Electrical Engineering Department
Midterm (062)
EE 662
Date: Monday April 16, 2007
Time: 2:00 PM -4:30 PM

Q1) *(15 Marks)*What is the "story" of the Adaptive Filtering course as you know it so far?

Q2) *(25 Marks)* Let $\boldsymbol{u}$ be a real Gaussian random row vector with zero mean and diagonal covariance matrix Λ.

a) Let $\boldsymbol{z}_a = \|\boldsymbol{u}\|^2$. Find the mean and variance of $\boldsymbol{z}_a$.

b) Now let $\boldsymbol{z}_b = \|\boldsymbol{u}\boldsymbol{U}\|^2$ where $\boldsymbol{U}$ is a unitary orthornormal matrix (i.e., $\boldsymbol{U}\boldsymbol{U}^* = \boldsymbol{U}^*\boldsymbol{U} = \boldsymbol{I}$). Find the mean and variance of $\boldsymbol{z}_b$.

c) (*Bounus: Do only if you have extra time*) Use a) and b) to find the mean and variance of $\boldsymbol{z}_c = \|\boldsymbol{u}\|^2$ when $\boldsymbol{u}$ has zero mean and a full (i.e. non-diagonal) autocorrelation matrix $\boldsymbol{R}$.

Q3) *(30 Marks)* A zero-mean stationary random process $\{\boldsymbol{u}(.)\}$ is generated by passing a zero-mean white sequence $\{\boldsymbol{v}(.)\}$ with variance σ_v^2 through a second order filter

$$\boldsymbol{u}(i) + \alpha\boldsymbol{u}(i-1) + \beta\boldsymbol{u}(i-2) = v(i) \quad i > -\infty$$

We would like to estimate $\boldsymbol{u}(i)$ using $\boldsymbol{u}(i-1)$ and $\boldsymbol{u}(i-2)$. To this end, define

$$\boldsymbol{u} = [\boldsymbol{u}(i-1) \;\; \boldsymbol{u}(i-2)] \;\text{ and }\; \boldsymbol{d} = \boldsymbol{u}(i)$$

a) Find the autocorrelation matrix $\boldsymbol{R}_u$ and the cross-correlation matrix $\boldsymbol{R}_{du}$

b) Prove that the weight vector that minimizes the mean-square estimation error is given by

$$\boldsymbol{w}^o = \begin{bmatrix} -\alpha \\ -\beta \end{bmatrix}$$

c) We would like to estimate $\boldsymbol{w}^o$ using the steepest decent algorithm. Write the corresponding recursion.

d) What is the condition on the step-size μ for the convergence of the steepest descent algorithm. Express this condition in terms of α and β.

Q4) *(30 Marks)* Consider the LMF algorithm

$$\boldsymbol{w}_i = \boldsymbol{w}_{i-1} + \mu\boldsymbol{u}_i^* e^3(i)$$

and assume that all the signals involved are real valued.

a) Starting from the energy relation, show that in the steady state, the adaptive filter satisfies

$$\mu E\left[\|\boldsymbol{u}_i\|^2 e^6(i)\right] = 2E\left[e_a(i)e^3(i)\right] \tag{1}$$

b) Assuming that $e_a(i)$ is Gaussian and $v(i)$ is Gaussian with variance σ_v^2, prove that $e(i)$ is Gaussian.

c) Assuming that $e_a(i)$ is Gaussian and $v(i)$ is Gaussian, evaluate the right-hand side of (1) (Remember that for a zero-mean Gaussian random variable x, $E[x^4] = 3\sigma_x^4$).

d) How would you evaluate the left-hand side of (1)? (For a zero-mean Gaussian random variable x, $E[x^6] = 15\sigma_x^6$).

e) (*Bounus: Do only if you have extra time*) Describe **briefly** how you would use the results of c) and d) to find the EMSE and the MSE.