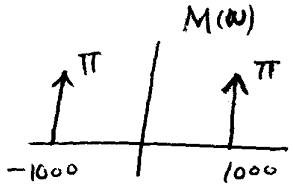
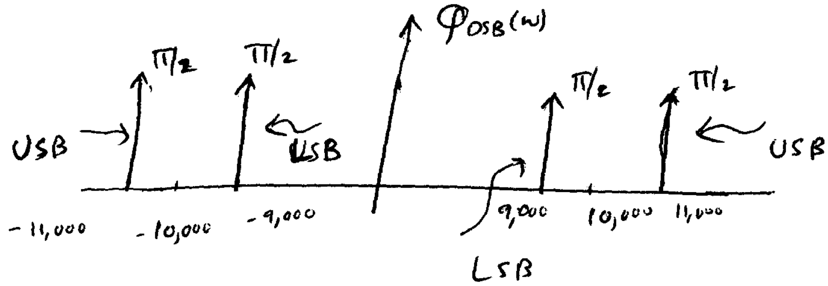
4.2-1

HW # 3

(i) $m(t) = \cos 1000t$





In general, to obtain the spectrum of $\varphi_{DSB}(t) = m(t) \cos 10{,}000t$, you have two options

(A) Time domain:

$$\varphi(t) = \cos 1000 \cos 10{,}000t$$

$$= \frac{1}{2}\left[\cos(10{,}1000 + 1{,}000)t + \cos(10{,}000 - 1{,}000)\right]$$

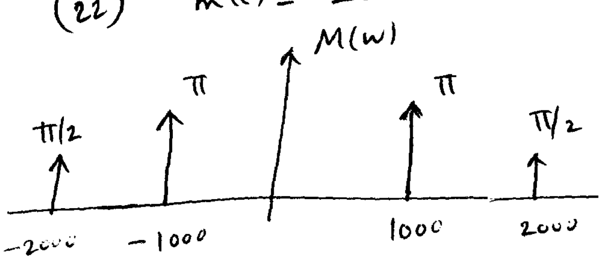
$$= \frac{1}{2} \cos(11{,}000t) + \frac{1}{2} \cos 9{,}000t$$

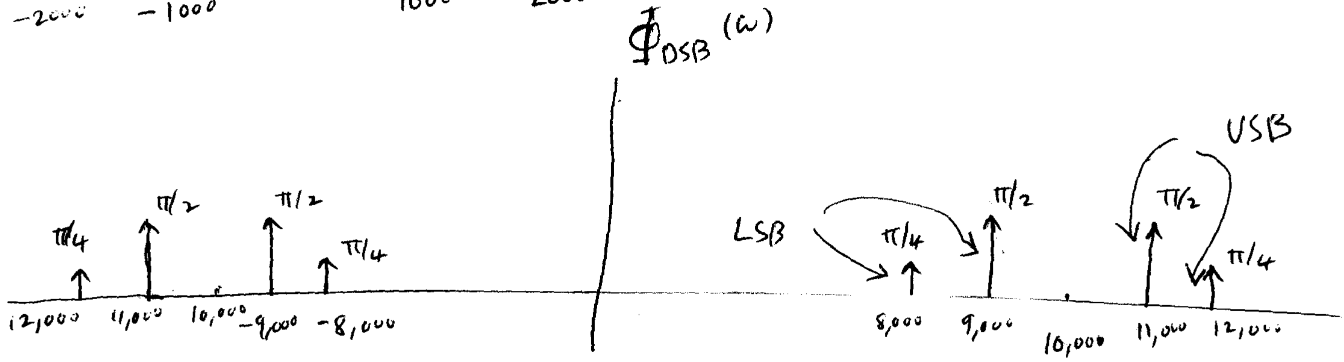
(B) Frequency domain

- Start with the spectrum of $m(t)$
- Shift it to the right by 10,000 & multiply amplitude by 1/2
- Shift it to the left by 10,000 & multiply amplitude by 1/2

Approach (B) is easier especially for case (iii).

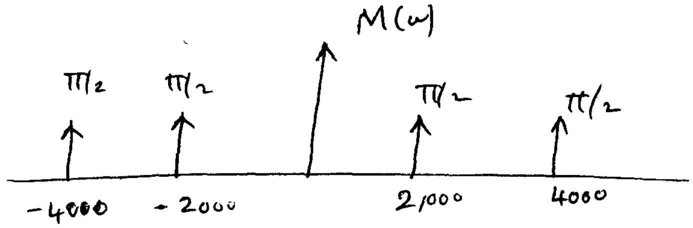
(ii) $m(t) = 2\cos 1000t + \cos 2000t$

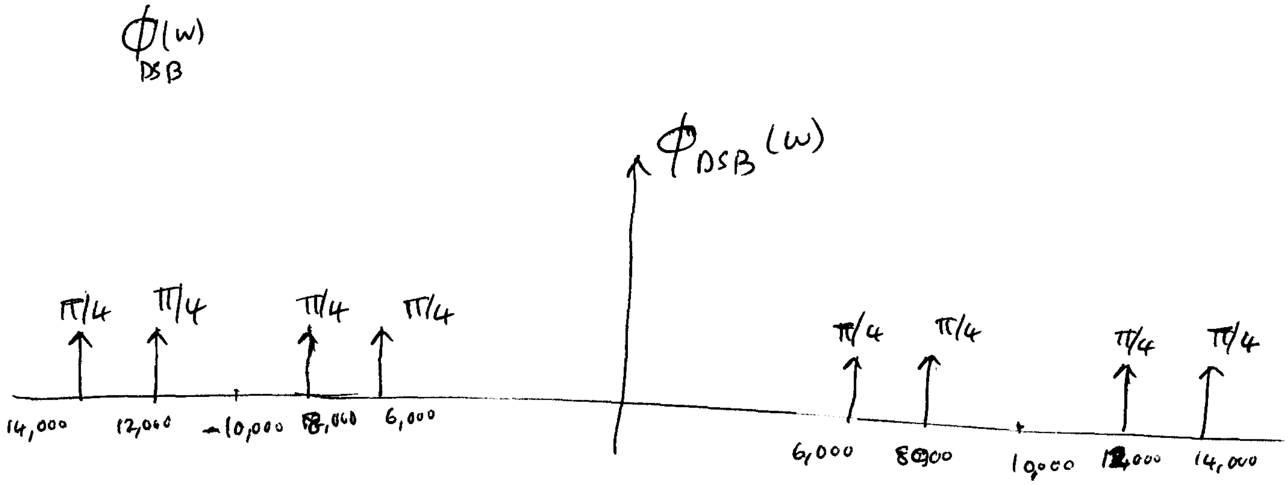




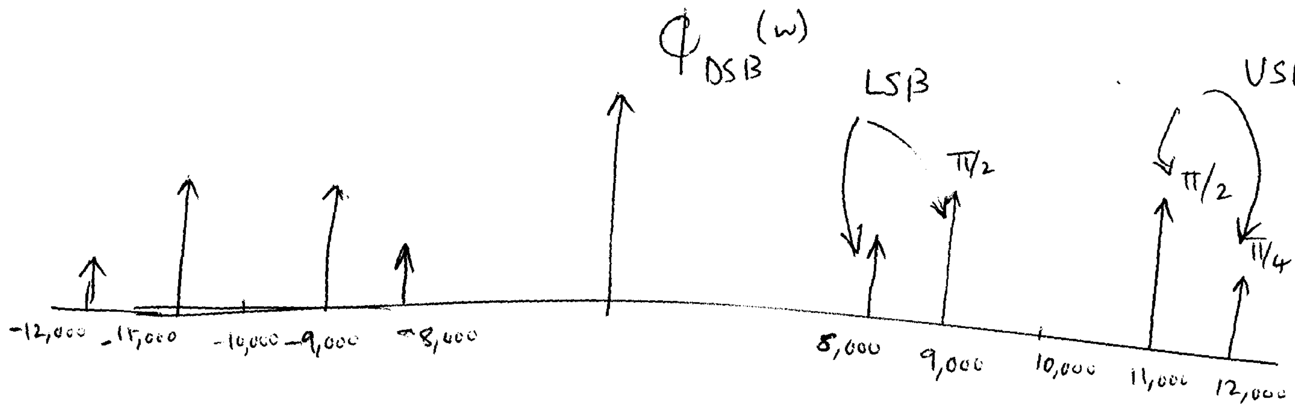
(iii) $m(t) = \cos 1000t \cos 3000t$

$= \frac{1}{2}\cos 2000t + \frac{1}{2}\cos 4000t$



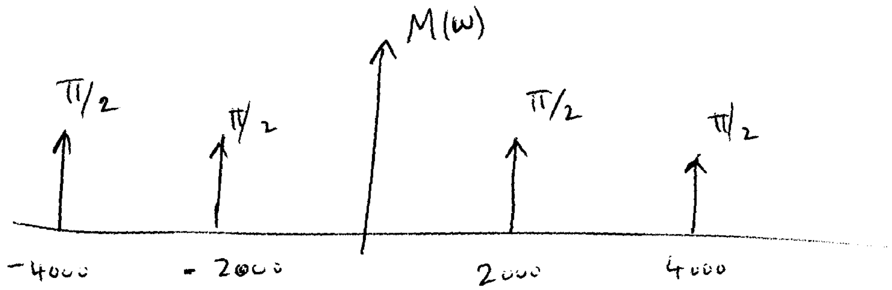


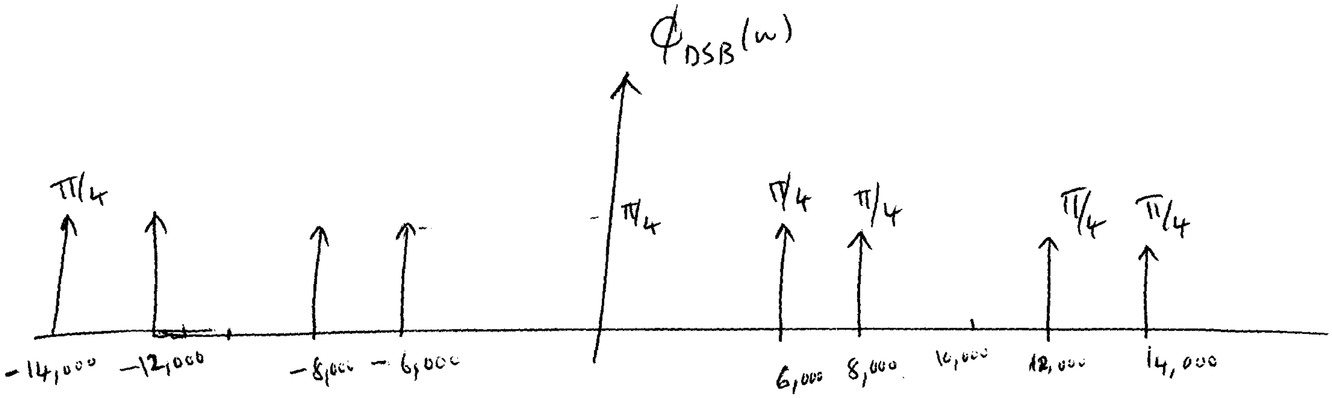
Obtaining the spectrum of $\Phi_{DSB}(\omega)$ is much easier using the Frequency domain method (B).



(iii) $m(t) = \cos 1000t \, \cos 3000t$

$$= \frac{1}{2}\cos 2000t + \frac{1}{2}\cos 4000t$$





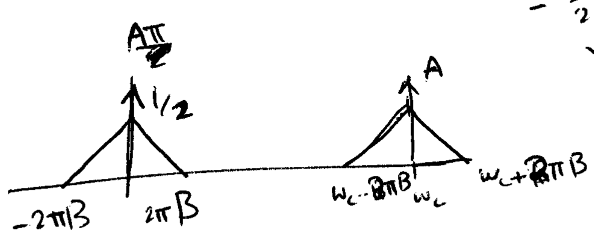
Obtaining the spectrum of $\phi_{DSB}(\omega)$ is much easier using the frequency domain method (B).

4-2-4(a)

Note that

$$(A+m(t))\cos\omega_c t \times \cos\omega_c t = \frac{1}{2}(A+mt)\left(1+\cos 2\omega_c t\right)$$

$$= \underbrace{\frac{1}{2}(A+m(t))}_{\text{Centered around "0" freq}} + \underbrace{\frac{1}{2}(A+m(t))\cos 2\omega_c t}_{\text{Centered around } 2\omega_c}$$



So if Low pass filter has BW $\gg$ BW $m(t) = B$ (but less than $\omega_c - 2\pi B$), then we get at the LPF output $\frac{1}{2}(A+m(t))$

The DC blocker allows us to get rid of the constant A and we get $\frac{1}{2}m(t)$ at the output.

The above operations and results are valid for all values of A. Compare this with the envelope detector which works only if $A \geq m_p$.

4-2-4

(a) First of all, note that

$$
\begin{aligned}
\cos^3 \theta &= \cos\theta\,(\cos^2\theta) \\
&= \cos\theta\left(\tfrac{1}{2} + \tfrac{1}{2}\cos 2\theta\right) \\
&= \tfrac{1}{2}\cos\theta + \tfrac{1}{2}\cos\theta\cos 2\theta \\
&= \tfrac{1}{2}\cos\theta + \tfrac{1}{4}\cos\theta + \tfrac{1}{4}\cos 3\theta \\
&= \tfrac{3}{4}\cos\theta + \tfrac{1}{4}\cos 3\theta
\end{aligned}
$$

So

$$
m(t)\cos^3\omega_c t = \underbrace{\frac{3}{4} m(t)\cos\omega_c t}_{\text{DSB SC at } \omega_c} + \underbrace{\frac{1}{4} m(t)\cos 3\omega_c t}_{\text{DSB SC at } 3\omega_c}
$$

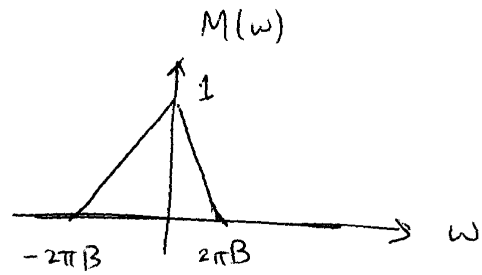
By choosing the filter to be a BPF with center freq. ω_c and bandwidth $2B_{m(t)}$, we would be able to obtain a DSB SC at ω_c.

4-2-1

b)

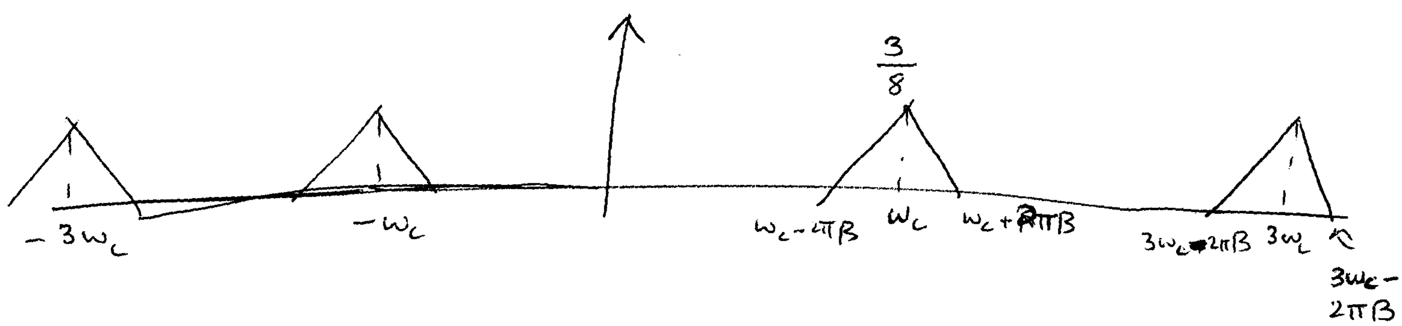
point ⓐ

m(t)

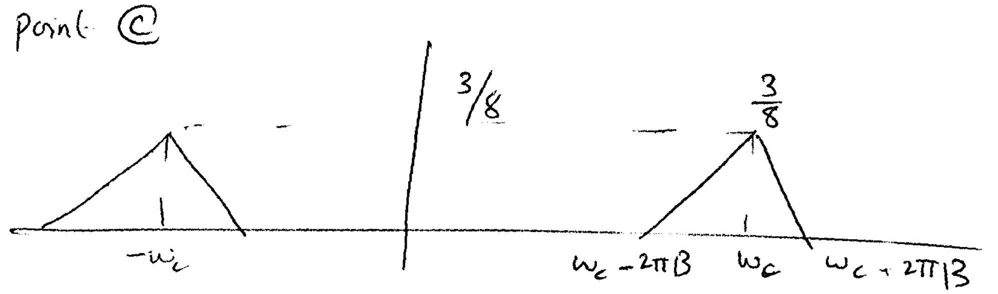


point ⓑ

m(t) cos³ ω_c t



point ⓒ



(c) For this to work, the various spectra should not overlap. This will happen provided

$$\left.\begin{aligned} \omega_c - 2\pi B &\geq 0 \\ 3\omega_c - 2\pi B &\geq \omega_c + 2\pi B \end{aligned}\right\} \Rightarrow \omega_c \geq 2\pi B$$

4-~~2~~-4

(d) The scheme would not work for $\cos^2 \omega_c t$.

$$m(t)\cos^2 \omega_c t = \frac{1}{2} m(t) + \frac{1}{2} m(t) \cos 2\omega_c t$$

As we can see, using $\cos^2 \omega_c t$, we generate a DSB SC signal at $2\omega_c$ (not at ω_c).

(e) It is easy to see that the scheme would work for $\cos^n \omega_c t$ provided n is odd (because the expansion of $\cos^n \omega_c t$ would contain the term $\cos \omega_c t$). If n is even, then the expansion of $\cos^n \omega_c t$ would not contain $\cos \omega_c t$ and so we can not generate the product $m(t) \cos \omega_c t$.

4.2-5

(a) Use the ring modulator with the carrier freq. $f_c = 100$ kHz $\Rightarrow \omega_c = 200\pi \times$

The output before filtering is given by

$$v_i(t) = m(t)\, w_0(t)$$

where w_0 is square wave train with frequency $f_c = 100$ kHz

$$= \frac{4}{\pi}\left[m(t)\cos\omega_c t - \frac{1}{3} m(t)\cos 3\omega_c t + \frac{1}{5} m(t)\cos 5\omega_c t + \cdots\right]$$

(see equation (4.7b))

The desired term is the one containing $m(t)\cos 3\omega_c t$.

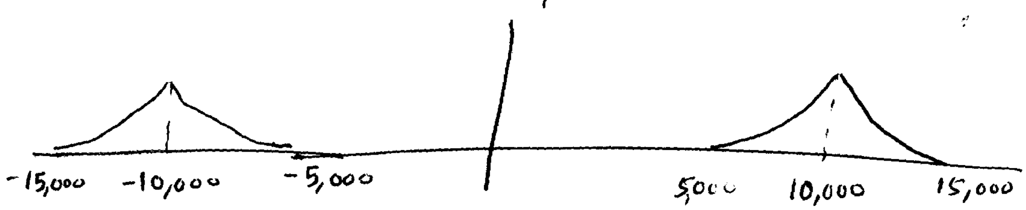
The bandpass filter (tuned to $3\omega_c$) will pass this signal & spress the remaining ones.

(b) The output of the modulator is $-\frac{4}{\pi}\frac{k}{3} m(t)\cos 3\omega_c t$,

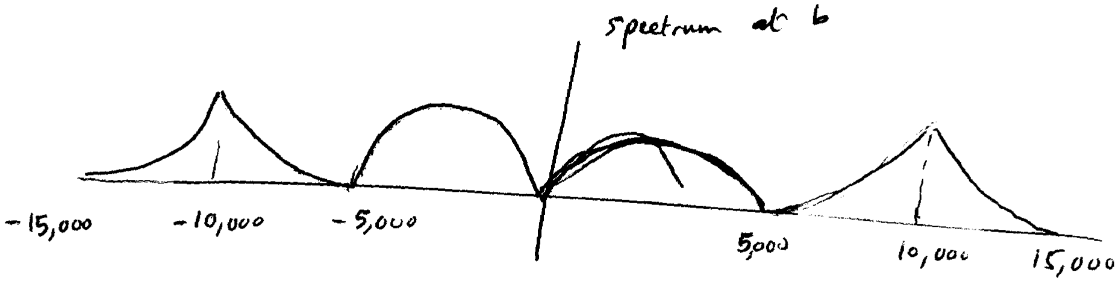
i.e. $\boxed{k = -\frac{4}{3\pi}}$

4.2-8

spectrum at a

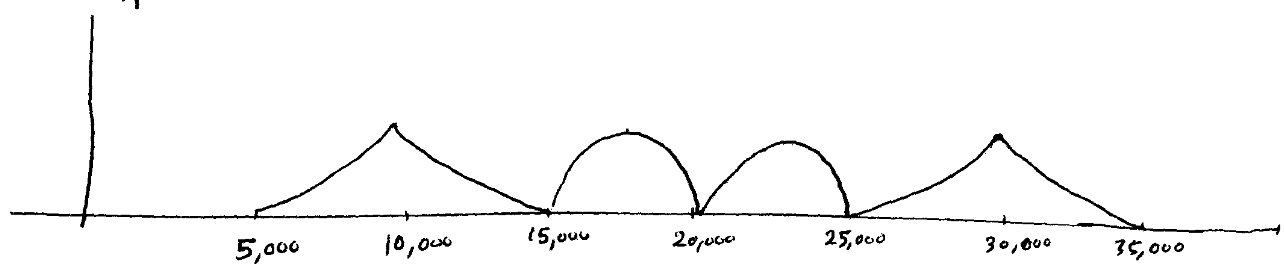


spectrum at b



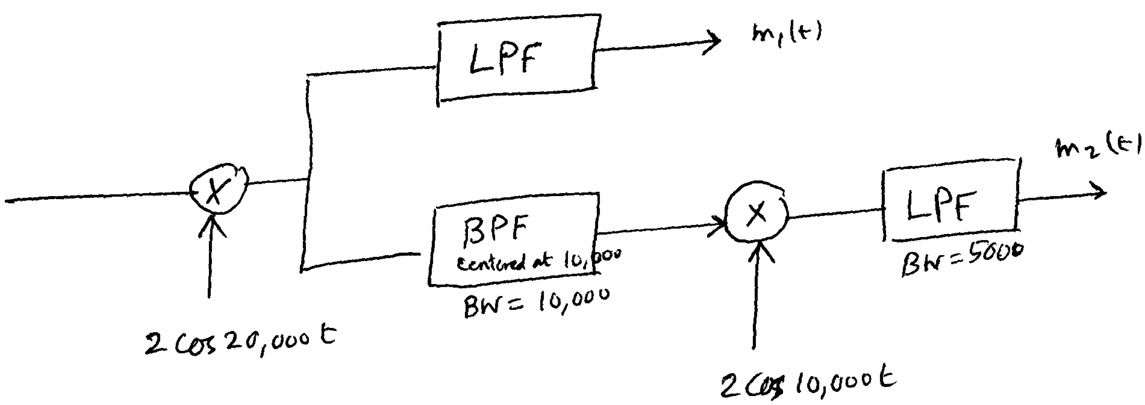
Spectrum at b is just the spectrum at a + $M_1(\omega)$

Spectrum at c : will show the positive part only. Negative freq. part is similar

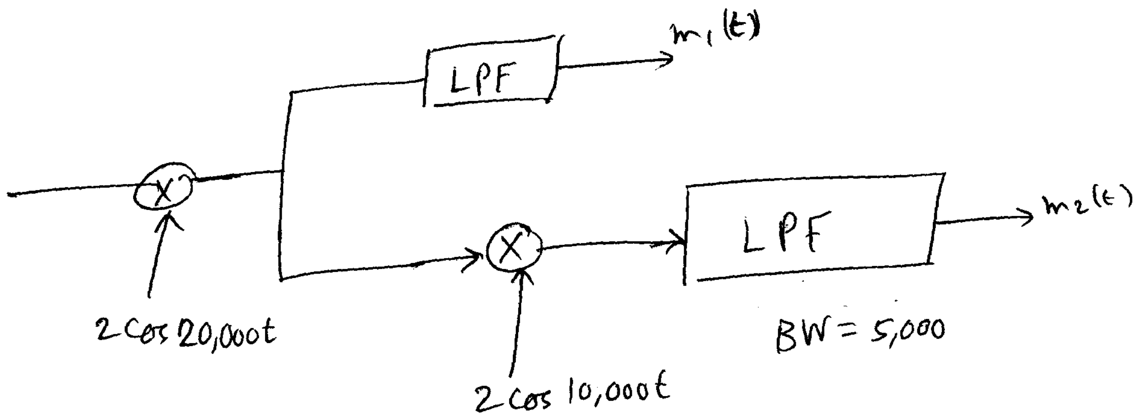


(b) From the spectrum at c figure, we can see that the channel BW should be 35,000 − 5,000 = 30,000 rad/sec.

(c) There are various ways to solve this problem. The simplest way is



A more economical way would be (i.e. using less components).



4-3-1

Note that

$$(A+m(t))\cos\omega_c t \times \cos\omega_c t =$$

$$(A+m(t))\left(\frac{1}{2}+\frac{1}{2}\cos 2\omega_c t\right) =$$

$$\underbrace{\frac{1}{2}(A+m(t))}_{\text{centered around "0" freq. with } BW = B_{m(t)}} + \underbrace{\frac{1}{2}(A+m(t))(\cos 2\omega_c t)}_{\text{centered around } 2\omega_c \text{ with bandwith } 2B_{m(t)}}$$

$\Rightarrow$ A low Pass filter will recover $\frac{1}{2}(A+m(t))$

The DC blocker will block the DC constant A.

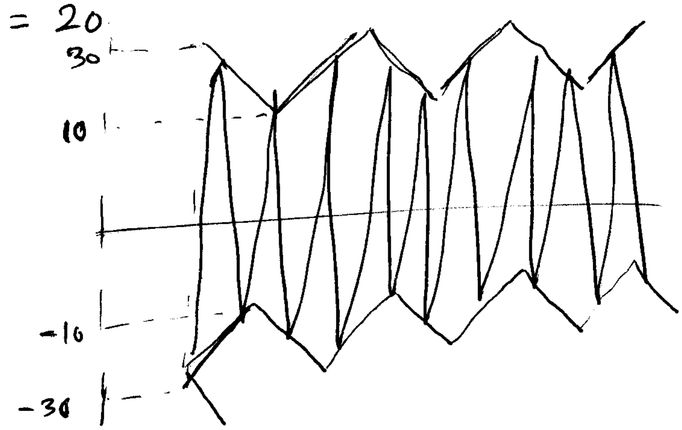
Note that the scheme works for any value of A. This is contrary to envelope detection which requires that $A \geq m_p$.

4.3-2

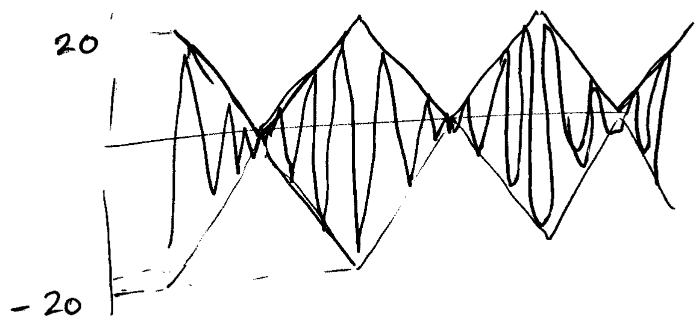
Note that $m_p = 10$.

$$\mu = \frac{m_p}{A} \Rightarrow \boxed{A = \frac{m_p}{\mu}}$$

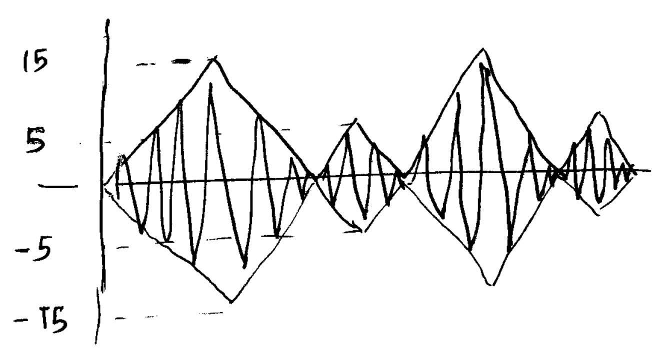
a) $\mu = 0.5 \Rightarrow A = \frac{10}{0.5} = 20$



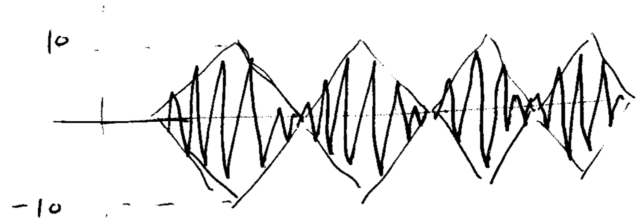
b) $\mu = 1.0 \Rightarrow A = \frac{10}{1.0} = 10$



c) $\mu = 2.0 \Rightarrow A = \frac{10}{2.0} = 5$



d) $\mu = \infty \Rightarrow A = \frac{10}{\infty} = 0$



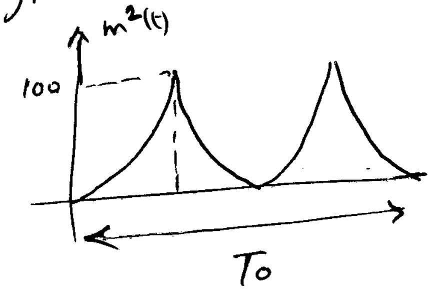
4.3-3

(a) $\mu = 0.8 \Rightarrow A = \frac{10}{0.8} = 12.8$

Carrier power $P_c = \frac{A^2}{2} = 78.125$

(b) Sideband power $P_s = \frac{1}{2} P_{m(t)}$

where $P_{m(t)}$ is the power of the signal $m(t)$. Since $m(t)$ is periodic with period $T_0 = 10^{-3}$, its power is given by



$$P_{m(t)} = \frac{1}{T_0} \int_0^{T_0} m^2(t)\, dt$$

$$= \frac{4}{T_0} \int_0^{T_0/4} \left(\frac{10t}{T_0/4}\right)^2 dt$$

$$= \frac{4}{T_0} \cdot \left(\frac{4}{T_0}\right)^2 \cdot \frac{100}{3}\ t^3 \Big|_0^{T_0/4}$$

$$= \frac{100}{3} = 33.3$$

$$\Rightarrow P_s = \frac{33.3}{2} = 16.67$$

Power efficiency $\eta = \frac{P_s}{P_c + P_s}$

$$= \frac{16.67}{78.125 + 16.67} \times 100 = 19.66\%$$