

10.40

SECTION 10.4

(a) $V_R = \frac{5/3}{5/3 + 5S + 25/S} V_1$

$V_R = \frac{5S V_1}{5S + 15S^2 + 75}$

$$V_R = \frac{5(-6+j3)(20)}{5(-6+j3) + 15(-6+j3)^2 + 75} = \frac{-600 + j300}{-30 + j15 + 15(36 - j36 - 9) + 75}$$

$$= \frac{-900(2-j)}{-30 + j15 + 405 - j540 + 75} = \frac{-900(2-j)}{450 - j525} = \frac{-20(2-j)}{30 - j35} = \frac{-20(2-j)}{5(6-j7)}$$

$$= \frac{(4 \angle 110^\circ)(\sqrt{5} \angle -26.565^\circ)}{\sqrt{85} \angle -49.4^\circ} = 0.97 \angle -157.17^\circ$$

$\therefore v_R(t) = 0.97 e^{-6t} \cos(3t - 157.17^\circ) \text{ V}$

(b) $V_L = \frac{5S V_1}{5/3 + 5S + 25/S} = \frac{15S^2 V_1}{5S + 15S^2 + 75}$

$$= \frac{15(-6+j3)^2 (20)}{5(-6+j3) + 15(-6+j3)^2 + 75} = \frac{300(36 - j36 - 9)}{450 - j525} = \frac{300(27 - j36)}{75(6-j7)} = \frac{8(3-j4)}{(6-j7)}$$

$$= \frac{36(5 \angle -53.13^\circ)}{\sqrt{85} \angle -49.4^\circ} = 19.52 \angle -3.73^\circ$$

$\therefore v_L(t) = 19.52 e^{-6t} \cos(3t - 3.73^\circ) \text{ V}$

(c) $V_C = \frac{25/S V_1}{5/3 + 5S + 25/S} = \frac{75 V_1}{5S + 15S^2 + 75} = \frac{75(20)}{75(6-j7)} = \frac{20}{6-j7}$

$$V_C = 2.17 \angle 49.4^\circ$$

$\therefore v_C(t) = 2.17 e^{-6t} \cos(3t + 49.4^\circ) \text{ V}$

If you find any mistake please send to Dr. Mugaibel
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10.42

By KCL,

$$\frac{V_C - V_1}{5/3} + \frac{V_C}{5S} + \frac{V_C}{25/S} = 0$$

$$\frac{3V_C - 3V_1}{5} + \frac{V_C}{5S} + \frac{5V_C}{25} = 0$$

$$15S V_C - 15S V_1 + 5V_C + 5^2 V_C = 0$$

$$(5^2 + 15S + 5) V_C = 15S V_1$$

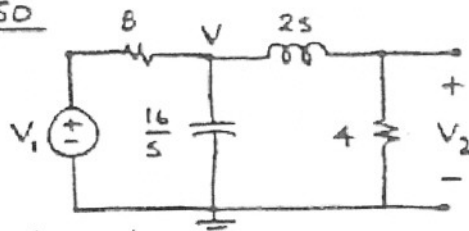
$$V_C = \frac{15S V_1}{5^2 + 15S + 5}$$

$$V_C = \frac{15(-6+j3)(20)}{(-6+j3)^2 + 15(-6+j3) + 5} = \frac{-900(2-j)}{36 - j36 - 9 - 90 + j45 + 5} = \frac{-900(2-j)}{-58 + j9}$$

$$= \frac{-900(2-j)}{-1(58 - j9)} = \frac{900(\sqrt{5} \angle -26.565^\circ)}{\sqrt{3445} \angle -8.82^\circ} = 34.29 \angle -17.745^\circ$$

$\therefore v_C(t) = 34.29 e^{-6t} \cos(3t - 17.745^\circ) \text{ V}$

10.50



By voltage division,

$$V_2 = \frac{4}{2s+4} V \Rightarrow V = \frac{2s+4}{4} V_2 \Rightarrow (s^2+2s+8)\left(\frac{2s+4}{4} V_2\right) - 8V_2 = 2sV_1$$

By KCL at node V_2 ,

$$\frac{V-V_2}{8} + \frac{V}{16/s} + \frac{V-V_2}{2s} = 0$$

$$2sV - 2sV_1 + s^2V + 8V - 8V_2 = 0$$

$$(s^2+2s+8)V - 8V_2 = 2sV_1$$

$$(s^2+2s+8)(s+2)V_2 - 16V_2 = 4sV_1$$

$$(s^3+2s^2+8s+2s^2+4s+16-16)V_2 = 4sV_1$$

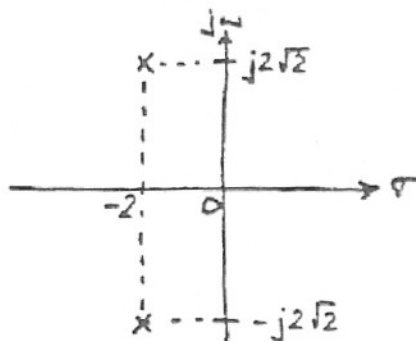
$$(s^3+4s^2+12s)V_2 = 4sV_1$$

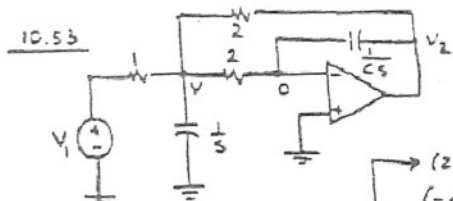
$$(s^2+4s+12)V_2 = 4V_1$$

$$\therefore H(s) = \frac{V_2}{V_1} = \frac{4}{s^2+4s+12}$$

$$s = \frac{-4 \pm \sqrt{16-48}}{2} = -2 \pm \sqrt{\frac{-32}{4}} = -2 \pm \sqrt{-8} = -2 \pm j2\sqrt{2}$$

$$H(s) = \frac{4}{(s+2+j2\sqrt{2})(s+2-j2\sqrt{2})}$$





By KCL at inverting input,

$$\frac{V_2}{1/s} + \frac{V_1}{2} = 0$$

$$-2Cs V_2 = V_1$$

$$(2s+4)(-2Cs V_2) = 2V_1$$

$$(-4Cs^2 - 8Cs - 4)V_2 = 2V_1$$

By KCL at node V1,

$$\frac{V_1 - V_1}{1} + \frac{V_1}{1/s} + \frac{V_1}{2} + \frac{V_1 - V_2}{2} = 0$$

$$2V_1 - 2V_1 + 2sV_1 + V_1 + V_1 - V_2 = 0$$

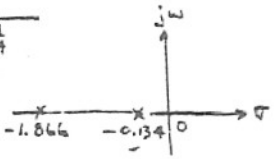
$$(2s+4)V_1 - V_2 = 2V_1$$

$$\therefore H(s) = \frac{V_2}{V_1} = \frac{-2}{4Cs^2 + 8Cs + 4}$$

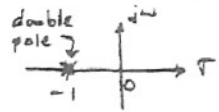
(a) $C = 1F \Rightarrow H(s) = \frac{-2}{4s^2 + 8s + 4} = \frac{-1/2}{s^2 + 2s + 1}$

$$s = \frac{-2 \pm \sqrt{4-4}}{2} = -1 \pm \frac{j0}{2} = -1 \pm 0.866j$$

$$H(s) = \frac{-1/2}{(s+0.134)(s+1.866)}$$



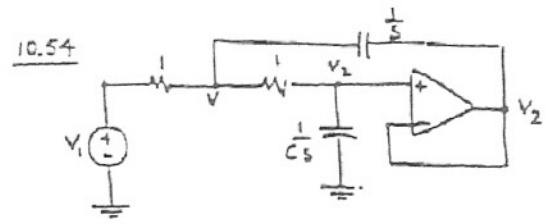
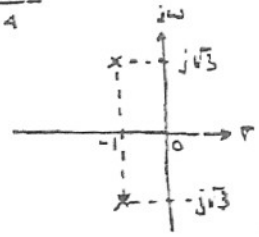
(b) $C = 1/4 F \Rightarrow H(s) = \frac{-2}{s^2 + 2s + 1} = \frac{-2}{(s+1)^2}$



(c) $C = 1/16 F \Rightarrow H(s) = \frac{-2}{\frac{1}{4}s^2 + \frac{1}{2}s + 1} = \frac{-8}{s^2 + 2s + 4}$

$$s = \frac{-2 \pm \sqrt{4-16}}{2} = -1 \pm j\sqrt{3}$$

$$H(s) = \frac{-8}{(s+1+j\sqrt{3})(s+1-j\sqrt{3})}$$



At node V1,

$$\frac{V_1 - V_1}{1} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_2}{1/s} = 0$$

$$V_1 - V_1 + V_1 - V_2 + sV_1 - sV_2 = 0$$

$$(s+2)V_1 - V_2 = 0 \Rightarrow (s+2)(Cs+1)V_2 = V_1$$

By voltage division,

$$V_2 = \frac{1/s}{1/s + 1} V_1 = \frac{1}{1+Cs} V_1$$

$$V = (1+Cs)V_2$$

$$(s+2)V_1 - V_2 = 0 \Rightarrow (s+2)(Cs+1)V_2 = V_1$$

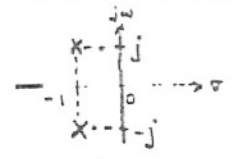
$$(Cs^2 + 2Cs + s + 2 - s - 1)V_2 = V_1$$

$$\therefore H(s) = \frac{V_2}{V_1} = \frac{1}{Cs^2 + 2Cs + 1} = \frac{1/C}{s^2 + 2s + 1/C}$$

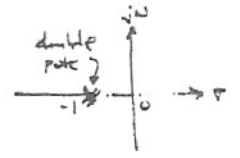
(a) $C = 1/2 F \Rightarrow \frac{V_2}{V_1} = \frac{2}{s^2 + 2s + 2}$

$$-2 \pm \frac{j\sqrt{4-4}}{2} = -1 \pm j$$

$$= \frac{2}{(s+1-j)(s+1+j)}$$



(b) $C = 1F \Rightarrow \frac{V_2}{V_1} = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$



(c) $C = 2F \Rightarrow \frac{V_2}{V_1} = \frac{1/2}{s^2 + 2s + 1/2}$

$$-2 \pm \frac{j\sqrt{4-2}}{2} = -1 \pm \frac{j}{\sqrt{2}}$$

$$-1 \pm 0.707j, -0.293, -1.707$$

