P 8.10 [a] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(10{,}000)(8)} = 12{,}500$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(1.25)(8)} = 10^8$

$s_{1,2} = -12{,}500 \pm \sqrt{(1.5625 - 1)10^8} = -12{,}500 \pm 7500$

$s_1 = -5000 \text{ rad/s}$

$s_2 = -20{,}000 \text{ rad/s}$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 10^8 - 36 \times 10^6 = 0.64 \times 10^8$

$\alpha = 0.8 \times 10^4 = 8000$

$\alpha = \dfrac{1}{2RC} = 8000; \qquad \therefore \quad R = \dfrac{10^9}{(16{,}000)(8)} = 7812.5\,\Omega$

[d] $s_1 = -8000 + j6000; \qquad s_2 = -8000 - j6000 \text{ rad/s}$

[e] $\alpha = 10^4 = \dfrac{1}{2RC}; \qquad \therefore \quad R = \dfrac{1}{2C(10^4)} = 6250\,\Omega$

P 8.11 $\alpha = 2000/2 = 1000$

$R = \dfrac{1}{2\alpha C} = \dfrac{10^6}{(2000)(2)} = 250\,\Omega$

$v(0^+) = -24 \text{ V}$

$i_R(0^+) = \dfrac{-24}{250} = -96 \text{ mA}$

$\dfrac{dv}{dt} = 2400e^{-200t} - 21{,}600e^{-1800t}$

$\dfrac{dv(0^+)}{dt} = 2400 - 21{,}600 = -19{,}200 \text{ V/s}$

$i_C(0^+) = 2 \times 10^{-6}(-19{,}200) = -38.4 \text{ mA}$

$i_L(0^+) = -[i_R(0^+) + i_C(0^+)] = -[-96 - 38.4] = 134.4 \text{ mA}$

P 8.12 [a] $2\alpha = 200; \qquad \alpha = 100 \text{ nepers}$

$2\sqrt{\alpha^2 - \omega_o^2} = 120; \qquad \omega_o = 80 \text{ rad/s}$

$C = \dfrac{1}{2\alpha R} = \dfrac{1}{200(200)} = 25\,\mu F$

$L = \dfrac{1}{\omega_o^2 C} = \dfrac{10^6}{(80)^2(25)} = 6.25 \text{ H}$

$i_C(0^+) = A_1 + A_2 = 15 \text{ mA}$

$\dfrac{di_C}{dt} + \dfrac{di_L}{dt} + \dfrac{di_R}{dt} = 0$

$\dfrac{di_C(0)}{dt} = -\dfrac{di_L(0)}{dt} - \dfrac{di_R(0)}{dt}$

$\dfrac{di_L(0)}{dt} = \dfrac{0}{6.25} = 0 \text{ A/s}$

$\dfrac{di_R(0)}{dt} = \dfrac{1}{R}\dfrac{dv(0)}{dt} = \dfrac{1}{R}\dfrac{i_C(0)}{C} = \dfrac{15 \times 10^{-3}}{(200)(25 \times 10^{-6})} = 3 \text{ A/s}$

$\therefore \quad \dfrac{di_C(0)}{dt} = -3 \text{ A/s}$

$\therefore \quad 160A_1 + 40A_2 = 3$

$4A_1 + A_2 = 75 \times 10^{-3}; \qquad \therefore \quad A_1 = 20 \text{ mA}; \qquad A_2 = -5 \text{ mA}$

$\therefore \quad i_C = 20e^{-160t} - 5e^{-40t} \text{ mA}, \qquad t \geq 0$

[b] By hypothesis

$v = A_3e^{-160t} + A_4e^{-40t}, \qquad t \geq 0$

$v(0) = A_3 + A_4 = 0$

$\dfrac{dv(0)}{dt} = \dfrac{15 \times 10^{-3}}{25 \times 10^{-6}} = 600 \text{ V/s}$

$-160A_3 - 40A_4 = 600; \qquad \therefore \quad A_3 = -5 \text{ V}; \qquad A_4 = 5 \text{ V}$

$v = -5e^{-160t} + 5e^{-40t} \text{ V}; \qquad t \geq 0$

[c] $i_R(t) = \dfrac{v}{200} = -25e^{-160t} + 25e^{-40t} \text{ mA}, \qquad t \geq 0^+$

[d] $i_L = -i_R - i_C = 25e^{-160t} - 25e^{-40t} - 20e^{-160t} + 5e^{-40t}$

$i_L = 5e^{-160t} - 20e^{-40t} \text{ mA}, \qquad t \geq 0$

P 8.3 [a] $\alpha = 4000; \quad \omega_d = 3000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^6 + 16 \times 10^6 = 25 \times 10^6$$

$$\frac{1}{LC} = 25 \times 10^6$$

$$L = \frac{1}{25 \times 10^6 \cdot 50 \times 10^{-9}} = 0.8\ \text{H} = 800\ \text{mH}$$

[b] $$\alpha = \frac{1}{2RC}$$

$$\therefore R = \frac{1}{2\alpha C} = \frac{10^9}{(8000)(50)} = 2500\ \Omega$$

[c] $V_o = v(0) = 125\ \text{V}$

[d] $I_o = i_L(0) = -i_R(0) - i_C(0)$

$$i_R(0) = \frac{V_o}{R} = \frac{125}{2.5} \times 10^{-3} = 50\ \text{mA}$$

$$i_C(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 125\{e^{-4000t}[-3000\sin 3000t - 6000\cos 3000t] - 4000e^{-4000t}[\cos 3000t - 2\sin 3000t]$$

$$\frac{dv}{dt}(0) = 125\{1(-6000) - 4000\} = -125 \times 10^4$$

$$C\frac{dv}{dt}(0) = -125 \times 10^4(40 \times 10^{-9}) = -6250 \times 10^{-5} = -62.5\ \text{mA}$$

$$\therefore I_o = -50 + 62.5 = 12.5\ \text{mA}$$

[e] $$\frac{dv}{dt} = 125e^{-4000t}[5000\sin 3000t - 10{,}000\cos 3000t]$$

$$= 625 \times 10^3 e^{-4000t}[\sin 3000t - 2\cos 3000t]$$

$$C\frac{dv}{dt} = 31{,}250 \times 10^{-6} e^{-4000t}(\sin 3000t - 2\cos 3000t)$$

$$i_C(t) = 31.25e^{-4000t}(\sin 3000t - 2\cos 3000t)\ \text{mA}$$

$$i_R(t) = 50e^{-4000t}(\cos 3000t - 2\sin 3000t)\ \text{mA}$$

$$i_L(t) = -i_R(t) - i_C(t) = e^{-4000t}(12.5\cos 3000t + 68.75\sin 3000t)\ \text{mA}$$

CHECK:

$$\frac{di_L}{dt} = \{-4000e^{-4000t}[12.5\cos 3000t + 68.75\sin 3000t] +$$

$$e^{-4000t}[-37.5 \times 10^3 \sin 3000t + 206.25 \times 10^3 \cos 3000t] \times 10^{-3}$$

$$= e^{-4000t}[156.25\cos 3000t - 312.5\sin 3000t]$$

$$L\frac{di_L}{dt} = e^{-4000t}[125\cos 3000t - 250\sin 3000t]$$

$$= 125e^{-4000t}[\cos 3000t - 2\sin 3000t]\ \text{V}$$

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