2.1-4 $g(t) = C_1 \cos(w_1 t + \theta_1) + C_2 \cos(w_2 t + \theta_2)$
and $w_1 = w_2$.

The power of $g(t)$ is

$$P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} g^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left[C_1\cos(w_1 t+\theta_1) + C_2\cos(w_2 t+\theta_2)\right]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} C_1^2\cos^2(w_1 t+\theta_1) + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} C_2^2\cos^2(w_2 t+\theta_2)$$

$$+ \lim_{T\to\infty} \frac{2C_1C_2}{T}\int_{-T/2}^{T/2} \cos(w_1 t+\theta_1)\cos(w_2 t+\theta_2)\,dt$$

The first two terms are the power of cosines which is equal to $P_1 = \frac{C_1^2}{2}$ and $P_2 = \frac{C_2^2}{2}$ $\longrightarrow$ see Example 2.2(a in text Book

- Since $w_1 = w_2$, the last term is not equal to zero [unlike Example 2.2(b)]

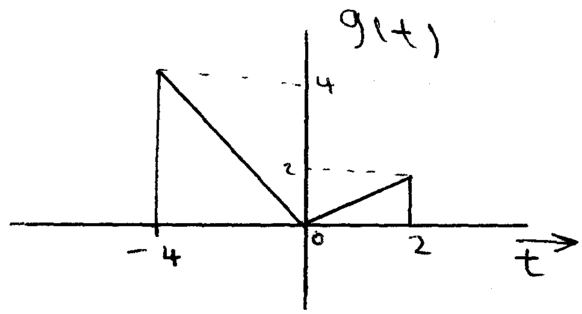
$$P_3 = \lim_{T\to\infty} \frac{2C_1C_2}{T}\int_{-T/2}^{T/2} \cos(w_1 t+\theta_1)\cos(w_1 t+\theta_2)\,dt$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T}\left[\int_{-T/2}^{T/2}\cos(\theta_2-\theta_1)\,dt + \int_{-T/2}^{T/2}\cos(2w_1 t+\theta_1+\theta_2)\,dt\right]$$

$$= \lim_{T\to\infty} \frac{C_1C_2}{T}\left[T\cos(\theta_1-\theta_2) + 0\right] = C_1C_2\cos(\theta_1-\theta_2)$$

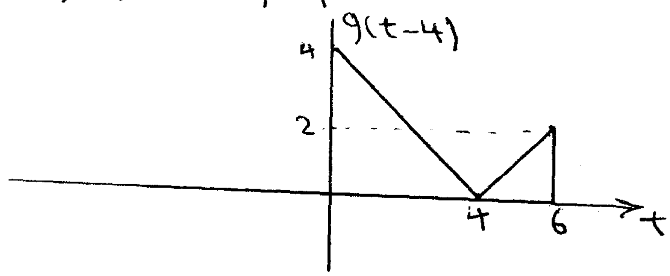
$$\therefore P_g = \frac{C_1^2}{2} + \frac{C_2^2}{2} + C_1C_2\cos(\theta_1-\theta_2)$$

2.3-3



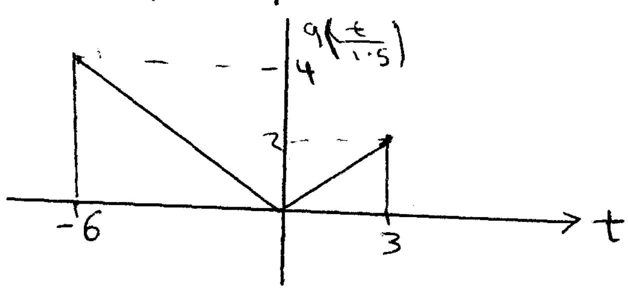
(a) $g(t-4)$

Delay $g(t)$ by 4



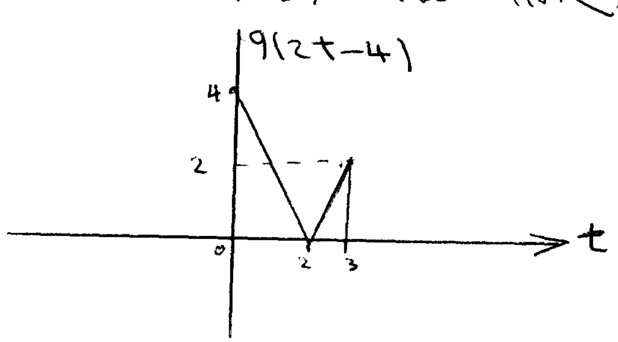
(b) $g\left(\frac{t}{1.5}\right)$

Expand $g(t)$ by 1.5



(c) $g(2t-4) = g[2(t-2)]$

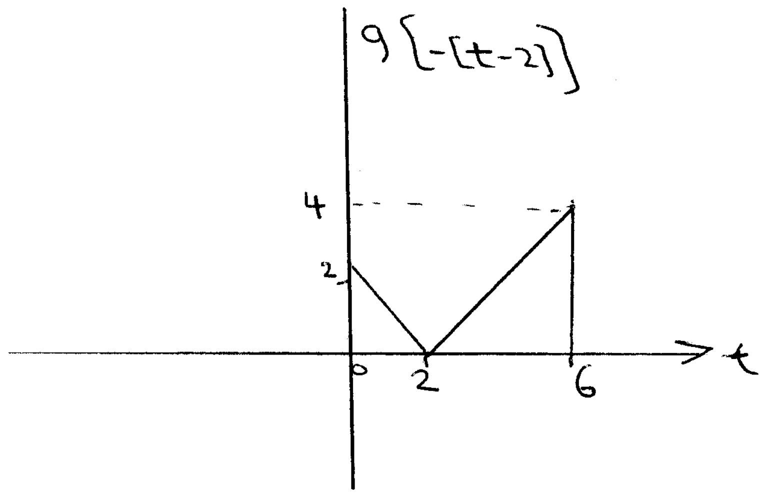
Compress by 2 and then delay by 2. Note that order is important.
Multiplication comes first, then addition.



2.3-3 Cont.

$\boxed{d}$ $g(2-t) = g\{-[t-2]\}$

Invert $g(t)$ then delay by 2.



$\boxed{2.4\text{-}1}$ Simplify.

Using the fact that $g(x)\delta(x) = g(0)\delta(x)$
we get

ⓐ $\left(\frac{\sin t}{t^2+2}\right)\delta(t) = \left(\frac{\sin 0}{0^2+2}\right)\delta(t)$

$= 0\,\delta(t) = 0$

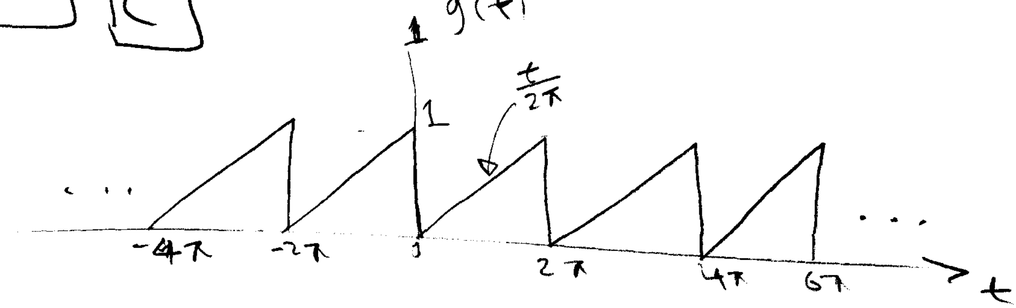
ⓔ $\left(\frac{1}{jw+2}\right)\delta(w+3)$

$= \frac{1}{-3j+2}\,\delta(w+3) = \frac{1}{2-3j}\,\delta(w+3)$

2.8-4

C

P.S / S



The period $T_0 = 2\pi \Rightarrow$ Fundmental Freq. $\omega_0 = \frac{2\pi}{T_0} = 1$ rad/s

$$g(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos nt + b_n \sin nt]$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{t}{2\pi}\, dt = 0.5 \rightarrow \text{DC component.}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi} \cos nt\, dt = 0 \rightarrow$$ because g(t) is odd function when we remove the dC component. Shift down by 0.5

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi} \sin nt\, dt = \frac{-1}{\pi n}$$

~~$\therefore g(t) =$~~ $\therefore g(t) = 0.5 - \frac{1}{\pi}\left(\sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \cdots\right)$

- in compact Form

$C_0 = a_0$ and $C_n = b_n$

The amplitude Spectrum is

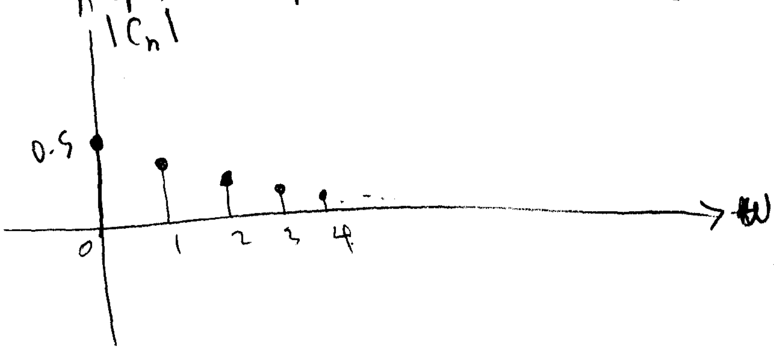
$|C_n| = |b_n| = \frac{1}{\pi n}$ for $n = 1, 2, \ldots$

$\therefore g(t) = 0.5 + \frac{1}{\pi}\left[\cos\left(t + \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2t + \frac{\pi}{2}\right) + \frac{1}{3}\cos\left(3t + \frac{\pi}{2}\right) + \cdots\right]$ in compact Form

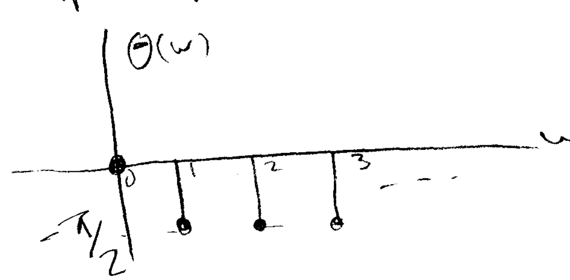
The phase Spectrum is

$$\theta_n = \tan^{-1}\frac{-b_n}{a_n} = -\tan^{-1}\frac{b_n}{0} = \left\{ -\frac{\pi}{2} \text{ for } n = 1, 2, \ldots \right.$$

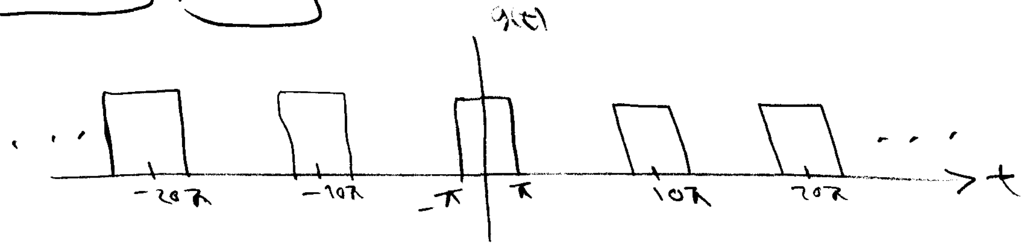
Amplitude spectrum



Phase spec



2.9-1 b



$T_0 = 10\pi$ and $\cancel{\omega_0 = \frac{\pi}{2}}$ $\omega_0 = \frac{1}{5}$ rad/s

$$g(t) = \sum_{n=-\infty}^{\infty} D_n e^{j\frac{n}{5}t}$$

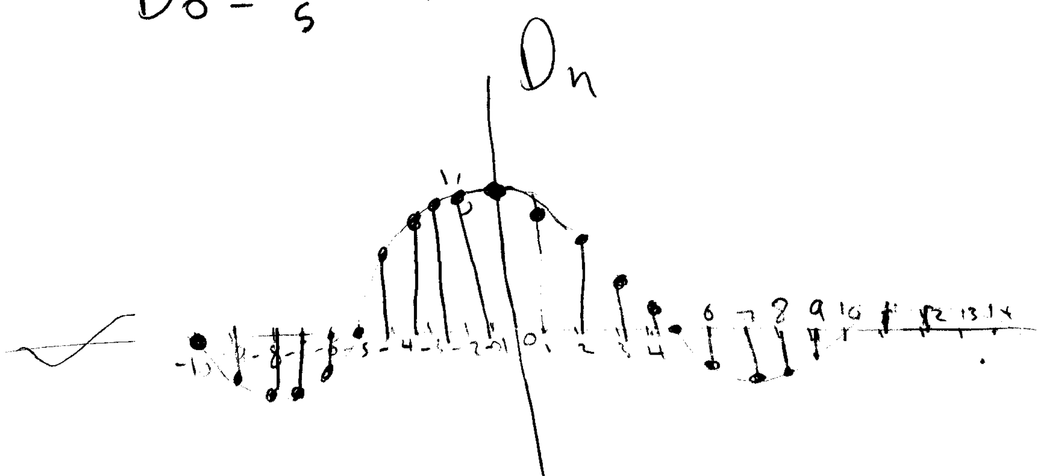
$$D_n = \frac{1}{10\pi}\int_{-\pi}^{\pi} e^{-j\frac{n}{5}t}\,dt$$

$$= \frac{1}{10\pi}\left[\frac{e^{-j\frac{n}{5}t}}{-j\frac{n}{5}}\right]_{-\pi}^{\pi}$$

$$= \frac{5}{10\pi n}\left[\frac{e^{-j\frac{n\pi}{5}} - e^{j\frac{n\pi}{5}}}{-j}\right]$$

$$D_n = \frac{1}{\pi n}\left[\frac{e^{jn\pi/5} - e^{-j\frac{n\pi}{5}}}{2j}\right] = \frac{1}{\pi n}\sin\left(\frac{n\pi}{5}\right)$$

$D_0 = \frac{1}{5}$ by L Hopitals Rule.

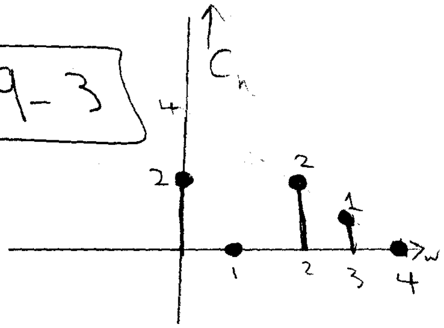


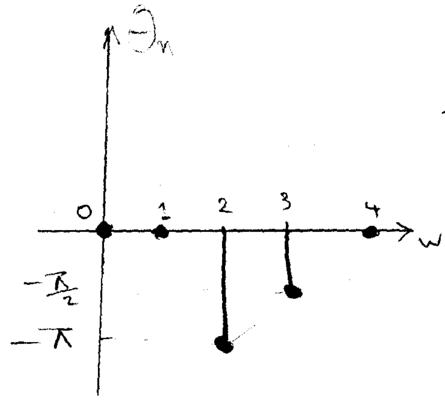
Note that $D_n = 0$ at $n = \pm 5, \pm 10, \pm 15, \ldots$

$D_n > 0$ (+ve) at $n = \pm 1, \pm 2, \pm 3, \pm 4, \pm 11, \pm 12, \ldots$

$D_n < 0$ (−ve) at $n = \pm 6, \pm 7, \pm 8, \pm 9, \pm 16, \pm 17, \ldots$

2.9-3





(a)

Recall that

$$|C_n| = \sqrt{a_n^2 + b_n^2}$$

$$C_0 = a_0$$

$$\theta_n = \tan^{-1}\left(-\frac{b_n}{a_n}\right)$$

$$\Rightarrow -\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

Thus, Similar to vectors,

$$a_0 = C_0$$

$$a_n = C_n \cos(-\theta_n) = C_n \cos(\theta_n)$$

$$b_n = C_n \sin(-\theta_n) = -C_n \sin\theta_n$$

Applying This, we found.

$$a_0 = 2$$

$$a_n = \begin{cases} 0 & , n=1 \\ 2\cos\pi = -2 & , n=2 \\ 1\cos\frac{\pi}{2} = 0 & , n=3 \\ 0 & , n=4 \end{cases}$$

$$b_n = \begin{cases} 0 & ; n=1 \\ -2\sin -\pi = 0 & , n=2 \\ -1\sin -\frac{\pi}{2} = 1 & , n=3 \\ 0 & , n=4 \end{cases}$$

Thus, the trigonometric Fourier Series is

$$g(t) = 2 - 2\cos 2\omega_0 t + \sin 3\omega_0 t$$

2.9-3 $\boxed{b}$

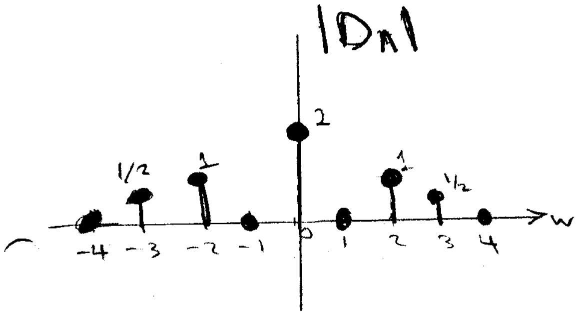
Recall that:

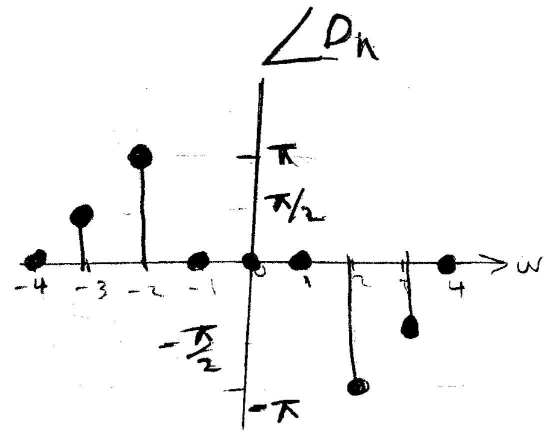
$|D_n| = |D_{-n}| = \frac{1}{2} C_n$

$D_0 = C_0$

$\angle D_n = \theta_n$ and $\angle D_{-n} = -\theta_n$

Thus,





$\boxed{c}$ $g(t) = D_0 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} D_n e^{j\omega_0 n t}$; recall that $D_n = |D_n| e^{j\theta_n}$

$= 2 + \frac{1}{2} e^{j\frac{\pi}{2}} e^{-j3\omega_0 t} + 1 e^{j\pi} e^{-j2\omega_0 t} + e^{-j\pi} e^{j2\omega_0 t} + \frac{1}{2} e^{-j\frac{\pi}{2}} e^{j3\omega_0 t}$

$= 2 + \frac{j}{2} e^{-j3\omega_0 t} - e^{-j2\omega_0 t} - e^{j2\omega_0 t} - \frac{j}{2} e^{j3\omega_0 t}$

$= 2 - 2\left[\frac{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{2}\right] + \left(\frac{e^{j3\omega_0 t} - e^{-j3\omega_0 t}}{2j}\right)$

$\therefore g(t) = 2 - 2\cos 2\omega_0 t + \sin 3\omega_0 t$

$\boxed{d}$ So, it is Equivalent to (a)