P 9.60 [a] $j\omega L_1 = j(50)(5) = j250\,\Omega$

$$j\omega L_2 = j(50)(20) = j1000\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^9}{j(50\times 10^3)(40)} = -j500\,\Omega$$

$$\therefore\; Z_{22} = 75 + 300 + j1000 - j500 = 375 + j500\,\Omega$$

$$\therefore\; Z_{22}^* = 375 - j500\,\Omega$$

$$M = k\sqrt{L_1L_2} = 10k\times 10^{-3}$$

$$\omega M = (50)(10k) = 500k$$

$$Z_r = \left[\frac{500k}{625}\right]^2(375 - j500) = k^2(240 - j320)\,\Omega$$

$$Z_{in} = 120 + j250 + 240k^2 - j320k^2$$

$$|Z_{in}| = [(120+240k)^2 + (250-320k)^2]^{\frac{1}{2}}$$

$$\frac{d|Z_{in}|}{dk} = \frac{1}{2}[(120+240k)^2 + (250-320k)^2]^{-\frac{1}{2}}\times$$

$$[2(120+240k^2)480k + 2(250-320k^2)(-640k)]$$

$$\frac{d|Z_{in}|}{dk} = 0 \text{ when}$$

$$960k(120+240k^2) - 1280k(250-320k^2) = 0$$

$$\therefore\; k^2 = 0.32; \qquad \therefore\; k = \sqrt{0.32} = 0.5657$$

[b]

$$Z_{in}\,(\text{min}) = 120 + 240(0.32) + j[250 - 0.32(320)]$$
$$= 196.8 + j147.6 = 246\underline{/36.87^\circ}\,\Omega$$

$$\mathbf{I}_1\,(\text{max}) = \frac{369\underline{/0^\circ}}{246\underline{/36.87^\circ}} = 1.5\underline{/-36.87^\circ}\,\text{A}$$

$$\therefore\; i_1\,(\text{peak}) = 1.5\,\text{A}$$

Note — You can test that the k value obtained from setting $d|Z_{in}|/dt = 0$ leads to a minimum by noting $0 \le k \le 1$. If $k = 1$,

$$Z_{in} = 360 - j70 = 366.74\underline{/-11^\circ}\,\Omega$$

Thus,

$$|Z_{in}|_{k=1} > |Z_{in}|_{k=\sqrt{0.32}}$$

If $k = 0$,

$$Z_{in} = 120 + j250 = 277.31\underline{/64.36^\circ}\,\Omega$$

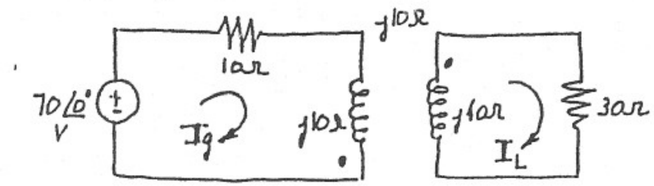
Thus,

$$|Z_{in}|_{k=0} > |Z_{in}|_{k=\sqrt{0.32}}$$

P 9.61 [a] $j\omega L_1 = j(5000)(2\times 10^{-3}) = j10\,\Omega$

$$j\omega L_2 = j(5000)(8\times 10^{-3}) = j40\,\Omega$$

$$j\omega M = j10\,\Omega$$



$$70 = (10 + j10)\mathbf{I}_g + j10\mathbf{I}_L$$

$$0 = j10\mathbf{I}_g + (30 + j40)\mathbf{I}_L$$

Solving,

$$\mathbf{I}_g = 4 - j3\,\text{A}; \qquad \mathbf{I}_L = -1\,\text{A}$$

$$i_g = 5\cos(5000t - 36.87^\circ)\,\text{A}$$

$$i_L = 1\cos(5000t - 180^\circ)\,\text{A}$$

[b]

$$k = \frac{M}{\sqrt{L_1L_2}} = \frac{2}{\sqrt{16}} = 0.5$$

HW 9

1/2

[c] When $t = 100\pi\ \mu s$,

$$5000t = (5000)(100\pi) \times 10^{-6} = 0.5\pi = \pi/2\,\text{rad} = 90^\circ$$

$$i_g(100\pi\mu s) = 5\cos(53.13^\circ) = 3\,\text{A}$$

$$i_L(100\pi\mu s) = 1\cos(-90^\circ) = 0\,\text{A}$$

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2 = \frac{1}{2}(2\times 10^{-3})(9) + 0 + 0 = 9\,\text{mJ}$$

When $t = 200\pi\ \mu s$,

$$5000t = \pi\,\text{rad} = 180^\circ$$

$$i_g(200\pi\mu s) = 5\cos(180 - 53.13) = -4\,\text{A}$$

$$i_L(200\pi\mu s) = 1\cos(180 - 180) = 1\,\text{A}$$

$$w = \frac{1}{2}(2\times 10^{-3})(16) + \frac{1}{2}(8\times 10^{-3})(1) + 2\times 10^{-3}(-4)(1) = 12\,\text{mJ}$$

P 9.63

$$j\omega L_1 = j(25\times 10^3)(3.2\times 10^{-3}) = j80\,\Omega$$

$$j\omega L_2 = j(25\times 10^3)(12.8\times 10^{-3}) = j320\,\Omega$$

$$\frac{1}{j\omega C} = \frac{10^9}{j(25\times 10^3)(250)} = -j160\,\Omega$$

$$j\omega M = j(25\times 10^3)k\sqrt{(3.2)(12.8)}\times 10^{-3} = j160k\,\Omega$$

$$Z_{22} = 40 + j320 - j160 = 40 + j160\,\Omega$$

$$Z_{22}^* = 40 - j160\,\Omega$$

$$Z_r = \left[\frac{160k}{|40 + j160|}\right]^2 (40 - j160) = 64k^2 - j256k^2$$

$$Z_{ab} = 10 + j80 + 64k^2 - j256k^2 = (10 + 64k^2) + j(80 - 256k^2)$$

Z_{ab} is resistive when

$$80 - 256k^2 = 0 \quad \text{or} \quad k^2 = 80/256 = 5/16$$

$$\therefore\ Z_{ab} = 10 + 64(5/16) = 30\,\Omega$$

P. 9.68

$$Z_{ab} = (50)^2 (1/25)^2 Z_L = 4 Z_L$$

$$\therefore Z_{ab} = 800 + j600\ \Omega$$

if you find any mistake please contact Dr. Ali Muqaibel

muqaibel@kfupm.edu.sa

Hw9