

Solution to Homework #7

1. Problem 7.1-12:

a- Valid

b- Not valid, since function is not an even function of ω .

c- Not valid, since negative at $\omega=0$.

d- Not valid, since complex.

2. Problem 7.1-16:

$$Y(t) = A_0 + B_0 X(t)$$

Since $R_X(e) \longleftrightarrow S_X(f)$, then:

$$\begin{aligned} R_Y(t, t+\tau) &= E[(A_0 + B_0 X(t))(A_0 + B_0 X(t+\tau))] \\ &= E[A_0^2 + A_0 B_0 X(t) + A_0 B_0 X(t+\tau) + B_0^2 X(t)X(t+\tau)] \end{aligned}$$

$$= A_0^2 + 2A_0 B_0 \bar{X} + B_0^2 R_X(e), \text{ since } X(t) \text{ is stationary}$$

$$\therefore S_Y(f) = [A_0^2 + 2A_0 B_0 \bar{X}] \delta(f) + B_0^2 S_X(f)$$

3. Problem 7.1-22:

$$R_X(\tau) = 3 + 2 \exp(-4\tau^2)$$

$$a) S_X(f) = 3\delta(f) + \frac{1}{2\sqrt{\pi}} e^{-f^2/16}$$

b) The average power in $X(t)$ corresponds to $R_X(0) = R_X(\infty) = 5W$

4. Problem 7.2-3:

$$P_x = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_x(\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{6\omega^2}{(1+\omega^2)^3} d\omega$$

$$= \frac{6}{\pi} \left[-\frac{\omega}{4(1+\omega^2)^2} + \frac{\omega}{8(1+\omega^2)} + \frac{1}{8} \tan^{-1}(\omega) \right]_0^{+\infty}$$

$$= \frac{3}{8}$$

5. Problem 7.2-10:

$$R_x(\tau) = A e^{-\alpha|\tau|} \cos(\omega_0 \tau)$$

$$A e^{-\alpha|\tau|} \longleftrightarrow \frac{2\alpha A}{\alpha^2 + \omega^2}$$

Since if $f(t) \leftrightarrow F(\omega)$, then
 $f(t) e^{\pm j\omega_0 t} \longleftrightarrow F(\omega \pm \omega_0)$

$$R_x(\tau) = A e^{-\alpha|\tau|} \left[\frac{e^{j\omega_0 \tau} + e^{-j\omega_0 \tau}}{2} \right]$$

$$\therefore S_x(\omega) = \frac{\alpha A}{\alpha^2 + (\omega - \omega_0)^2} + \frac{\alpha A}{\alpha^2 + (\omega + \omega_0)^2}$$