

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS  
ELECTRICAL ENGINEERING DEPARTMENT

EE 315

Quiz #6

Name: Solution

ID#: \_\_\_\_\_

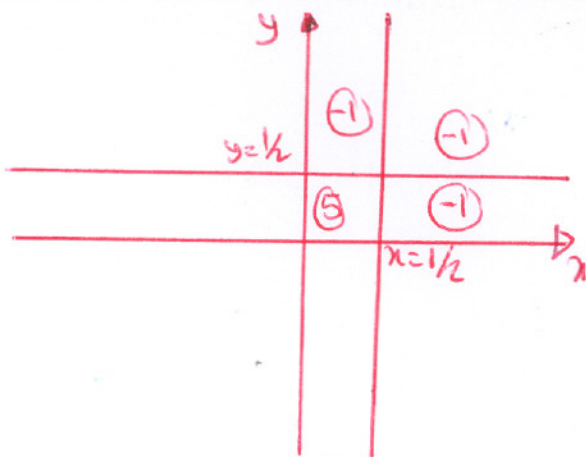
Section No: \_\_\_\_\_

Q1: The joint density of the random variables X and Y is given by

$$f_{X,Y}(x,y) = 16 \exp[-4(x+y)] u(x)u(y)$$

Find the mean value of the function:

$$g(X,Y) = \begin{cases} 5 & 0 < X \leq \frac{1}{2} \quad \text{and} \quad 0 < Y \leq \frac{1}{2} \\ -1 & X > \frac{1}{2} \quad \text{and/or} \quad Y > \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{aligned} E[g(X,Y)] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y) dx dy \\ &= \int_0^{1/2} \int_0^{1/2} 5 \times 16 e^{-4(x+y)} dx dy \\ &\quad + \int_0^{1/2} \int_{1/2}^{\infty} (-1) \times 16 e^{-4(x+y)} dx dy \\ &\quad + \int_{1/2}^{\infty} \int_0^{1/2} (-1) \times 16 e^{-4(x+y)} dx dy \end{aligned}$$

$$\begin{aligned} &+ \int_{1/2}^{\infty} \int_{1/2}^{\infty} (-1) \times 16 e^{-4(x+y)} dx dy \\ &= \int_0^{1/2} \int_0^{1/2} 80 e^{-4(x+y)} dx dy \\ &\quad - \int_0^{1/2} \int_{1/2}^{\infty} 16 e^{-4(x+y)} dx dy \\ &\quad - \int_{1/2}^{\infty} \int_0^{1/2} 16 e^{-4(x+y)} dx dy \\ &= 6e^{-4} - 12e^{-2} + 5 \end{aligned}$$