King Fahd University of Petroleum and Minerals
Electrical Engineering Department

EE570 Stochastic Processes

Major Examination #2	Time allowed: 75 minutes	Date: January 10, 2008

Student Name:	Student Number:

Part I - Short Questions (15 Marks)

Question 1 **(1 Mark)**

The expected value of a real random variable refers to................, rather than the most probable single value,

a)average,

b)mean,

c)average of mean with probability distribution as weight,

d)none of the above,

e)all of the above.

Question 2 **(2 Mark)**

(i) Write the expression for the expectation of an arbitrary function of a continuous random variable, **x**.

(ii) Suppose the random variable in (i) is discrete, then rewrite the expression for the expectation of function of a discrete random variable.

Question 3 **(1 Marks)**

The concept of expectation can be extended to linearity of expectation and conditional density function. Complete the following equation by entering the missing terms:

$$f_y(y|m) = \sum_{?} \frac{f_{\boldsymbol{x}}(x_i|?)}{g'(?)} = \qquad (1)$$

Question 4 **(1 Mark)**

The expectation $\mathcal{E}[\boldsymbol{x}]$ does not provide enough information about a random variable. What additional information about the random variable is usually needed to describe the random variable more precisely?

Question 5 **(2 Mark)**

The mean and variance of a random variable can be used to bound the probability of occurrence of an event. Identify one of such bounds and write an expression for the bound.

Question 6 **(1 Mark)**

What is the mathematical difference between the characteristic function and the Fourier transform of a function?

Question 7 **(2 Marks)**

Enlist at least three properties of joint distribution function.

a)

b)

c)

d)

e)

Question 8 **(1 Mark)**

Two jointly random variables my be independent if and only a certain parameter is equal to zero. Identify the parameter?

Question 9 **(2 Marks)**

Define (k+r)th order joint moments of random variables **x** and **y**.

Question 10 **(2 Marks)**

Suppose **x** and **y** are two random variables with joint density function $f_{xy}(x, y)$ and $g(u, v)$ and $h(u,v)$ are used to define two random variables **z** and **w** as **z** = g(**x**, **y**) and **w** = h(**x**, **y**). Let the solution to the simultaneous equations in (**x**,**y**) for **z** and **w** (defined above) be the countable set

$$\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n), \ldots\} \tag{2}$$

then $f_{zw}(z, w)$ is obtained as

$$f_{zw}(z, w) = \frac{f_{xy}(x_1, y_1)}{?} + \frac{?}{|J(x_2, y_2)|} + \ldots + \frac{f_{xy}(x_n, y_n)}{?} + \ldots \tag{3}$$

Complete the above equation.

Part II Long Questions (35 Marks)

Question 1 **(12 Marks)**

A random variable describing the radius, **x**, has *pdf*

$$f_x(x) = \begin{cases} 6x(1-x) & 0 \le x \le 1 \\ 0 & \text{eleswhere} \end{cases} \tag{4}$$

(a) Find the pdf of the area covered by a disc with radius **x**.

(b) Find the pdf of the volume of a sphere with radius **x**.

Question 2 **(10 Marks)**

Let **x** and **y** denote the amplitude of noise signal at two antennas. The random vector (**x**,**y**) has the joint pdf

$$f_{\mathbf{x},\mathbf{y}}(x, y) = axe^{-ax^2/2} bye^{-ay^2/2} \qquad x > 0, y > 0, a > 0, b > 0 \tag{5}$$

(a) Find the joint *CDF*.

(b) Find the marginal *pdf*'s.

Question 3 **(13 Marks)**

The random variables **x** and **y** have joint *pdf*

$$f_{xy}(x, y) = C\sin(x + y) \qquad 0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2} \tag{6}$$

(a) Find the value of constant C.

(b) Find the joint *CDF* of **x** and **y**.

(c) Find the marginal *pdf*s of **x** and **y**.

(d) Find the mean, variance, and co-variance of **x** and **y**.