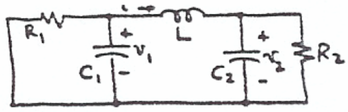
EE205: HW#5 Final Answers — Dr. Ali Muqaibel

If you think there is a mistake in the solution please send to muqaibel@kfupm.edu.sa

7.6



By KVL,

$$L\frac{di}{dt}+v_2-v_1=0$$

$$\therefore \frac{di}{dt}=\frac{1}{L}v_1-\frac{1}{L}v_2$$

By KCL,

$$C_1\frac{dv_1}{dt}+\frac{v_1}{R}+i=0$$

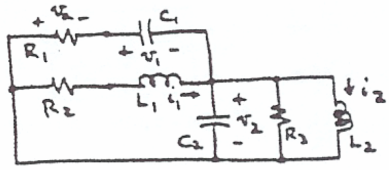
$$\frac{dv_1}{dt}=-\frac{1}{C_1}i-\frac{1}{RC_1}v_1$$

By KCL,

$$C_2\frac{dv_2}{dt}+\frac{v_2}{R_2}=i$$

$$\frac{dv_2}{dt}=+\frac{1}{C_2}i-\frac{1}{R_2C_2}v_2$$

7.9



By KVL,

$$L_1\frac{di_1}{dt}+v_2+R_2i_1=0$$

$$\frac{di_1}{dt}=-\frac{R_2}{L_1}i_1-\frac{1}{L_1}v_2$$

By KVL,

$$L_2\frac{di_2}{dt}=v_2$$

$$\frac{di_2}{dt}=\frac{1}{L_2}v_2$$

By KCL,

$$C_2\frac{dv_2}{dt}+\frac{v_2}{R_3}+i_2=i_1+\frac{v_a}{R_1}=i_1+\frac{-v_2-v_1}{R_1}$$

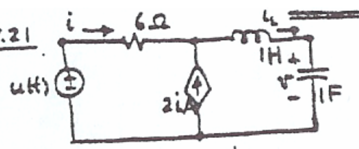
$$C_2\frac{dv_2}{dt}=i_1-\frac{1}{R_1}v_1-\left(\frac{1}{R_1}+\frac{1}{R_3}\right)v_2-i_2$$

$$\frac{dv_2}{dt}=\frac{1}{C_2}i_1-\frac{1}{C_2}i_2-\frac{1}{R_1C_2}v_1-\left(\frac{1}{R_1C_2}+\frac{1}{R_3C_2}\right)v_2$$

By KCL, $C_1\frac{dv_1}{dt}=\frac{v_a}{R_1}=\frac{-v_1-v_2}{R_1}\Rightarrow \frac{dv_1}{dt}=-\frac{1}{R_1C_1}v_1-\frac{1}{R_1C_1}v_2$

$$\frac{d}{dt}\begin{bmatrix}i_1\\ i_2\\ v_1\\ v_2\end{bmatrix}=\begin{bmatrix}-\frac{R_2}{L_1} & 0 & 0 & -\frac{1}{L_1}\\ 0 & 0 & 0 & \frac{1}{L_2}\\ 0 & 0 & -\frac{1}{R_1C_1} & -\frac{1}{R_1C_1}\\ \frac{1}{C_2} & -\frac{1}{C_2} & -\frac{1}{R_1C_2} & \frac{-1}{R_1C_2}-\frac{1}{R_3C_2}\end{bmatrix}\begin{bmatrix}i_1\\ i_2\\ v_1\\ v_2\end{bmatrix}$$

7.21



By KCL, $3i=i_L\Rightarrow i_L/3=i$

By KVL,

$$u(t)=6i+1\frac{di_L}{dt}+v$$

$$u(t)=2i_L+\frac{di_L}{dt}+v$$

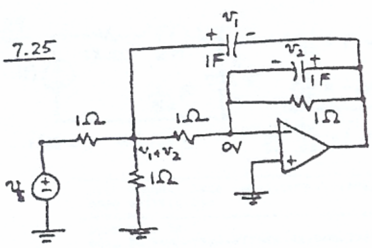
$$\frac{di_L}{dt}=-2i_L-v+u(t)$$

By KCL, $i_L=1\frac{dv}{dt}$

$$\frac{dv}{dt}=i_L$$

$$\frac{d}{dt}\begin{bmatrix}i_L\\ v\end{bmatrix}=\begin{bmatrix}-2 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}i_L\\ v\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}u(t)$$

7.25



By KCL, $\frac{(v_1+v_2)-v_s}{1}+\frac{v_1+v_2}{1}+\frac{v_1+v_2}{1}+1\frac{dv_1}{dt}=0$

$$\therefore \frac{dv_1}{dt}=-3v_1-3v_2+v_s$$

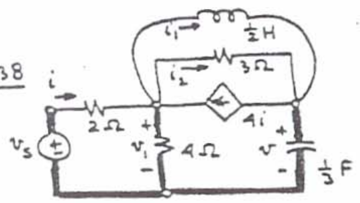
By KCL,

$$\frac{v_1+v_2}{1}+\frac{v_2}{1}+1\frac{dv_2}{dt}=0$$

$$\therefore \frac{dv_2}{dt}=-v_1-2v_2$$

$$\frac{d}{dt}\begin{bmatrix}v_1\\v_2\end{bmatrix}=\begin{bmatrix}-3&-3\\-1&-2\end{bmatrix}\begin{bmatrix}v_1\\v_2\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}v_s$$

7.38



1. 4Ω resistor chosen for tree

2. i_1 and v are the state variables. v_1 and i_2 are assigned.

3. KVL: $\frac{1}{2}\frac{di_1}{dt}+v-v_1=0 \Rightarrow \frac{di_1}{dt}=-2v+2v_1$

4. KCL: $\frac{1}{3}\frac{dv}{dt}+4i=i_1+i_2 \Rightarrow \frac{dv}{dt}=-12i+3i_1+3i_2$

5. KVL: $2i+v_1-v_s=0 \Rightarrow 2i+v_1=v_s$

$3i_2+v-v_1=0 \Rightarrow 3i_2-v_1=-v$

6. KCL: $i+4i=i_1+i_2+\frac{v_1}{4} \Rightarrow 20i-4i_2-v_1=4i_1$

7. $\Delta=\begin{vmatrix}2&0&1\\0&3&-1\\20&-4&-1\end{vmatrix}=-6-60-8=-74$ $\quad \Delta_i=\begin{vmatrix}v_s&0&1\\-v&3&-1\\4i_1&-4&-1\end{vmatrix}=-3v_s+4v-12i_1-4v_s=-12i_1+4v-7v_s$

$\Delta_{i_2}=\begin{vmatrix}2&v_s&1\\0&-v&-1\\20&4i_1&-1\end{vmatrix}=2v-20v_s+20v+8i_1=8i_1+22v-20v_s$ $\quad \Delta_{v_1}=\begin{vmatrix}2&0&v_s\\0&3&-v\\20&-4&4i_1\end{vmatrix}=24i_1-60v_s-8v$

$i=\frac{\Delta_i}{\Delta}=\frac{12}{74}i_1-\frac{4}{74}v+\frac{7}{74}v_s \qquad i_2=\frac{\Delta_{i_2}}{\Delta}=-\frac{8}{74}i_1-\frac{22}{74}v+\frac{20}{74}v_s$

$v_1=\frac{\Delta v_1}{\Delta}=-\frac{24}{74}i_1+\frac{8}{74}v+\frac{60}{74}v_s$

8.

$\frac{di_1}{dt}=-2v+2\left(-\frac{24}{74}i_1+\frac{8}{74}v+\frac{60}{74}v_s\right)=-\frac{24}{37}i_1-\frac{66}{37}v+\frac{60}{37}v_s=\frac{di_1}{dt}$

$\frac{dv}{dt}=-12\left(\frac{12}{74}i_1-\frac{4}{74}v+\frac{7}{74}v_s\right)+3i_1+3\left(-\frac{8}{74}i_1-\frac{22}{74}v+\frac{20}{74}v_s\right)$

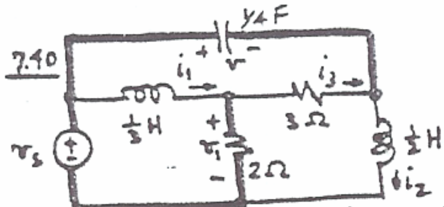
$=-\frac{144}{74}i_1+\frac{48}{74}v-\frac{84}{74}v_s+\frac{222}{74}i_1-\frac{24}{74}i_1-\frac{66}{74}v+\frac{60}{74}v_s$

$\frac{dv}{dt}=\frac{54}{74}i_1-\frac{18}{74}v-\frac{24}{74}v_s=\frac{27}{37}i_1-\frac{9}{37}v-\frac{12}{37}v_s=\frac{dv}{dt}$

7.38 (cont'd)

$$\frac{d}{dt}\begin{bmatrix}i_1\\v\end{bmatrix}=\begin{bmatrix}-\frac{24}{37}&-\frac{66}{37}\\\frac{27}{37}&-\frac{9}{37}\end{bmatrix}\begin{bmatrix}i_1\\v\end{bmatrix}+\begin{bmatrix}\frac{60}{37}\\-\frac{12}{37}\end{bmatrix}v_s$$

7.40



1. 2Ω resistor chosen for tree

2. i_1, i_2 and v are the state variables. i_3 and v_1 are assigned.

3. KVL: $\frac{1}{3}\frac{di_1}{dt} + v_1 - v_s = 0 \Rightarrow \frac{di_1}{dt} = -3v_1 + 3v_s$

$\frac{1}{2}\frac{di_2}{dt} - v_s + v = 0 \Rightarrow \underline{\frac{di_2}{dt} = -2v + 2v_s}$

4. KCL: $\frac{1}{4}\frac{dv}{dt} + i_3 = i_2 \Rightarrow \frac{dv}{dt} = 4i_2 - 4i_3$

5. KVL: $3i_3 - v + v_s - v_1 = 0 \Rightarrow 3i_3 - v_1 = v - v_s$

6. KCL: $i_1 = \frac{v_1}{2} + i_3 \Rightarrow \underline{2i_3 + v_1 = 2i_1}$

$5i_3 = 2i_1 + v - v_s$ 7.

$\Leftarrow i_3 = \frac{2}{5}i_1 + \frac{1}{5}v - \frac{1}{5}v_s$

$v_1 = 2i_1 - 2i_3$

$= 2i_1 - 2\left(\frac{2}{5}i_1 + \frac{1}{5}v - \frac{1}{5}v_s\right)$

$v_1 = \frac{6}{5}i_1 - \frac{2}{5}v + \frac{2}{5}v_s$

8. $\frac{di_1}{dt} = -3\left(\frac{6}{5}i_1 - \frac{2}{5}v + \frac{2}{5}v_s\right) + 3v_s = \underline{-\frac{18}{5}i_1 + \frac{6}{5}v + \frac{9}{5}v_s = \frac{di_1}{dt}}$

$\frac{dv}{dt} = 4i_2 - 4\left(\frac{2}{5}i_1 + \frac{1}{5}v - \frac{1}{5}v_s\right) = \underline{-\frac{8}{5}i_1 + 4i_2 - \frac{4}{5}v + \frac{4}{5}v_s = \frac{dv}{dt}}$

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} = \begin{bmatrix} -\frac{18}{5} & 0 & \frac{6}{5} \\ 0 & 0 & -2 \\ -\frac{8}{5} & 4 & -\frac{4}{5} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ v \end{bmatrix} + \begin{bmatrix} \frac{9}{5} \\ 2 \\ \frac{4}{5} \end{bmatrix} v_s$$