3.1-4 (b):

$$g(t) = e^{a(t-T)}, \quad 0 \le t \le T$$

$$G(w) = \int_0^T e^{a(t-T)} e^{-jwt} dt$$

$$= e^{-aT} \int_0^T e^{t(a-jw)} dt$$

$$= \frac{e^{-aT}}{a-jw} \left[e^{(a-jw)T} - 1 \right]$$

OR: By applying the Time-Shifting property:

Let $v(t) = e^{at}, \quad -T \le t \le 0$

$$\Rightarrow g(t) = v(t-T)$$

$$\Rightarrow G(w) = V(w) e^{-jwT}$$

$$V(w) = \int_{-T}^{0} e^{at} e^{-jwt} dt$$

$$= \frac{1}{a-jw} \left[1 - e^{-T(a-jw)} \right]$$

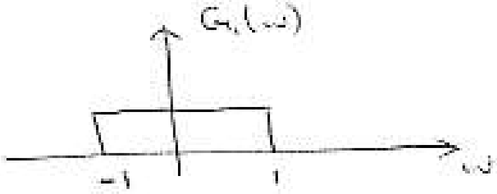
$$\Rightarrow G(w) = \frac{e^{-jwT}}{a-jw} \left[1 - e^{-T(a-jw)} \right]$$

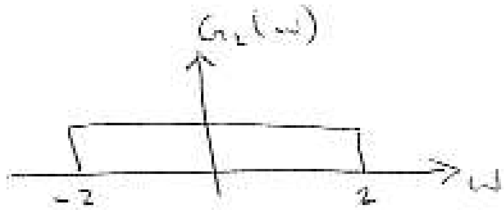
$$= \frac{e^{-aT}}{a-jw} \left[e^{(a-jw)T} - 1 \right]$$

Note that the two methods have the same answer!

3.1-6 (b)

The solution can be simplified by noting that $G(\omega) = G_1(\omega) + G_2(\omega)$, where $G_1(\omega)$ & $G_2(\omega)$ are shown below:





Therefore:

$$g(t) = \frac{1}{2\pi}\int_{-2}^{2}\left[G_1(\omega) + G_2(\omega)\right]e^{j\omega t}\,d\omega$$

$$= \frac{1}{2\pi}\left\{\int_{-2}^{2} e^{j\omega t}\,d\omega + \int_{-1}^{1} e^{j\omega t}\,d\omega\right\}$$

$$= \frac{\sin 2t + \sin t}{\pi t}$$

3.3-6(a) $g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t$ where $\Delta\left(\frac{t}{2\pi}\right)$ is triangle function defined in pg 79 of text book.

From table

$$\Delta\left(\frac{t}{2\pi}\right) \longleftrightarrow \pi\,\mathrm{sinc}^2\left(\frac{\pi\omega}{2}\right)$$ cont.

Using the modulation property:

$$g(t) = \Delta\left(\frac{t}{2\pi}\right)\cos 10t \longleftrightarrow \frac{\pi}{2}\left\{\mathrm{sinc}^2\left[\frac{\pi(\omega-10)}{2}\right] + \mathrm{sinc}^2\left[\frac{\pi(\omega+10)}{2}\right]\right\}$$

3.3-7(b):

$$G(\omega) = \Delta\left(\frac{\omega+4}{4}\right) + \Delta\left(\frac{\omega-4}{4}\right)$$

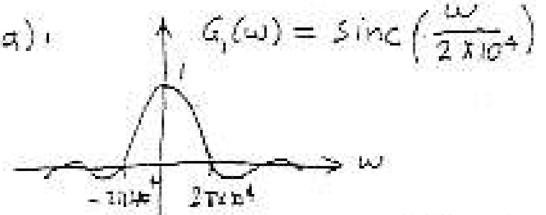
Also, we know that

$$\frac{1}{\pi}\,\text{sinc}^2(t) = \Delta\left(\frac{\omega}{4}\right)$$

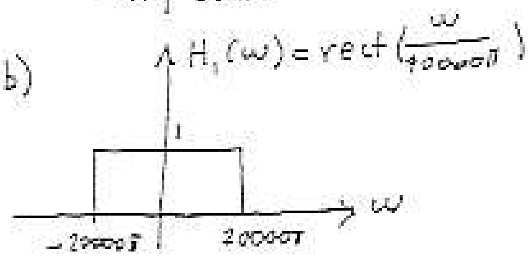
therefore,

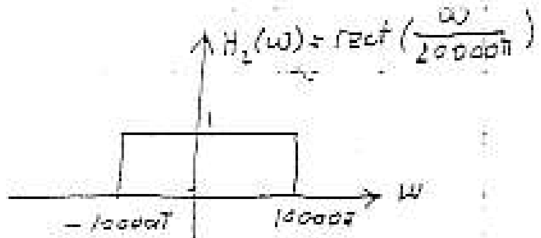
$$g(t) = \frac{2}{\pi}\,\text{sinc}^2(t)\cos 4t$$

3.4-1 a):

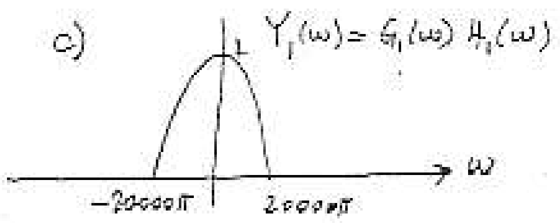


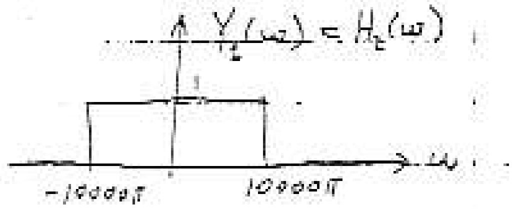
b)





c)





d) Bandwidth of $y_1(t)$ is 20000π rad/s $= 10$ KHz.

" " $y_2(t)$ is 10000π rad/s $= 5$ KHz

$y(t) = y_1(t)\,y_2(t) \longleftrightarrow Y(\omega) = Y_1(\omega) \circledast Y_2(\omega)$

From property of convolution, bandwidth of $y(t)$ is the sum of bandwidths Y_1 & Y_2 and it is 15 kHz.