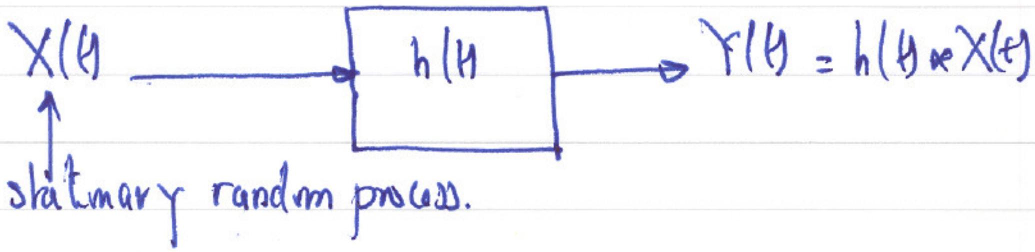
Solution to Homework # 8

1_ Problem 8.2_20 :



$$h(t) = 3t^2 e^{-8t} u(t)$$

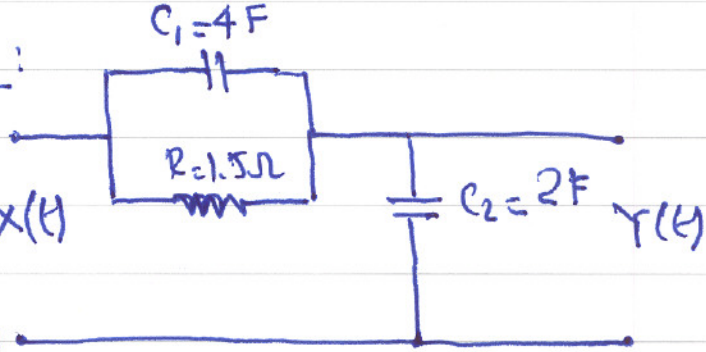
$$E[Y(t)] = E\left[\int_{-\infty}^{+\infty} h(\tau) X(t-\tau) d\tau\right]$$

$$= \mu_X \int_{-\infty}^{+\infty} h(t)\, dt \qquad , \mu_X = 2$$

$$= 2 \int_0^{+\infty} 3t^2 e^{-8t} dt$$

$$= \frac{3}{128}$$

2_ Problem 8.4_8 :



$$R_X(\tau) = 2 e^{-4|\tau|}$$

a) $S_X(\omega)$

$$R_x(\tau) \longleftrightarrow S_x(\omega) = \frac{16}{16+\omega^2}$$

b) $|H(\omega)|^2$

$$H(\omega) = \frac{1/j\omega C_2}{\frac{1}{j\omega C_2} + \frac{R(1/j\omega C_1)}{R + (1/j\omega C_1)}}$$

$$= \frac{1+j\,6\omega}{1+j\,9\omega}$$

$$|H(\omega)|^2 = \frac{1+36\omega^2}{1+81\omega^2}$$

c) $S_Y(\omega) = |H(\omega)|^2 S_x(\omega)$

$$= \frac{16(1+36\omega^2)}{(16+\omega^2)(1+81\omega^2)}$$

3_ Problem 8_4_9:

$$E[x(t)] = 5$$

$$S_x(\omega) = 50\pi\delta(\omega) + \frac{3}{\left[1+\left(\frac{\omega}{2}\right)^2\right]}$$

$$h(t) = 4e^{-4|t|}$$

a) $H(\omega)$ for the network

$$H(\omega) = \frac{32}{16+\omega^2}$$

b) The mean of $Y(t)$:

$$E[Y(t)] = E\left[\int_{-\infty}^{+\infty} h(\theta)\, X(t-\theta)\, d\theta\right]$$

$$= \mu_X \int_{-\infty}^{+\infty} h(t)\, dt$$

$$= 20 \int_{-\infty}^{+\infty} e^{-4|t|}\, dt$$

$$= 40 \int_{0}^{+\infty} e^{-4t}\, dt$$

$$= 10 .$$

c) $S_Y(\omega)$

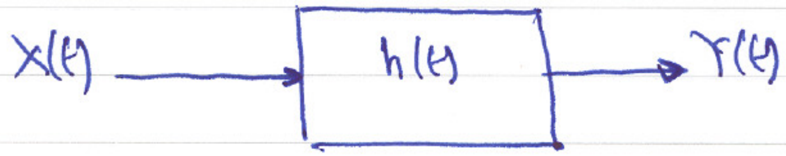
$$S_Y(\omega) = |H(\omega)|^2\, S_X(\omega)$$

$$|H(\omega)|^2 = \left[\frac{32}{16+\omega^2}\right]^2$$

$$S_Y(\omega) = 50\pi\delta(\omega)\, |H(0)|^2 + \frac{3}{\left[1+\left(\frac{\omega}{2}\right)^2\right]} \times \frac{(32)^2}{(16+\omega^2)^2}$$

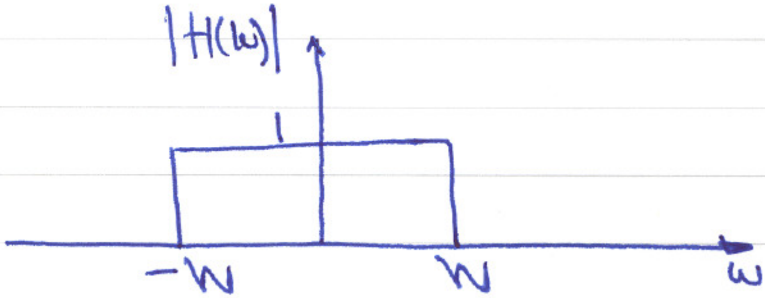
$$= 200\pi\delta(\omega) + \frac{12288}{(4+\omega^2)(16+\omega^2)^2}$$

4- Problem 8.4-11:



$$S_X(\omega) = \frac{N_0}{2}$$

$$= 6 \times 10^{-6} \text{ W/Hz}$$



$$S_Y(\omega) = |H(\omega)|^2 S_X(\omega)$$

$$= \frac{N_0}{2} \quad , \text{since } H(\omega) = 1, -W \leq \omega \leq W$$

$$= 6 \times 10^{-6} \text{ W/Hz}$$

$$P_Y = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_Y(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-W}^{W} 6 \times 10^{-6}\, d\omega$$

$$= \frac{6 \times 10^{-6} W}{\pi}$$

$$= 15 \quad \Rightarrow W = \frac{5\pi}{2} \times 10^{6} \text{ rad/sec}$$