Homework Solutions # 4

Problem # 4.4-1:

From Figure 4.14 of your textbook, when the receiver carrier is:
$\cos[(\omega_c + \Delta\omega)t + \delta]$ or $\sin[(\omega_c + \Delta\omega)t + \delta]$, we can see that $x_1(t)$ becomes:

$$x_1(t) = \varphi_{QAM}(t) \times 2\cos[(\omega_c + \Delta\omega)t + \delta]$$

where:

$$\varphi_{QAM}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t.$$

Hence,

$$\begin{aligned} x_1(t) &= 2m_1(t)\cos\omega_c t\cos[(\omega_c + \Delta\omega)t + \delta] \\ &\quad + 2m_2(t)\sin\omega_c t\cos[(\omega_c + \Delta\omega)t + \delta] \\ &= m_1(t)\{\cos[\Delta\omega t + \delta] + \cos[(2\omega_c + \Delta\omega)t + \delta]\} \\ &\quad + m_2(t)\{\sin[(2\omega_c + \Delta\omega)t + \delta] - \sin[\Delta\omega t + \delta]\} \end{aligned}$$

Similarly,

$$\begin{aligned} x_2(t) &= m_1(t)\{\sin[(2\omega_c + \Delta\omega)t + \delta] + \sin[\Delta\omega t + \delta]\} \\ &\quad + m_2(t)\{\cos[\Delta\omega t + \delta] - \cos[(2\omega_c + \Delta\omega)t + \delta]\} \end{aligned}$$

Low pass filtering these two signals, $x_1(t)$ and $x_2(t)$, the outputs of the low pass filters are:

$$m_1(t)\cos[(\Delta\omega)t + \delta] - m_2(t)\sin[(\Delta\omega)t + \delta] \quad \text{instead of } m_1(t)$$

and

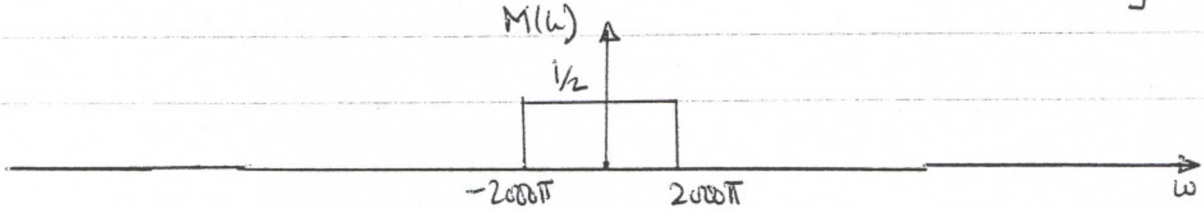
$$m_1(t)\sin[(\Delta\omega)t + \delta] + m_2(t)\cos[(\Delta\omega)t + \delta] \quad \text{instead of } m_2(t).$$

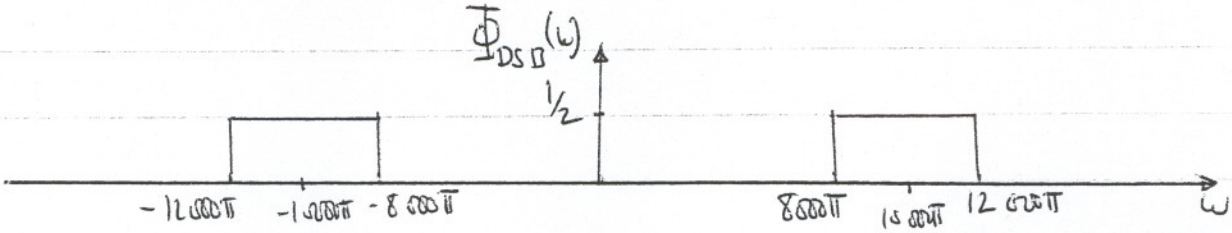
Problem # 4.5-3:

$$m(t) = B\,\text{sinc}\,[2\pi B t] \quad , \quad B = 1000 \text{ and } \omega_c = 10\,000\pi \text{ rad/s}$$

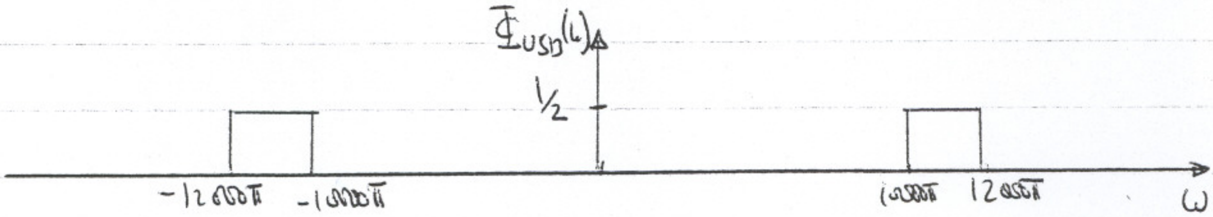
Remember that $\quad B \sin[2\pi B t] \longleftrightarrow \frac{1}{2}\,\text{rect}\left[\frac{\omega}{4\pi B}\right]$

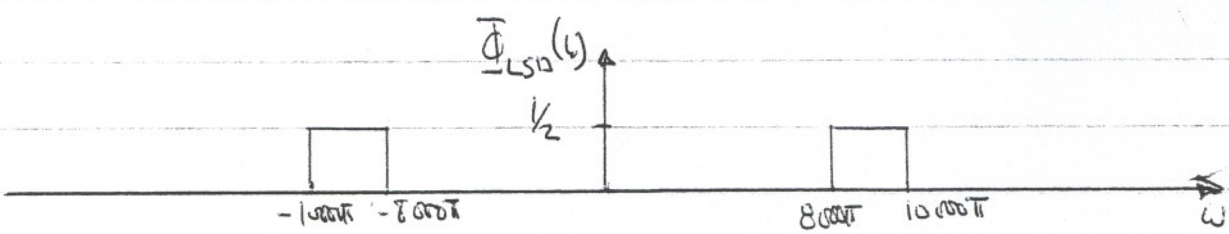
a)





b)





c) The inverse Fourier transform of the spectra shown in part b) are, respectively, given by:

$$\varphi_{USB}(t) = 1000\,\text{sinc}(1000\pi t)\cos(11{,}000\pi t)$$

$$\varphi_{LSB}(t) = 1000\,\text{sinc}(1000\pi t)\cos(9\,000\pi t)$$

Problem # 4.5-5:

$$\varphi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

The local carrier is given by: $\cos[(\omega_c + \Delta\omega)t + \delta]$.

The product of these signals is denoted $z(t)$ and is given by:

$$z(t) = 2\varphi_{LSB}(t) \times \cos[(\omega_c + \Delta\omega)t + \delta]$$

Low pass filtering $z(t)$ will result in $y(t)$, and after some mathematical manipulations, $y(t)$ is found to be:

$$y(t) = m(t)\cos[(\Delta\omega)t + \delta] - m_h(t)\sin[(\Delta\omega)t + \delta].$$

observe that if $\Delta\omega$ and δ are both zero, the output is

$$y(t) = m(t).$$

a) when $\delta = 0$, $y(t) = m(t)\cos[(\Delta\omega)t] - m_h(t)\sin[(\Delta\omega)t]$.

b) when $\Delta\omega = 0$, $y(t) = m(t)\cos\delta - m_h(t)\sin\delta$

let $y(t) \Leftrightarrow Y(\omega)$

Then $Y(\omega) = [M(\omega)\cos\delta - M_h(\omega)\sin\delta]$

Since $M_h(\omega) = -j\,\mathrm{sgn}(\omega)M(\omega) = \begin{cases} -jM(\omega) & \omega > 0 \\ jM(\omega) & \omega < 0 \end{cases}$

$$\therefore \quad Y(\omega) = M(\omega)\cos\delta + jM(\omega)\sin\delta$$
$$= M(\omega)e^{j\delta} \quad \text{for } \omega > 0$$

and
$$Y(\omega) = M(\omega)\cos\delta - jM(\omega)\sin\delta$$
$$= M(\omega)e^{-j\delta} \quad \text{for } \omega < 0.$$