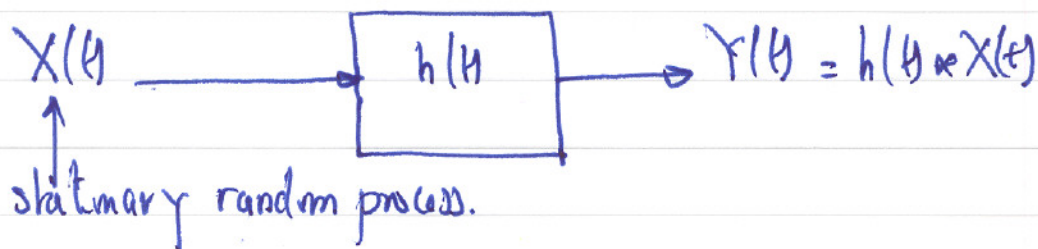


Solution to Homework # 8

1. Problem 8.2-20:



$$h(t) = 3t^2 e^{-8t} u(t)$$

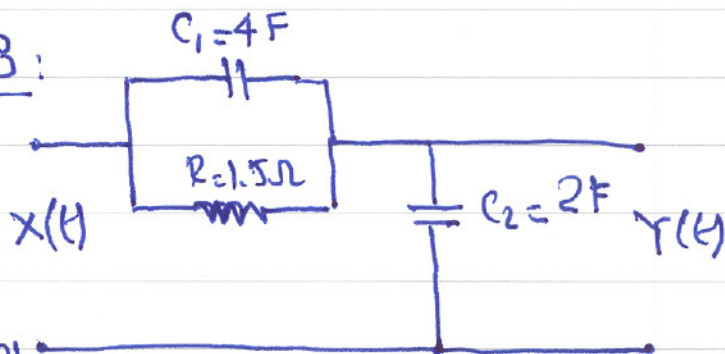
$$E[Y(t)] = E\left[\int_{-\infty}^{+\infty} h(\tau) X(t-\tau) d\tau\right]$$

$$= \mu_X \int_{-\infty}^{+\infty} h(t) dt, \quad \mu_X = 2$$

$$= 2 \int_0^{+\infty} 3t^2 e^{-8t} dt$$

$$= \frac{3}{128}$$

2. Problem 8.4-8:



$$R_X(\tau) = 2 e^{-4|\tau|}$$

a) $S_X(\omega)$

$$R_x(e) \longleftrightarrow S_x(\omega) = \frac{16}{16 + \omega^2}$$

b) $|H(\omega)|^2$

$$H(\omega) = \frac{1/j\omega c_2}{\frac{1}{j\omega c_2} + \frac{R(1/j\omega c_1)}{R + (1/j\omega c_1)}}$$

$$= \frac{1 + j6\omega}{1 + j9\omega}$$

$$|H(\omega)|^2 = \frac{1 + 36\omega^2}{1 + 81\omega^2}$$

c) $S_y(\omega) = |H(\omega)|^2 S_x(\omega)$

$$= \frac{16(1 + 36\omega^2)}{(16 + \omega^2)(1 + 81\omega^2)}$$

3. Problem 8.4.9:

$$E[x(t)] = 5$$

$$S_x(\omega) = 50\pi\delta(\omega) + \frac{3}{[1 + (\frac{\omega}{2})^2]}$$

$$h(t) = 4e^{-4|t|}$$

a) $H(\omega)$ for the network

$$H(\omega) = \frac{32}{16 + \omega^2}$$

b) The mean of $Y(t)$:

$$E[Y(t)] = E\left[\int_{-\infty}^{+\infty} h(\tau) X(t-\tau) d\tau\right]$$

$$= \mu_X \int_{-\infty}^{+\infty} h(\tau) d\tau$$

$$= 20 \int_{-\infty}^{+\infty} e^{-4|\tau|} d\tau$$

$$= 40 \int_0^{+\infty} e^{-4t} dt$$

$$= 10.$$

c) $S_Y(\omega)$

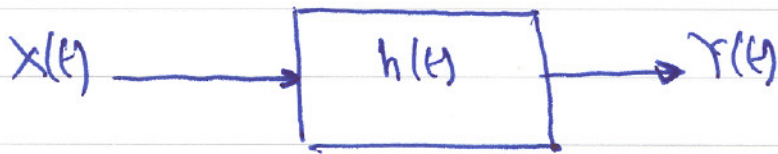
$$S_Y(\omega) = |H(\omega)|^2 S_X(\omega)$$

$$|H(\omega)|^2 = \left[\frac{32}{16 + \omega^2}\right]^2$$

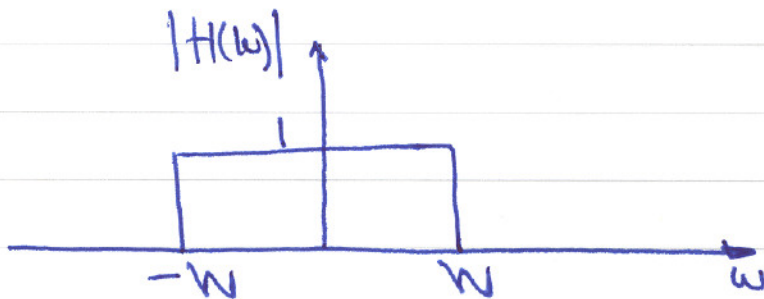
$$S_Y(\omega) = 50\pi\delta(\omega) |H(0)|^2 + \frac{3}{\left[1 + \left(\frac{\omega}{2}\right)^2\right]^2} \times \frac{(32)^2}{(16 + \omega^2)^2}$$

$$= 200\pi\delta(\omega) + \frac{12288}{(4 + \omega^2)(16 + \omega^2)^2}$$

4. Problem 8.4-11:



$$S_X(\omega) = \frac{N_0}{2} \\ = 6 \times 10^{-6} \text{ W/Hz}$$



$$S_Y(\omega) = |H(\omega)|^2 S_X(\omega) \\ = \frac{N_0}{2}, \text{ since } H(\omega) = 1, -W \leq \omega \leq W \\ = 6 \times 10^{-6} \text{ W/Hz}$$

$$P_Y = \frac{1}{2\pi} \int_{-\omega}^{+\omega} S_Y(\omega) d\omega \\ = \frac{1}{2\pi} \int_{-W}^W 6 \times 10^{-6} d\omega \\ = \frac{6 \times 10^{-6} W}{2\pi} \\ = 15 \pi \Rightarrow W = \frac{5\pi}{2} \times 10^6 \text{ rad/sec}$$