which tracks $k\cos 4\pi f_c t$.

* Carrier Acquisition in DSB-SC:

We shall now discuss two methods of carrier regeneration at the receiver in DSB-SC: signal squaring and Costas loop:

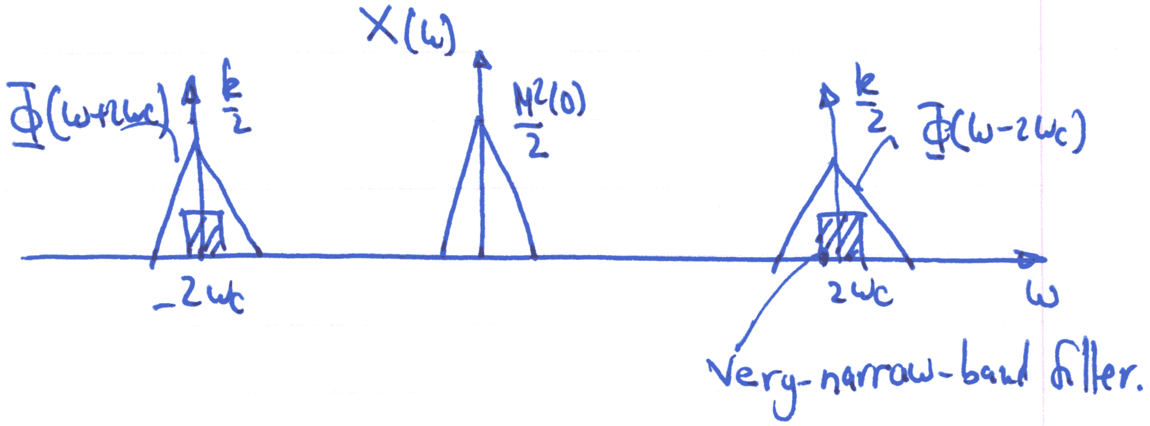
- Signal-Squaring Method:



$$x(t) = \frac{1}{2} m^2(t) + \frac{1}{2} m^2 \cos 2\omega_c t \quad , \quad \frac{m^2(t)}{2} = k + \phi(t)$$

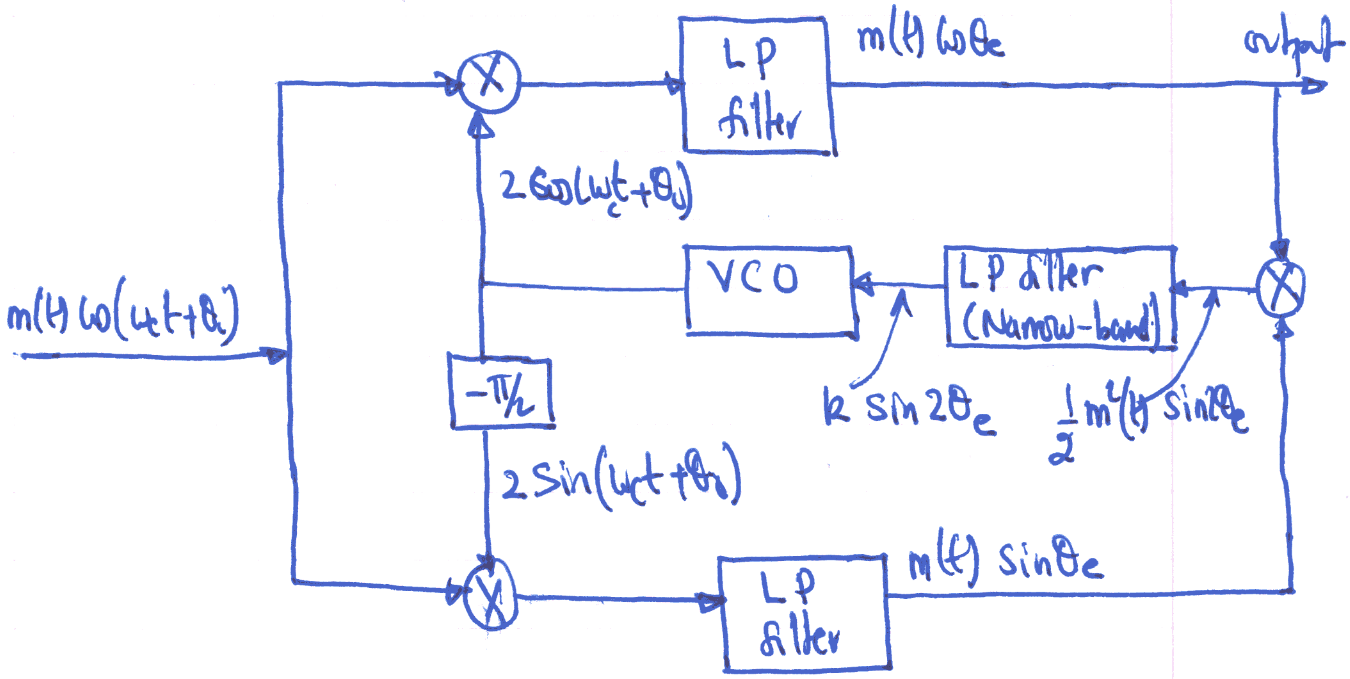
zero-mean baseband signal

$$x(t) = \frac{1}{2} m^2(t) + k\cos 2\omega_c t + \phi(t)\cos 2\omega_c t$$

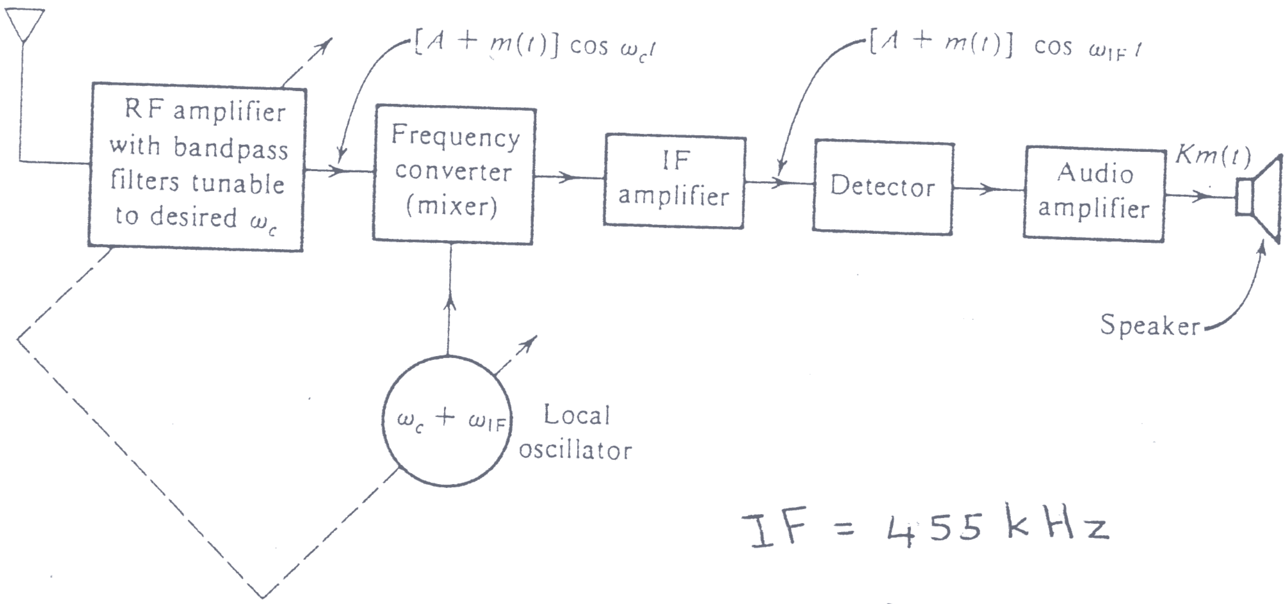


- Costas loop:

  see block diagram.



$$\theta_e = \theta_i - \theta_o$$



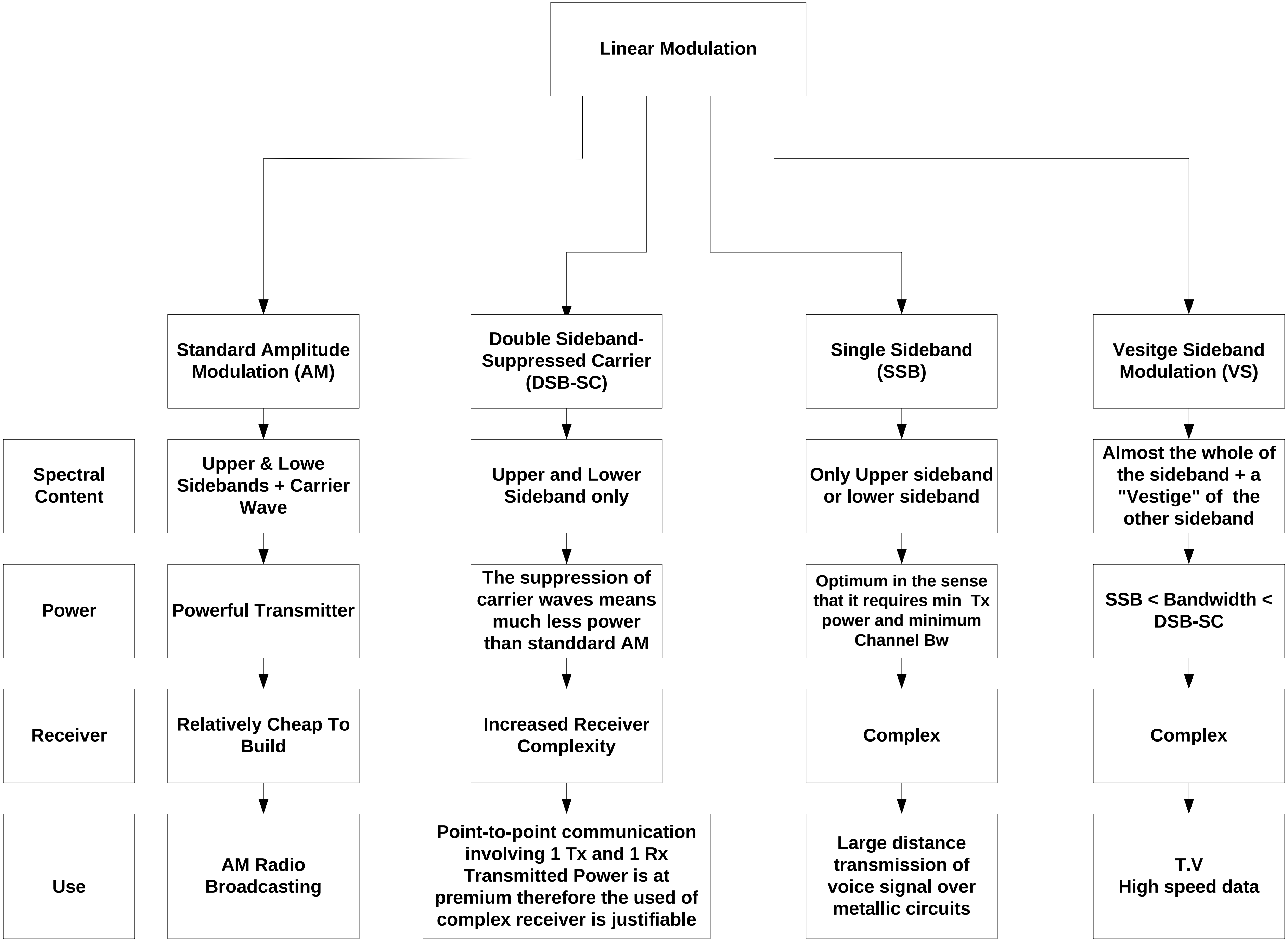
Superheterodyne receiver.

$$IF = 455\ kHz$$

$$f_{LO} = f_c + f_{IF}$$

For AM radio broadcast, the bandwidth of the IF amplifier is 10 kHz.

The IF amplifier provides adequate selectivity.



Linear Modulation
Standard Amplitude Modulation (AM)
Double Sideband-Suppressed Carrier (DSB-SC)
Single Sideband (SSB)
Vesitge Sideband Modulation (VS)
Spectral Content
Upper & Lowe Sidebands + Carrier Wave
Upper and Lower Sideband only
Only Upper sideband or lower sideband
Almost the whole of the sideband + a "Vestige" of the other sideband
Power
Powerful Transmitter
The suppression of carrier waves means much less power than standdard AM
Optimum in the sense that it requires min Tx power and minimum Channel Bw
SSB < Bandwidth < DSB-SC
Receiver
Relatively Cheap To Build
Increased Receiver Complexity
Complex
Complex
Use
AM Radio Broadcasting
Point-to-point communication involving 1 Tx and 1 Rx Transmitted Power is at premium therefore the used of complex receiver is justifiable
Large distance transmission of voice signal over metallic circuits
T.V
High speed data