

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS  
ELECTRICAL ENGINEERING DEPARTMENT

EE 315

Quiz #5

Name: Solution  
ID#: \_\_\_\_\_  
Section No: \_\_\_\_\_

Q1: The joint density of the random variables X and Y is given by

$$f_{X,Y}(x,y) = 2 \exp\left[-\left(\frac{x}{2} + 4y\right)\right] u(x)u(y)$$

Are X and Y independent?

Let's find the marginal pdf's of X and Y:

$$\begin{aligned} f_X(x) &= \int_{-\infty}^{+\infty} f_{X,Y}(x,y) dy \\ &= 2 u(x) \int_0^{+\infty} e^{-x/2} e^{-4y} dy \\ &= \frac{1}{2} e^{-x/2} u(x) \end{aligned}$$

$$\begin{aligned} f_Y(y) &= \int_{-\infty}^{+\infty} f_{X,Y}(x,y) dx \\ &= 2 u(y) \int_0^{+\infty} e^{-x/2} e^{-4y} dx \\ &= 4 e^{-4y} u(y) \end{aligned}$$

Now, let's evaluate the product  $f_X(x) f_Y(y)$ :

$$\begin{aligned} f_X(x) f_Y(y) &= \frac{1}{2} x^4 e^{-x/2} e^{-4y} u(x) u(y) \\ &= 2 e^{-(x/2 + 4y)} u(x) u(y) \\ &= f_{X,Y}(x,y) \end{aligned}$$

$\therefore$  X and Y are  
Statistically independent.