Solution to Homework #7

1- Problem 7.1-12:

a- Valid
b- Not valid, since function is not an even function of w.
c- Not valid. since negative at $w=0$.
d- Not valid. Since complex.

2- Problem 7.1-16:

$$Y(t) = A_0 + B_0 X(t)$$

Since $R_X(\tau) \longleftrightarrow S_X(f)$, then:

$$R_Y(t, t+\tau) = E\left[(A_0 + B_0 X(t))(A_0 + B_0 X(t+\tau))\right]$$

$$= E\left[A_0^2 + A_0 B_0 X(t) + A_0 B_0 X(t+\tau) + B_0^2 X(t) X(t+\tau)\right]$$

$$= A_0^2 + 2A_0 B_0 \overline{X} + B_0^2 R_X(\tau)$$, since $X(t)$ is stationary

$$\therefore \quad S_Y(f) = \left[A_0^2 + 2A_0 B_0 \overline{X}\right]\delta(f) + B_0^2 S_X(f)$$

3- Problem 7.1-22:

$$R_X(\tau) = 3 + 2\exp(-4t^2)$$

a) $$S_X(f) = 3\delta(f) + \frac{1}{2\sqrt{\pi}} e^{-f^2/16}$$

b) The average power in $X(t)$ corresponds to $R_X(\tau) = R_X(0) = 5W$

4. Problem 7.2_3:

$$P_x = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_x(\omega)\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{6\omega^2}{(1+\omega^2)^3}\, d\omega$$

$$= \frac{6}{\pi} \left[-\frac{\omega}{4(1+\omega^2)^2} + \frac{\omega}{8(1+\omega^2)} + \frac{1}{8} \tan^{-1}(\omega) \right]_0^{+\infty}$$

$$= \frac{3}{8}$$

5. Problem 7.2_10:

$$R_x(\tau) = A e^{-\alpha|\tau|} \cos(\omega_0 \tau)$$

$$A e^{-\alpha|\tau|} \longleftrightarrow \frac{2\alpha A}{\alpha^2 + \omega^2}$$

Since if $f(t) \Leftrightarrow F(\omega)$, then

$$f(t) e^{\pm j\omega_0 t} \longleftrightarrow F(\omega \pm \omega_0)$$

$$R_x(\tau) = A e^{-\alpha|\tau|} \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right]$$

$$\therefore S_x(\omega) = \frac{\alpha A}{\alpha^2 + (\omega - \omega_0)^2} + \frac{\alpha A}{\alpha^2 + (\omega + \omega_0)^2}$$