KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
ELECTRICAL ENGINEERING DEPARTMENT
EE 315
Quiz #7

Name: Solution
ID#: _____
Section No: _____

Q1: Zero-mean Gaussian random variables X and Y have variances 3 and 4, respectively, and a correlation coefficient (-1/4). $\rho = -1/4$ $\sigma_X^2 = 3$ $\sigma_Y^2 = 4$

1. Write an expression for the joint density function of X and Y.
2. Find the conditional density functions $f_X(x \mid Y = y)$ and $f_Y(y \mid X = x)$.

1-

$$f_{X,Y}(x,y) = \frac{1}{3\pi\sqrt{5}} e^{-\frac{8}{15}\left(\frac{x^2}{3} + \frac{xy}{4\sqrt{3}} + \frac{y^2}{4}\right)}$$

2- $$f_X(x) = \frac{1}{\sqrt{6\pi}} e^{-\frac{x^2}{6}}$$

$$f_Y(y) = \frac{1}{2\sqrt{4\pi}} e^{-\frac{y^2}{8}}$$

$$\therefore f_X(x \mid Y=y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

$$= \frac{2}{3}\sqrt{\frac{2}{5\pi}} e^{-\frac{8}{45}\left(x + \frac{\sqrt{3}}{8}y\right)^2}$$

and

$$f_Y(y \mid X=x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$= \sqrt{\frac{2}{15\pi}} e^{-\frac{2}{15}\left(y + \frac{x}{2}\right)^2}$$