6.1-1

Nyquist interval $\quad T_s = 1/2B$

Nyquist freq. $\quad f_s = 2B$

$g_1(t)$: $B_{g_1} = 100\,kHz \Rightarrow \quad T_s = 5\,\mu sec.$

$\qquad f_s = 200\,kHz$

$g_2(t)$: $B_{g_2} = 150\,kHz \quad \Rightarrow \quad T_s = \frac{10}{3}\,\mu sec$

$\qquad f_s = 300\,kHz$

$g_1^2(t)$: $B_{g_1^2} = 2 \times B_{g_1} = 200\,kHz \quad \Rightarrow \quad T_s = 2.5\,\mu sec$

$\qquad f_s = 400\,kHz$

$g_2^3(t)$: $B_{g_2^3} = 3 \times B_{g_2} = 450\,kHz \quad \Rightarrow \quad T_s = \frac{10}{9}\,\mu sec.$

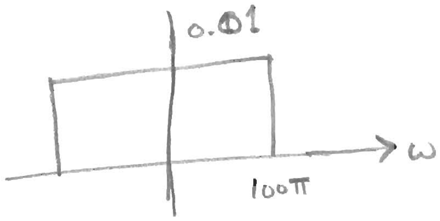
$\qquad f_s = 900\,kHz$

$g_1(t)g_2(t)$: $B_{g_1 g_2} = B_{g_1} + B_{g_2} = 250\,kHz \quad \Rightarrow \quad T_s = 2\,\mu sec$

$\qquad f_s = 500\,kHz$

6.1-2

(a) $\quad \text{sinc}(100\pi t) \longleftrightarrow 0.01\ \text{rect}\left(\frac{\omega}{200\pi}\right)$



$\Rightarrow BW = 50\,Hz$

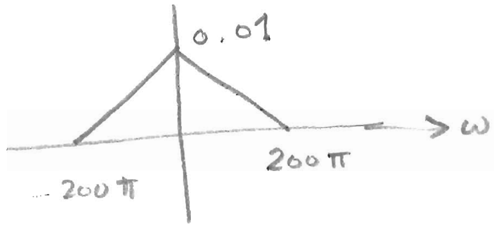
$f_s = 100\,Hz \quad$ (Nyquist rate)

(c) $\text{sinc}(100\pi t) + \text{sinc}(50\pi t) \longleftrightarrow 0.01\,\text{rect}\left(\frac{\omega}{200\pi}\right) + 0.02\,\text{rect}\left(\frac{\omega}{100\pi}\right)$

$$\begin{aligned} BW &= \text{highest freq} \\ &= 100\pi \text{ rad/sec} \\ &= 50 \text{ Hz} \end{aligned}$$

$f_s = 100$ Hz (samples/sec)

(b) $\text{sinc}^2(100\pi t) \longleftrightarrow 0.01\,\Delta\left(\frac{\omega}{400\pi}\right)$



$$\begin{aligned} BW &= 200\pi \\ &= 100 \text{ Hz} \end{aligned}$$

$\Rightarrow f_s = 200$ Hz (samples/sec)

(d) $\text{sinc}(100\pi t) + 3\,\text{sinc}^2(60\pi t) \longleftrightarrow 0.01\,\text{rect}\left(\frac{\omega}{200\pi}\right) + \frac{1}{20}\Delta\left(\frac{\omega}{240\pi}\right)$

$$\begin{aligned} BW &= \text{highest freq} \\ &= \max\{100\pi, 120\pi\} \text{ rad/sec} \\ &= \max\{50, 60\} \text{ Hz} \\ &= 60 \text{ Hz} \end{aligned}$$

$\Rightarrow f_s = 120$ Hz

(e) $\text{sinc}(150\pi t)\,\text{sinc}(100\pi t)$

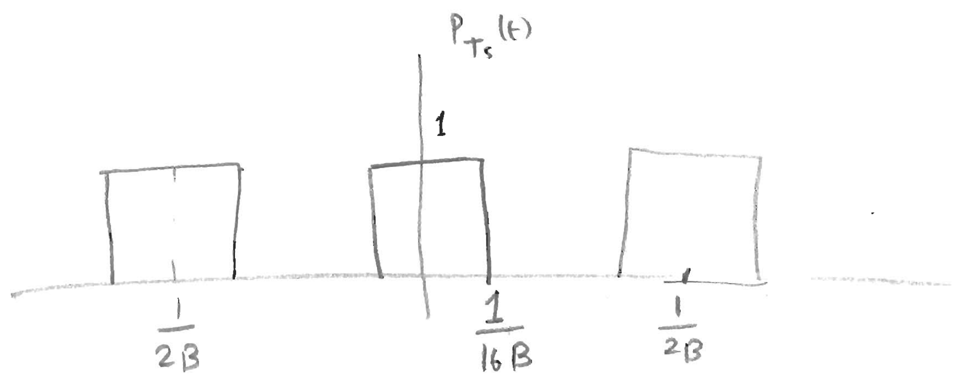
$\text{sinc}(50\pi t) \longleftrightarrow 0.02\,\text{rect}\left(\frac{\omega}{100\pi}\right)$ $\quad BW = 50\pi$

$\text{sinc}(100\pi t) \longleftrightarrow 0.01\,\text{rect}\left(\frac{\omega}{200\pi}\right)$ $\quad BW = 100\pi$

$$\begin{aligned} BW &= BW_1 + BW_2 \\ &= 150\pi \text{ rad/sec} = 75 \text{ Hz} \end{aligned}$$

$f_s = 150$ Hz (samples/sec)

6.1-3



Lets' first represent $P_{T_s}(t)$ using the compact form of the Fourier Series

$$P_{T_s}(t) = C_0 + \sum C_n \cos(\omega_s t + \theta_n)$$

$$C_0 = a_0 = \frac{1}{T_s} \int_{-1/16B}^{1/16B} 1 \, dt = 1/4 \qquad T_s = \frac{1}{2B}$$

$$a_n = \frac{2}{T_s} \int_{-1/16B}^{1/16B} \cos n\omega_s t \, dt = \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right)$$

$b_n = 0$ because $P_{T_s}(t)$ is even

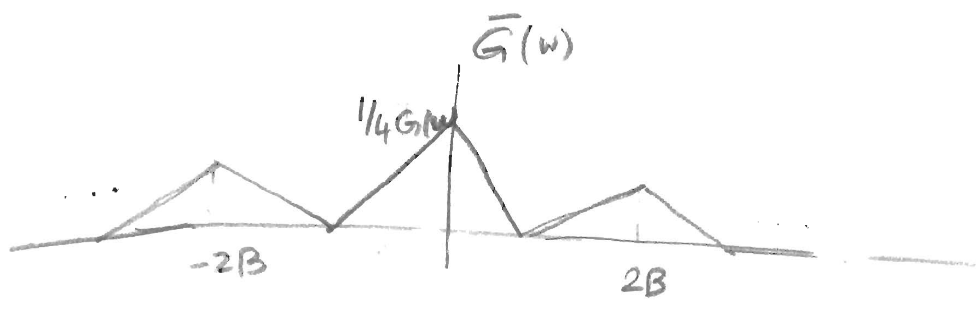
So $C_n = a_0 = \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right)$

$\theta_n = 0$

and
$$\boxed{P_{T_s}(t) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) \cos n\omega_s t}$$

$\bar{g}(t) = g(t) \; P_{T_s}(t)$

$$\frac{1}{4} g(t) + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\left(\frac{n\pi}{4}\right) g(t) \cos n\omega_s t$$

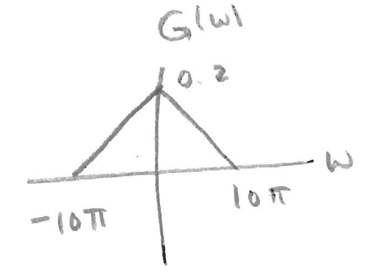


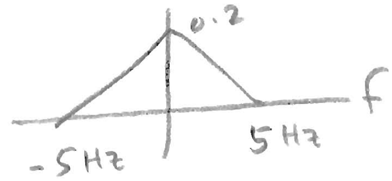
So if we pass $\bar{g}(t)$ through a LPF with cut off freq. B and gain 4, we get $G(\omega)$ (or $g(t)$ in the time domain)

6.1-4

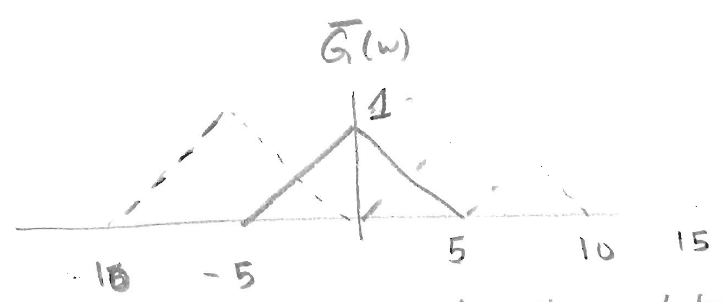
$g(t) = \text{sinc}^2(5\pi t)$

$\leftrightarrow 0.2\, \Delta\left(\frac{\omega}{20\pi}\right)$





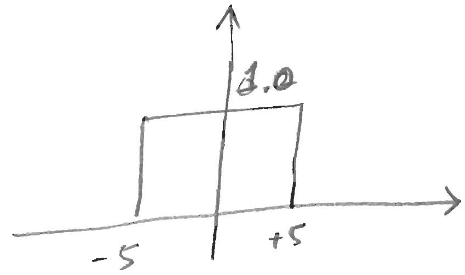
(i)



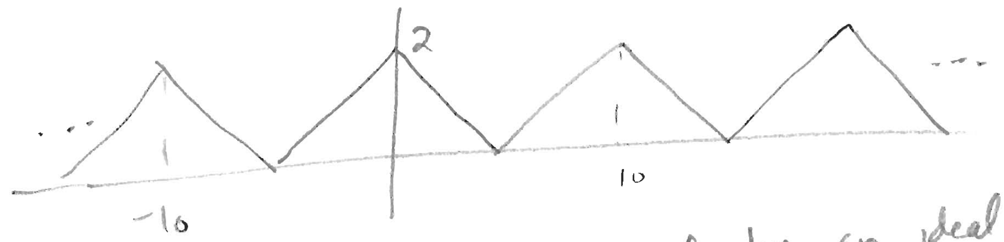
sampling rate = 5 Hz < 10 Hz (Nyquist rate)

Spectrum of $\bar{G}(\omega)$ also shows that we can not recover the $g(t)$

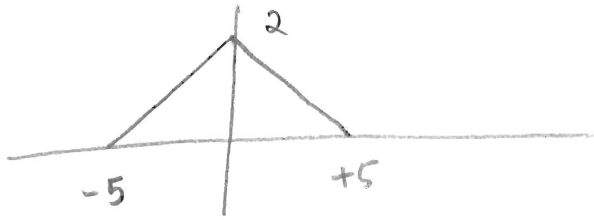
output of ideal LPF



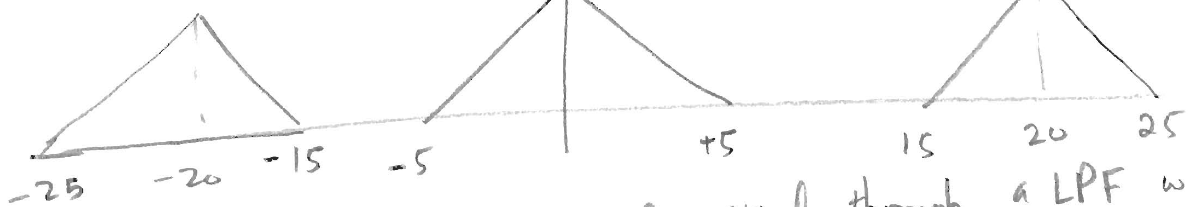
(ii)



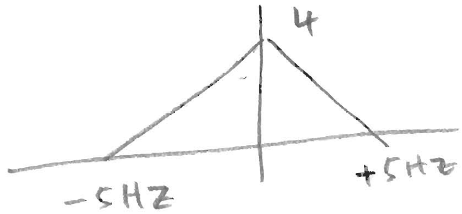
We can recover the signal by an ideal LPF with cutoff freq. at 5 Hz to get the following output



(iii)



We can recover original signal through a LPF with cutoff freq. < 15 Hz. The output when the LPF is ideal with cutoff freq. 5 Hz is



6.1-5

$g_1(t) = 10^4 \text{rect}(10^4 t)$ has infinite frequency content

When passed through $H_1(\omega) = \text{rect}\left(\frac{\omega}{40{,}000\pi}\right)$, we get a signal with $BW = 20{,}000\pi$ rad/se

$= 10$ kHz

i.e. $\boxed{BW_{Y_1} = 10 \text{ kHz}}$

$\Rightarrow$ $\boxed{f_1 = 20 \text{ kHz} \text{ (samples/sec)}}$

$g_2(t)$ has infinite freq. content too

When passed through $H_2(\omega) = \text{rect}(\omega/20{,}000\pi)$, we get a signal with $BW = 10{,}000\pi$ rad/sec

$= 5$ kHz

i.e $\boxed{BW_{Y_2} = 5 \text{ kHz}}$

$\boxed{f_2 = 10 \text{ kHz} \text{ (samples/sec)}}$

$BW_{Y_3} = BW_{Y_1} + BW_{Y_2} = 15$ kHz

$\Rightarrow$ sampling rate $\boxed{f_3 = 30 \text{ kHz}}$