

**KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS**  
**ELECTRICAL ENGINEERING DEPARTMENT**

EE 315

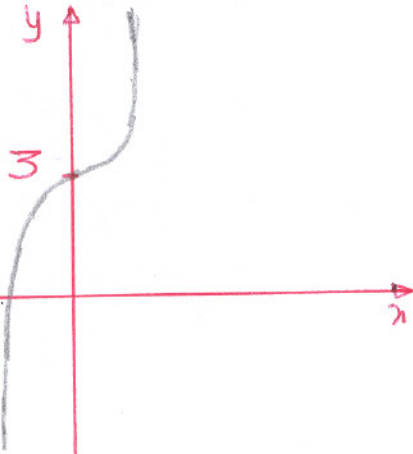
Quiz #4

Name: Solution  
ID#: \_\_\_\_\_  
Section No.: \_\_\_\_\_

Q1: The probability density function of a certain variable X is given by

$$f_X(x) = \frac{1}{2}|x| \exp\{-|x|\}.$$

- Find the pdf of Y if  $Y=2X^3+3$ .



\* If  $y \leq 3$ ,  $x \leq 0$

$$f_X(x) = -\frac{1}{2}x \exp\{x\}$$

$$y = 2x^3 + 3 \Rightarrow x = \sqrt[3]{\frac{y-3}{2}}$$

$$\frac{dy}{dx} = 6x^2$$

$$\Rightarrow f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} \Big|_{x=x_0}$$

$$= \frac{-1}{12 \sqrt[3]{\frac{y-3}{2}}} \exp\left\{ \sqrt[3]{\frac{y-3}{2}} \right\}$$

\* If  $y \geq 3$ ,  $x \geq 0$

$$f_X(x) = \frac{1}{2}x \exp\{-x\}$$

$$y = 2x^3 + 3 \Rightarrow x = \sqrt[3]{\frac{y-3}{2}}$$

$$\frac{dy}{dx} = 6x^2$$

$$f_Y(y) = \frac{1}{12 \sqrt[3]{\frac{y-3}{2}}} \exp\left\{ -\sqrt[3]{\frac{y-3}{2}} \right\}$$

$$f_Y(y) = \begin{cases} -\frac{1}{12 \sqrt[3]{\frac{y-3}{2}}} \exp\left\{ \sqrt[3]{\frac{y-3}{2}} \right\} & y \leq 3 \\ \frac{1}{12 \sqrt[3]{\frac{y-3}{2}}} \exp\left\{ -\sqrt[3]{\frac{y-3}{2}} \right\} & y \geq 3 \end{cases}$$