

EE315 EQUATION SHEET

CH 1

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = \sum_{n=1}^N P(A|B_n)P(B_n)$$

$$P(B_n|A) = \frac{P(A|B_n)P(B_n)}{P(A|B_1)P(B_1) + \dots + P(A|B_n)P(B_n)}$$

$$\binom{N}{k} = \frac{N!}{k!(N-k)!}$$

$$P(A \text{ occurs } k \text{ times}) = \binom{N}{k} p^k (1-p)^{N-k}$$

CH 2

$$F_X(x) = P\{X \leq x\}$$

$$F_X(x) = \sum_{i=0}^N P(x_i)u(x-x_i)$$

$$f_X(x) = \frac{dF_X(x)}{dx}$$

$$F_X(x) = \int_{-\infty}^x f_X(\xi) d\xi$$

$$F_X(x) = F\left(\frac{x-a_x}{\sigma_x}\right)$$

$$F_X(x|B) = P\{X \leq x|B\} = \frac{P\{X \leq x \cap B\}}{P(B)}$$

$$f_X(x|B) = \frac{dF_X(x|B)}{dx}$$

CH 3

$$E[X] = \bar{X} = \sum_{i=1}^N x_i P(x_i)$$

$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

$$m_n = E[X^n]$$

$$\mu_n = E[(X - \bar{X})^n]$$

$$\Phi_X(\omega) = \int_{-\infty}^{\infty} f_X(x) e^{j\omega x} dx$$

$$m_n = (-j)^n \left. \frac{d^n \Phi_X(\omega)}{d\omega^n} \right|_{\omega=0}$$

$$M_X(v) = \int_{-\infty}^{\infty} f_X(x) e^{vx} dx$$

$$m_n = \left. \frac{d^n M_X(v)}{dv^n} \right|_{v=0}$$

$$f_Y(y) = \sum_n \left. \frac{f_X(x_n)}{dx} \right|_{x=x_n}$$

CH 4

$$F_{X,Y}(x,y) = P\{X \leq x, Y \leq y\}$$

$$P\{x_1 < X \leq x_2, y_1 < Y \leq y_2\} =$$

$$F_{X,Y}(x_2, y_2) + F_{X,Y}(x_1, y_2) - F_{X,Y}(x_1, y_2) - F_{X,Y}(x_2, y_1)$$

$$F_{X,Y}(x, \infty) = F_X(x);$$

$$f_{X,Y}(x,y) = \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y}$$

$$P\{x_1 < X \leq x_2, y_1 < Y \leq y_2\} = \int_{y_1}^{y_2} \int_{x_1}^{x_2} f_{X,Y}(x,y) dx dy$$

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dy; \quad f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx$$

$$f_X(x) = \frac{dF_X(x)}{dx}; \quad f_Y(y) = \frac{dF_Y(y)}{dy}$$

$$f_X(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$$

CH 5

$$\bar{g} = E[g(x)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y) dx dy$$

$$m_{nk} = E[X^n Y^k] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^n y^k f_{X,Y}(x,y) dx dy$$

$$R_{XY} = m_{11} = E[XY]$$

$$\mu_{nk} = E[(X - \bar{X})^n (Y - \bar{Y})^k] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (X - \bar{X})^n (Y - \bar{Y})^k f_{X,Y}(x,y) dx dy$$

$$C_{XY} = \mu_{11} = E[(X - \bar{X})(Y - \bar{Y})] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (X - \bar{X})(Y - \bar{Y}) f_{X,Y}(x,y) dx dy$$

$$C_{XY} = R_{XY} - \bar{X}\bar{Y} = R_{XY} - E[X]E[Y]$$

$$\rho = \frac{C_{XY}}{\sigma_X \sigma_Y}; \quad -1 \leq \rho \leq 1$$

$$f_{u,v}(u,v) = f_{X,Y}(x,y) |J|; \quad \text{where } J = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

CH 6,7&8

$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x f_{X(t)}(x) dx$$

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1), X(t_2)}(x_1, x_2) dx_1 dx_2$$

$$\langle X(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T X(t) dt; \quad \mu_Y(t) = E[Y(t)] = \mu_Y H(0)$$

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft) df; \quad H(f) = \int_{-\infty}^{\infty} h(t) \exp(-j2\pi ft) dt$$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) \exp(-j2\pi f\tau) d\tau \Leftrightarrow R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) \exp(j2\pi f\tau) df$$

$$S_Y(f) = |H(f)|^2 S_X(f)$$